University of Michigan PhD Microeconomics Prelim
Fall 2000

The exam consists of nineteen (possibly compound) assertions that are true or false. Begin each answer by asserting '**true**', '**false**'. This alone is completely worthless unless you then briefly and correctly **justify** your answer. Don't get tricked. Be rigorous! Whenever you can give a brief *analytic* argument or counterexample or cite a named theorem, you must do so. Graphical arguments are better than purely verbal ones, but you still must explain your analytics. Focus your efforts; guesses or loose handwaving have expected payoff ≈ 0. Show us you understand what is going on.

There are 150 points (a point every 2 minutes = perfect score).

Consumer Theory (22 points)

1. Michael Milken consumes no strict luxuries. Then Michael cannot consume any good with income elasticity equal to 0.7. [4 pts]

2. If the CPI rises 3%, and the representative consumer earns 2% more income, then *the cost* of his luxury purchases rises more than 3%. [4 pts]

3. Lones and Ennio each love fruits, and have constant preferences each week. One week bananas and apples are each 50 cents a pound, Lones buys four pounds of apples and two pounds of bananas, while Ennio, being more fond of juicy fruit, buys three pounds of each. The next week bananas are on special at 25 cents, while apples now cost 75 cents, and both just go bananas over bananas: Each buys nine pounds of bananas, and only one pound of apples. Then the welfare of each has been strictly improved by the price change. [7 points]

4. Rockafellar (or Rocky, as his friends call him) has *strictly convex* preferences over leisure and one other composite consumption good. Rocky has a flex-hours job, with a constant hourly wage rate, for however many hours he works. If Rocky faces a *progressive* income tax (i.e. the tax rate he pays is absolutely higher at greater incomes), then Rocky will never be indifferent about two levels of leisure; however, with a regressive income tax, he might well be. [7 points]

Producer Theory (20 points)

5. Dmitry's Hairdo's is both a monopolistic seller of his chic new ex-post-gulag Russian hair-styling and a monopsonistic buyer of Russian-speaking hair stylists in Ann Arbor. It follows that compared to the efficient outcome, Dmitry hires too little labour, and pays too high a wage. [6 points]

6. The city of Ann Arbor sells its fleet of buses to a private firm that runs the business as a profit-maximizing monopoly. The implied capital cost to the firm per hour of

owning a bus is \$50. Each bus picks up 50 passengers per hour. The marginal cost — in addition to the capital cost — per pick-up equals \$2. The demand for buses varies depending on the time of day. Half the hours are $peak$, at which the demand price equals $P_H = 4 - Q_H/1000$. The inverse demand at $off\text{-}peak$ equals $P_L = 3 - Q_L/1000$. Then the firm charges \$1 more at peak than off-peak hours, and purchases ten buses. [14 pts]

Partial Equilibrium (21 points)

7. Assume that firm $k = 1, 2, \ldots$ in a competitive industry has cost $k^2 + q + q^2$ of output level q. (Here, k^2 is an escapable cost in the long run.) Then the long run industry "saw tooth" supply curve has positive output only for prices at least 3. [6 pts]

8. There are two expressways but countless backroads from Eastern Michigan into Detroit. The commute by backroads always takes 120 minutes (no traffic congestion). With M motorists on the expressways, commuting takes $40 + .01M$ minutes. Every weekday morning 10,000 commuters make the trek using either the expressways or the backroads. Then twice as many motorists use the expressways as is efficient (i.e. minimizes the sum of all commute times). [9 pts]

9. In question #8, suppose the government charges a toll for using the expressway. Suppose every motorist is willing to pay exactly \$1 for every 20 minutes saved on the commute. Assume the toll is set to minimize the sum of commute times; also, toll revenue is burned. Then commuters strictly prefer this situation to the decentralized one with no toll imposed. [6 pts]

Decision Theory (10 points)

10. Assume that u is Leia's increasing and continuous von Neumann Morgenstern utility function. Assume that Luke's utility of any lottery with prizes x_i having chances p_i is $\sum_i p_i u(x_i) + [\sum_i p_i u(x_i)]^2$. Then Luke violates the expected utility hypothesis. [4 pts]

11. Assume the GEA head spends his Thursday nights sequentially searching for a keg of beer (supposedly for the Friday afternoon meeting). [Strangely, he always forgets the previous best price.] Each trip costs \$4 in gas and hassle and wear-and-tear on his snazzy new BMW. Prices for kegs are uniformly distributed between \$1 and \$9. Then the GEA head buys at the first store he visits. [6 pts]

