Economics 610 – Quiz #2 (March 30, 2001)

- Closed book quiz. You have 50 minutes. Justify what you claim.
- No anti-Bellman conspiracies allowed.
- Anyone writing the quiz on March 29 must hand in this sheet to Katya, and not hint to any other Michigan students of its contents.
- Problem 1 counts also half towards the assignment 1 grade, even if this quiz does not count towards your final grade.
- Have fun: After all, if the quiz hurts your grade, it doesn't count. Besides, you'll do better without getting stressed out.

1. Assume that a grad student is forever trapped in the PhD program, and is either unmatched and searching, or matched and not searching. While unmatched, she enjoys a flow payoff of $b \in (0,1)$ per unit time (her singleness contentment level). Matches — which all last forever — yield a flow payoff of 1 per unit time. (So all partners are created equal.) Assume that she must engage in costly search for matches, visiting bars and rip-roaring Econ coffee hours, commonly known as pick-up places. At a flow cost ρ^2, one can secure a Poisson arrival rate $\rho > 0$ of matches per unit time. Let the individual strive to maximize her expected present value of payoffs, discounted at the interest rate $r > 0$.

   (a) Analytically describe the student's behaviour. [7]

   (b) Assume an interior solution. Does the optimal level of search intensity depend on the singleness contentment level b? [3]

   (c) Bonus: For which $b \in (0,1)$ does the student search? [5]

2. These analytical exercises concern the standard unit variance Wiener process $\langle W_t \rangle$.

   (a) What is the expected value of W_t^2 for $t > s > 0$ *given* that $W_s = x$? [3]

   (b) Evaluate $\int_0^t (W_s)^n dW_s$. (Full credit requires an answer in 'final' form, able to be entered into a computer, given the Wiener path realization on $[0,t]$.) [7]

3. Consider the following discrete time model of industry evolution. Each period an industry either gains three firms or loses one firm. Each event has chance 1/2. (Ignore the problems of a negative number of firms, by assuming a large enough number of initial firms.)

   (a) Justify a diffusion approximation to this discrete time stochastic process, by a suitable rescaling of its domain and range. Be precise. [10]

   (b) Then, crucially, give a useful approximation of the chance that the number of firms at time $k = 1000$ is between 2000 and 3000 more than at time 0. [5]