Fall 2000 Lones Smith

Decision Theory and Game Theory 602 — Final Exam

There are 125 points in three hours. Points are indicated in the margins, and 48 underlined points are easier to get. (This would be enough to pass). Aim for *quality answers* rather than quantity answers, *if you think you're guessing.* Prove that you understand what's going on, but be brief. Mathematics alone, if wrong, counts for little, as we cannot read your mind.

"What is a magician but a practising theorist?" – Obi-Wan Kenobi

1. Consider the standard graphical probability simplex over prizes \$1, \$2, \$4.

   (a) Precisely illustrate the lottery that gives \$1 and \$2 with weight 1/4, and \$4 with weight 1/2 (i.e. show how it is found geometrically). [2]

   (b) Carefully and precisely illustrate the indifference curves corresponding to: "lottery p is preferred to q iff p has a higher mean than q." [3]

2. Consider the Battle of the Sexes where both players have the same strictly increasing vNM utility function u.

	Football	Opera
Football	u(3),u(1)	u(0),u(0)
Opera	u(0),u(0)	u(1),u(3)

   (a) Compute the mixed strategy equilibria with $u(x) = ax + b$, for $a > 0$. [3]

   (b) Now assume strict risk aversion. Determine whether the mixing chance on one's preferred action in this equilibrium rises or falls. [Hint: Use a ranking theorem.] [12]

3. Give two extensive form representations of this Prisoners' Dilemma. [5]

	C	D
C	3,3	0,4
D	4,0	1,1

4. Colonels Blotto and Otto are getting ready to do battle. Blotto will attack Otto at one of *two possible battle sites S and T*. Blotto has two indivisible troop regiments, and Otto has just one. The colonels play a *zero-sum game.* If Otto has $k \in \{0, 1\}$ regiments at a site, and Blotto has more, then Blotto receives a payoff of $k + 1$. If Blotto has no regiments at a site, and Otto has one, then Blotto receives a payoff of -1. If Blotto and Otto have the same number of regiments at a site, both receive a payoff of 0. Both are proud soldiers – retreat is never an option for them; they must use all their regiments.

   (a) Represent this game as a normal form. [2]

   (b) Find **all** Nash equilibria. (Can Blotto play a pure strategy?) [8]

5. Consider the following two player game of incomplete information played by friends A and B on the telephone who live twenty miles away from each other. The radio correctly predicts that it will rain on at least one house. It will either rain on both houses (chance 1/3), on just A's house (chance 1/3), or just B's house (chance 1/3). Each player observes the weather outside his door (rainy or dry), but has no other information about his neighbour's weather. They then play the game of chicken below.

	Chicken	Tough
Chicken	5,5	2,6
Tough	6,2	1,1

(a) Prove: It is a Bayes-Nash equilibrium for each player to play chicken when it rains, and play tough otherwise. [13]

(b) Show that this expected payoff has a greater payoff sum than the best Nash payoff. [2]

6. The payoffs in the Battle of Sexes are $(0, 0)$ if husband and wife choose different activities (football and opera), while if they agree on an activity, then the (husband, wife) payoffs are $(3, 1)$ for football, and $(1, 3)$ for opera. Assume that the husband spies on the wife's decision, *discovering her chosen action before he makes his own action choice,* and that this fact is common knowledge among husband and wife.

Represent this game in its correct extensive form, and find all subgame perfect equilibria of this game. (P.S. What's the lesson here?) [10]

7. The Prisoner's Dilemma of question 3 is played repeatedly by different players, labeled $\{1, 2, 3, \ldots\}$. In period $k = 1, 2, 3 \ldots$, the game below is played by players k and $k+1$. (The game is symmetric so it doesn't matter who are the row and column players.) The total payoff to player k is his period $k-1$ payoff plus *delta* times his payoff in period k.

(a) Argue that in any subgame perfect equilibrium, player k must play D in period k. [3]

(b) Find the least $\delta \in (0, 1]$ such that it is a subgame perfect equilibrium for player $k = 2, 3, \ldots$ to play C in period $k-1$. [12]

8. Consider a two period bargaining game. In period 1, A makes an offer $(x, 1-x)$, where $0 \leq x \leq 1$, and B says 'yes' or 'no'. If 'yes', the payoffs are given by A's offer. If 'no', then we go to period 2, where A and B make simultaneous demands on the pie. If A says $(x, 1-x)$ and B says $(1-y, y)$, then the payoff vector is (x, y) if $x + y \leq 1$, and otherwise, it is $(0, 0)$. Payoffs are not discounted between periods 1 and 2.

(a) Represent this game as an extensive form. [2]

(b) Find all subgame perfect equilibria. [8]

9. Consider a two-player, zero-sum game in which player 1 consists of two people, Peter and Katya. Player two is Lones. Two cards, one king and one deuce, are dealt at random to Peter and Lones (with equal chance). The person with the king receives \$1 from the deuce-holder, and then may stop or continue. If the play continues, Katya barges in and takes control, *not knowing the outcome of the original deal.* (As usual, Peter and Katya do not talk.) She tells Peter and Lones either to exchange cards, or keep them. Again, the holder of the king receives \$1 from the holder of the deuce, and the game ends.

    (a) Represent this game as an extensive form. [5]

    (b) Represent this game as an normal form, and find the Nash equilibrium. [10]

10. This question is about venture capital funding. A risk neutral entrepreneur E knows the value V of a potential project. Specifically, E learns either $V = L$ or $V = H$, with $H > L$. The chance $p \in (0, 1)$ of $V = H$ is common knowledge. The project needs a seed money investment of $I > 0$ dollars. A risk-neutral venture capitalist C has the I dollars, and wishes to invest wisely. After learning his type, E's action is the equity stake $e \in [0, 1]$ to give to C. Then C either declines to fund the project, and earns a return i on his money, or accepts the project. Thus the payoffs to (E, C) are $((1 - e)V, eV - I)$ if C accepts, and if the project pays out V. If C declines, the payoffs are $(0, (1 + i)I)$.

    (a) Carefully depict this as an extensive form. [2]

    (b) Suppose that when C moves, his posterior on $V = H$ is q. What is the condition on q required for him to accept offer e? [3]

    (c) Assume $L - I > (1 + i)I$. Find *all* sequential equilibria. [15]
    Hint: Use a weaker equilibrium concept to help eliminate candidate equilibria.