Game Theory (35 points)

12. Two individuals A, B are bargaining over a pie of size 1. Each has the discount factor $\delta = 1/2$. The timing is modified so that A offers in periods $1, 4, 7, \ldots$; B offers in periods $2, 5, 8, \ldots$; and in periods $3, 6, 9, \ldots$ the pie gets tossed in their faces and self-destructs with chance 1/2. Then A demands at least 7/8 of the pie in period 1 in the unique subgame perfect equilibrium. [10 pts]

13. *The Silence of the Lions?* There are ten hungry lions in a circle around a lamb. Sequentially and repeatedly each lion is given the chance to eat the lamb. But if a lion eats the lamb, he becomes a lamb himself, and thus risks becoming the dinner of another lion in a later round. When a lion eats the lamb and becomes a lamb, the other lions continue to get sequentially the chance to eat the (new) lamb. All the lions enjoy the same strict preferences: [11 pts]

    $\{$eat the lamb and stay alive$\} \succ \{$stay hungry$\} \succ \{$be the meal of another lion$\}$

    Then in any subgame perfect equilibrium, nobody eats the lamb and the ten lions remain hungry.

14. Two players chip in two dollars each to the pot. The "house" deals a card to each player: The card can be A (ace) or K (king) with equal chance. After privately observing his card, player 1 decides whether to fold (F) or raise the bet (R) by three dollars. If he folds, player 2 gets the pot (with four dollars) and the game ends. If he raises, the house deals a card to player 2, which again can be either A or K with equal chance. After observing her card, player 2 decides whether to quit (Q) or to match (M) player 1's bet by chipping in another three dollars herself. If player 2 quits, player 1 gets the pot (with seven dollars). Otherwise, the players compare their cards. If the cards coincide (both are A's or both are K's), then the players get back their bets (five dollars each). If the cards differ, the player with the A gets the pot (with ten dollars). [14 pts]

    Then the (unique) equilibrium of the game requires player 1 to raise the bet with probability $1/2$ when he gets a K and player 2 to match the bet with probability $1/2$ when she gets a K. That is, in equilibrium, when player 1 is dealt a K, he bluffs with chance $1/2$.

General Equilibrium (20 points)

15. Consider an endowment economy with two agents Stackelberg Steve and Pacman Kuhn with smooth indifference curves. Steve is a monopolist who decides on the price ratio at which the two goods are traded. Pacman ("the follower") can trade any amount he likes at that price ratio. Then the equilibrium outcome is efficient. [8 pts]

16. Consider a two-good, two consumer exchange economy. Assume that both consumers have identical homothetic preferences. Then the equilibrium price ratio does not depend on distribution of endowments. [6 pts]

17. Consider the 2×2 production model for a small open economy. Let factors be capital and labour. Assume that good X has a homogeneous Leontief production function (perfect complements), and good Y has a homogeneous perfect substitutes production function, with X relatively more capital intensive. Assume that an asteroid wipes out some of the capital-stock $\bar{K}$, but that some of each good is still produced afterwards. Then production of X is unchanged. [6 pts]

Information and Incentives (22 points)

18. A firm is considering offering a binding outcome-contingent contract. If the worker accepts the contract, there are two possible outcomes: L and H, with corresponding gross profits for the firm of 10 and 20. If the firm doesn't hire the worker, it makes no profits. The worker can put in additional care in his work, which is unobservable by the firm. The worker has a von-Neuman/Morgenstern utility function $u(x, c) = \log(x) - c\log(4)$, where x is his salary and $c = 1$ when he is careful and $c = 0$ otherwise. The worker has an alternative offer with reservation utility $r = \log(2)$. Let $p_{c\omega}$ denote the probability that outcome $\omega \in \{L, H\}$ occurs when the worker takes action $c \in \{0, 1\}$. Assume [14 pts]

$$P \equiv \begin{pmatrix} p_{0L} & p_{0H} \\ p_{1L} & p_{1H} \end{pmatrix} = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$$

Then the firm will hire the worker and offer him a nonconstant salary (i.e. that depends on the outcome) to induce him to be careful.

19. A car is being sold by its owner. The car may be in good condition (an orange, chance $1 - q$) or in bad condition (a lemon, chance q). The owner knows the condition of his car, and it is common knowledge that he values lemons at \$3,000 and oranges at \$9,000. A buyer has no way of learning the condition of the car until he/she owns it. Buyers value a lemon at \$6,000 and an orange at \$12,000. [8 pts]

Then the car will be sold for sure if $q \leq 1/2$.