The functions are also eigenfunctions of \( \hat{a} \) and \( \hat{a}^\dagger \) with the same eigenvalues \( n \). The operators \( \hat{a} \) and \( \hat{a}^\dagger \) are both eigenfunctions of \( \hat{a}^2 \) with the same eigenvalues \( n \).

\[
\begin{align*}
0 &= \hat{a} (\hat{a}^\dagger + \hat{a}) x = \hat{a} \hat{a}^\dagger x + \hat{a}^\dagger \hat{a} x = [\hat{a}, \hat{a}^\dagger] x \equiv [\hat{a}, \hat{a}^\dagger] = [\hat{a}, \hat{a}^\dagger]
\end{align*}
\]

This is the commutator of \( \hat{a} \) and \( \hat{a}^\dagger \). Therefore, the simultaneous eigenfunctions of \( \hat{a} \) and \( \hat{a}^\dagger \) are both eigenfunctions of \( \hat{a}^2 \) with the same eigenvalues \( n \).

\[
(\hat{a}^\dagger \hat{a}^\dagger x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x)
\]

So

\[
(\hat{a}^\dagger \hat{a}^\dagger x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x)
\]

Part (c)

These functions are also eigenfunctions of \( \hat{a}^\dagger \). With eigenvalues \( n \), the normalized eigenfunctions are

\[
\langle \{\hat{a}^\dagger \hat{a}^\dagger x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x)\rangle = \langle \{\hat{a}^\dagger \hat{a}^\dagger x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x)\rangle
\]

The normalized eigenfunctions are

\[
\langle \{\hat{a}^\dagger \hat{a}^\dagger x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x)\rangle = \langle \{\hat{a}^\dagger \hat{a}^\dagger x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x = (\hat{a}^\dagger)^2 x)\rangle
\]

**Problem 2.4 A Simple Function Space**

---

**Solution for Problem 2.4**

\[
\begin{align*}
1.6021892 \times 10^{-19} &\text{ kg} \\
= &\text{ proton mass}
\end{align*}
\]

Part (a) The proton is about

\[
1.6021892 \times 10^{-19} \text{ kg} = \text{ proton mass}
\]

Part (c) The proton is about

\[
1.6021892 \times 10^{-19} \text{ kg} = \text{ proton mass}
\]

Part (d) The proton is about

\[
1.6021892 \times 10^{-19} \text{ kg} = \text{ proton mass}
\]

Part (e) The proton is about

\[
1.6021892 \times 10^{-19} \text{ kg} = \text{ proton mass}
\]

Part (f) The proton is about

\[
1.6021892 \times 10^{-19} \text{ kg} = \text{ proton mass}
\]
\[
\int_1^\infty \frac{z}{1 + z^2} \, dz = C
\]

where, from orthogonality and the normalization condition,

\[
\langle x \mid y \rangle \sum_{i=1}^{n} = (x)_{i}f
\]

Any function has an expansion

\[
0 = z p(z) \int_1^\infty \sim z p(z) \delta f(z) \int_1^\infty
\]

relation says that \( a = -\frac{d}{dz} \). Again, you can check that, since \( a = 0 \) when \( a \neq 0 \), the coefficients of equal powers vanish. With \( l = 0 \) and \( m = 1 \), the expansion

\[
e = \sum_{n=0}^{\infty} (z)^{2n+2} + \sum_{n=0}^{\infty} (z)^{2n+1} + \sum_{n=0}^{\infty} (z)^{2n} = (z)^2f
\]

Finally,

\[
0 = z p(z) \int_1^\infty \sim z p(z) \delta f(z) \int_1^\infty
\]

the orthonormal function. You can check that

By a similar argument, \( a = 0 \) and the orthonormal function gives \( a = -\frac{d}{dz} \). So for \( l = 1 \), \( m = 0 \). So for \( l = 0 \), \( m = 0 \).

Next one is

\[
\sum_{n=0}^{\infty} (z)^{2n+2} + \sum_{n=0}^{\infty} (z)^{2n+1} + \sum_{n=0}^{\infty} (z)^{2n} = (z)^2f
\]

Since \( f \) is a polynomial of degree \( \lambda \), \( \lambda > 0 \) for \( l = 1 \). Set \( m = 1 \) in the above.

\[
e = \sum_{n=0}^{\infty} (z)^{2n+2} + \sum_{n=0}^{\infty} (z)^{2n+1} + \sum_{n=0}^{\infty} (z)^{2n} = (z)^2f
\]

\[
\sum_{n=0}^{\infty} (z)^{2n+2} + \sum_{n=0}^{\infty} (z)^{2n+1} + \sum_{n=0}^{\infty} (z)^{2n} = (z)^2f
\]

