Economics 676: Macroeconometrics

Requirements:
The course covers topics in time series analysis with an emphasis on applications in
macroeconomics. The aim of the course is to equip students with a working knowledge of
important econometric techniques used in monetary economics, financial economics,
international economics, and econometric theory. Substantial emphasis will be placed on the
development of programming skills in MATLAB (a matrix algebra program).

Students taking 676 are required to have completed the first-year Ph.D. sequence in
econometrics (Economics 671/672). Ph.D. students taking international finance or advanced
macroeconomics are strongly advised to take Economics 676 concurrently. The course is not
open to Master students.

Grading:
Course grades for Economics 676 will be based on a course paper (40%) and regular homework
assignments (60%). This may not sound like much, but this course is quite work-intensive and
will involve long hours at the computer. If you do not have the time to give this course your full
attention, you may want to take the course at some other time. The investment will be worth it
once you embark on your thesis.

The problem sets typically consist of programming exercises in the matrix algebra software
MATLAB. They may be prepared in groups of up to three students, but must be written up and
handed in individually. Everyone is responsible for writing their own code. Please indicate the
other group members, as applicable, and include all of the code along with your interpretation of
the results. All problem sets must be stapled. Electronic submissions are not acceptable. The
problem sets are due in class (or under my door by the end of class) on the due date. There will
be no extensions.

All problem sets for this course must be coded in MATLAB. There are no exceptions. One of the
aims of this course is to make you proficient in MATLAB programming, so you can tackle new
challenges on your own, when you start writing your thesis. MATLAB is used extensively
among practitioners and among researchers and is indispensable for your career whether you
plan to go to Wall Street, the Federal Reserve Board or a research university. It might not be the
only software you will have to master, but it will be the most useful and versatile software.
MATLAB is available on UNIX and on the PCs in the department’s econometrics lab. In addition, the university provides virtual access to MATLAB.

The term project involves identifying an econometric technique for time series that has not been covered in class. You are supposed to write MATLAB code implementing this technique. The code should be well documented and accompanied by a readme.txt file, by a description of how this technique works and what each file accomplishes. Most papers will focus on an empirical application to actual data. The empirical application may replicate some findings in the literature, but it has to be of substantive interest. The empirical analysis should be concisely written and clearly spell out the question of interest and the findings. You may also substitute a methodological question for the empirical application. All topics are subject to my approval.

The course paper should not normally exceed 15-20 pages in length and is due at the end of the course without fail. Electronic submissions are not acceptable. The format of the papers should adhere to the standards required for submission to a journal. Papers that do not meet these standards will not be accepted. Please consult my homepage for examples of acceptable formats. A short, but polished paper is vastly preferred to a longer, but shoddy one. Papers must not be co-authored. The paper is due on April 23 at noon. Please drop them off at my office. There will be no extensions of this deadline.

I will be out of town for a conference on Wednesday, March 23. I plan to have a double make-up class on Friday, January 15, 4:00-7:00PM. The details are to be confirmed.

Readings:
There will be a coursepack for this course (available for purchase at Dollar Bill and online at ctools). You are expected to bring the coursepack to class. The coursepack will form the backbone of this course. In addition, there are selective readings from journal articles on each topic and there are two required textbooks:


Both books are worthwhile having on your shelf, whether you are interested in finance, macroeconomics, international finance or econometrics. Hamilton (1994) is best thought of as a reference book. It is somewhat dated, but still the only graduate-level textbook that covers all aspects of time series econometrics. Lütkepohl’s book is a substantially revised version of his earlier book Introduction to Multiple Time Series Analysis. Rather than cover a wide range of time series methods, it focuses on multivariate time series models only. This includes the vector autoregressive framework which has become the workhorse model of applied time series analysis. For the purpose of this course, either version of this book will do. Lütkepohl’s book is especially useful for this course in that it is very explicit, which facilitates the programming of econometric procedures in a matrix algebra software such as MATLAB. Lütkepohl’s book is also available online through mirlyn. You should nevertheless buy the book in my view. It is a worthwhile investment.
Another good resource is the Handbook of Econometrics, the Handbook of Statistics and the Handbook of Economic Forecasting. The selective list below contains additional textbooks and monographs that you may find useful:

**General Books on Macroeconometrics:**

**Spectral Analysis:**

**Nonlinear Models:**

**Unit Roots and Cointegration:**

**Forecasting:**

**Applications:**


**Historical Perspective:**


**Econometrics Background:**


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