This is supposed to be the same as

\[
\sum_{n=0}^{\infty} (z)^{2n+2} + \sum_{n=0}^{\infty} (z)^{2n+1} + \sum_{n=0}^{\infty} (z)^{2n} = (z)^2f
\]

Then

\[
\sum_{n=0}^{\infty} (z)^{2n+2} + \sum_{n=0}^{\infty} (z)^{2n+1} + \sum_{n=0}^{\infty} (z)^{2n} = (z)^2f
\]

Part (b) Set \( z = \chi \). The endpoint term vanishes because of the \( z = \chi \). The last identity holds because the integrand is real.
\[ e^{\frac{i\pi}{t} - \frac{3\pi}{2}} N = \langle x \rangle \phi = \langle \phi | x \rangle \]

Let \( |\phi\rangle \) be a normalized Gaussian wave packet.

### Problem 2.7 Time Evolution of a Free Gaussian Wave Function

\[ (S.2.41) \]

\[ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \mathbb{1} \]

Part (a) The eigenvalues satisfy \( \lambda = \{1, -\lambda\} \).

The solutions are

\[ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} (1 - \theta \cos \theta) + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \theta \sin \theta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ \begin{bmatrix} \cdots + \gamma(\theta) i\rho \I + \epsilon(\theta) i\pi \I \\ \cdots + \gamma(\theta) i\rho \I + \epsilon(\theta) i\pi \I \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \]

\[ \begin{bmatrix} \cdots + \gamma(\theta) i\rho \I + \epsilon(\theta) i\pi \I - \theta \\ \cdots + \gamma(\theta) i\rho \I + \epsilon(\theta) i\pi \I - \theta \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \]

\[ \begin{bmatrix} \cdots + \gamma(\theta) i\rho \I + \epsilon(\theta) i\pi \I + \theta \\ \cdots + \gamma(\theta) i\rho \I + \epsilon(\theta) i\pi \I + \theta \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \]

\[ (S.2.49) \]

\[ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathbb{1} \]

\[ (S.2.39) \]

Part (b) The rotation matrices

\[ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \mathbb{1} \]

### Problem 2.6 Some Properties of the Rotation Matrices

\[ (S.2.38) \]

\[ (z - \bar{z}) \phi = \langle z \rangle \mathcal{D}(z) \phi = \frac{z - \bar{z}}{1 + \bar{z}} \sum_{a} \]

Since \((z - \bar{z}) \phi \) cannot depend on \((z) \phi \)

\[ \therefore \lambda(z) \phi = \sum_{a} \]

\[ \therefore \int_{1}^{z} \frac{z - \bar{z}}{1 + \bar{z}} \sum_{a} = \langle z \rangle \phi \]

Therefore
\[
\left[ \frac{w/\eta + e\frac{d}{x}}{x} \right] \frac{dx}{e^x} \frac{(w/\eta + e)^{\Lambda}}{I} \frac{q_N}{I} = \]

\[
dp \left[ \left( \frac{u/\eta + e\frac{d}{x}}{x} \right) \right] \frac{dx}{e^x} \frac{\mu^{\Lambda}}{q_N} = \]

\[
dp \left[ \left( \frac{u/\eta + e\frac{d}{x}}{x} \right) \right] \frac{dx}{e^x} \frac{\mu^{\Lambda}}{q_N} = \]

\[
dp \left( \frac{u/\eta + e\frac{d}{x}}{x} \right) \frac{dx}{e^x} \frac{\mu^{\Lambda}}{q_N} = \]

The wave function in position space is the Fourier transform.

\[
\frac{\mu^{\Lambda}}{I} = \langle \langle \phi \mid \mathcal{A} \mid \phi \rangle \rangle
\]

Since the dispersion in \( d \) is independent of the time:

\[
\langle (0) \phi | d \rangle \langle d | (0) \phi \rangle = \langle (0) \phi | (0) \phi \rangle
\]

with the momentum space wave function,

\[
dp \left( \frac{u/\eta + e\frac{d}{x}}{x} \right) \frac{dx}{e^x} \frac{\mu^{\Lambda}}{q_N} = \]

Time Development

\[
\left( \frac{u/\eta + e\frac{d}{x}}{x} \right) \frac{dx}{e^x} \frac{\mu^{\Lambda}}{q_N} = \]

which is the minimum allowed by the uncertainty principle.

\[
\frac{\mu^{\Lambda}}{I} = \frac{\mu^{\Lambda}}{\eta} = \frac{\mu^{\Lambda}}{N} = \]

The product is

\[
\left( \frac{u/\eta + e\frac{d}{x}}{x} \right) \frac{dx}{e^x} \frac{\mu^{\Lambda}}{q_N} = \]

The dispersions are

\[
\frac{\mu^{\Lambda}}{N} = \frac{\mu^{\Lambda}}{N} \]

The normalized momentum space wave function is

\[
\frac{\mu^{\Lambda}}{q_N} = \frac{\mu^{\Lambda}}{N} = \]

Then

\[
\frac{\mu^{\Lambda}}{N} = \frac{\mu^{\Lambda}}{N} = \]
The conjugate momenta are

\[ \frac{\varepsilon w + i w}{\varepsilon w/\varepsilon + i w/\varepsilon} = \eta \quad \text{and} \quad \varepsilon w + i w = \eta \]

where

\[ (\xi + i \lambda) \frac{\varepsilon w}{\varepsilon} + \frac{\varepsilon w}{\varepsilon} = \eta \]

Part (c) Write \( \eta \) in terms of \( \eta \) and \( \xi \) and their time derivatives. By direct substitution, the considered quantity is \( \phi + \phi \frac{d}{dt} \), while \( \phi \frac{d}{dt} \) is invariant, and therefore so is \( \eta \).

Under a translation \( \xi \to \xi + \zeta \), write \( \phi \frac{d}{dt} \) as

\[ \varepsilon w = \varepsilon w \quad \text{and} \quad \eta = \eta \]

Part (a) Two-Particle Systems

\[ - \varepsilon \frac{d}{dp} \left( \phi \Delta \phi - \phi \Delta \phi \right) \cdot \psi \int = \int \psi \Delta \phi \frac{d}{dp} \left( \phi \Delta \phi - \phi \Delta \phi \right) \cdot \psi \int \]

The first two terms are

\[ \int \psi \Delta \phi \frac{d}{dp} \left( \phi \Delta \phi - \phi \Delta \phi \right) \cdot \psi \int = \int \psi \Delta \phi \frac{d}{dp} \left( \phi \Delta \phi - \phi \Delta \phi \right) \cdot \psi \int \]

The radius of change of the probability that the particle is inside \( \Delta t \) is

\[ \int \frac{dp}{d\phi} \left( \frac{\phi}{\phi \Delta \phi - \phi \Delta \phi} \right) \cdot \psi \int = \int \frac{dp}{d\phi} \left( \frac{\phi}{\phi \Delta \phi - \phi \Delta \phi} \right) \cdot \psi \int \]

The dispersion in \( x \) is

\[ \int e^{i x (\phi \Delta \phi - \phi \Delta \phi)} \cdot \psi \int = \int e^{i x (\phi \Delta \phi - \phi \Delta \phi)} \cdot \psi \int \]

Chapter II: Fundamentals of Quantum Mechanics
When \( x \neq 2 \), another representation of the Delta function is

\[
\delta_{x} = \frac{\left( x - z \right)}{I} + \frac{\left( z - x \right)}{I} - \frac{\left( z + x - z \right)}{I} + \frac{(1 - 0)\left( z + x - z \right)}{I} = \frac{\delta_{x} - \delta_{z}}{I} \}
\]

for \( z \neq x \).

**Problem 2.12 Another Representation of the Delta Function**

\[
\delta_{x} = \frac{\left( x - z \right)}{I} + \frac{\left( z - x \right)}{I} - \frac{\left( z + x - z \right)}{I} + \frac{(1 - 0)\left( z + x - z \right)}{I} = \frac{\delta_{x} - \delta_{z}}{I} \}
\]

Part (b) The answer scales the inverse square root of the mass, so if \( M \) is one kilogram, which is 1000

\[
\text{grams,} \quad \frac{\delta_{x}}{\text{sec} \cdot \text{g} \cdot \text{cm}} \approx \frac{9.8 \times 10^{-3}}{\text{sec} \cdot \text{g} \cdot \text{cm}} \times 10000 \times 9 \times g \approx \frac{9.8 \times 10^{-3}}{\text{sec} \cdot \text{g} \cdot \text{cm}} \times 10000 \times 9 \times g \approx v
\]

Part (a) Since according to equation (2.20), the spread is

\[
\text{V^\wedge}
\]

the correct result. This relation is satisfied by the given expression, which also has the correct \( a \), so by induction this is

\[
\sum \frac{a^{n}}{n!} = e^{a}
\]

**Problem 2.11 Spread of Wave Function**