

**Syllabus
Lutz Kilian
Fall 2009**

Economics 671: Econometric Analysis I

Location:

Lectures: Monday-Wednesday 8:30-10:00AM in Dennison 130.
Office hours: Monday 5:00-6:00PM in Lorch 309.
Discussion: Tuesday 6:00-7:30PM in Dennison 110.

Graduate Student Instructor:

Breno Braga Email: bgbraga@umich.edu (NEW!)

Important dates:

First day of class: Wednesday, September 9.
Last day of class: Monday, December 14.
Midterm: TBA
Final exam: Tuesday, December 22, 8:00-10:00AM.

Course requirements:

The course covers basic topics in probability theory, descriptive statistics, estimation, and statistical inference, including the classical linear regression model. The aim of the course is to lay the foundations for more advanced econometrics courses such as Economics 672, 675 and 676. Course grades for Economics 671 will be based on one midterm (40 percent), a final requirement (40 percent) and regular homework assignments (20 percent). There will be no extensions for homework assignments. Some assignments involve use of the matrix algebra software MATLAB which is available in the department's econometrics lab and online through the university.

This course is not appropriate for Master students. In particular, students from the Master programs in Economics and in Public Policy are not allowed to take this course. Please consult your adviser for alternatives.

Ph.D. students from fields other than economics or finance should obtain my permission at the beginning of the term, since not everybody will have the prerequisites required for this course.

Required textbook:

Casella, G., and R.L. Berger (2002), *Statistical Inference*, 2nd ed., Pacific Grove, CA: Duxbury.

Coursepack:

Includes lecture notes, old exams, problem sets and MATLAB instructions. Available at Ulrich's Bookstore and on ctools. The revised version will become available in early August probably. Please bring your copy to class.

Further Background Readings:

- Greene, W.H. (2000), *Econometric Analysis*, 4th ed., Upper Saddle River, NJ: Prentice Hall.
- Hogg, R.V., J.W. McKean, and A.T. Craig (2005), *Introduction to Mathematical Statistics*, 6th ed., Upper Saddle River, NJ: Prentice Hall.
- Judge, G. G., R.C. Hill, W.E. Griffiths, H. Lütkepohl, and T.-C. Lee (1988), *Introduction to the Theory and Practice of Econometrics*, 2nd ed., New York: Wiley. (“Baby” Judge)
- Judge, G. G., W.E. Griffiths, R.C. Hill, H. Lütkepohl, and T.-C. Lee (1985), *The Theory and Practice of Econometrics*, 2nd ed., New York: Wiley. (“Big” Judge)
- Schmidt, P. (1976), *Econometrics*, New York: Marcel Dekker.

Table of Contents

Part 1: Probability Theory

1. Probability	2
1.1. Introductory Concepts	2
1.2. Probability of Events	4
1.3. Conditional Probability and Independent Events	7
1.4. Bayes Rule	10
2. Combinatorial Methods (Counting Techniques)	12
2.1. Binomial Coefficients	14
2.2. Rules for Binomial Coefficients	15
3. Random Variables	15
3.1. Probability Distributions of Discrete Random Variables	16
3.2. Probability Distributions for Continuous Random Variables	19
4. Multivariate Distributions	22
4.1. Discrete Random Variables (Bivariate Case)	23
4.2. Continuous Random Variables (Bivariate Case)	24
4.3. Some Examples	24
4.4. Marginal Distributions	28
4.5. Conditional Distributions	30
4.6. Independence of Random Variables	31
5. Mathematical Expectations	32
5.1. Unconditional Expectations	32
5.2. Unconditional Expectation of a Function of a Single Random Variable	34
5.3. Unconditional Expectation of a Function of Several Random Variables	35
6. Moments	36
6.1. Moments of Linear Functions of Random Variables	40
6.2. Digression: Expectations involving multivariate random variables	41
6.3. Moments of Nonlinear Functions of Random Variables	43
7. Conditional Expectations	45
7.1. Law of Iterated Expectations	47
8. Some Useful Inequalities	48
8.1. Markov’s Inequality	48
8.2. Chebychev’s Inequality	49
9. Moment Generating Function	50
10. Finding the Distribution of Functions of Random Variables	54
10.1. Distribution-Function Technique	54
10.2. Change-of-Variable Technique: One-Variable Case	55
10.3. Change-of-Variable Technique: Several Variables	57

10.4. Transformation Technique: Joint Density of Several Functions of Several Variables	59
11. Special Probability Distributions and Densities	61
11.1. Special Probability Distributions	61
11.1.1. Discrete Uniform Distribution	61
11.1.2. Bernoulli Distribution	62
11.1.3. Binomial Distribution	62
11.1.4. Negative Binomial Distribution and Geometric Distribution	64
11.1.5. Hypergeometric Distribution	66
11.1.6. Poisson Distribution	68
11.1.7. Multinomial Distribution	71
11.1.8. Multivariate Hypergeometric Distribution	72
11.2. Special Probability Densities	72
11.2.1. Uniform Density	72
11.2.2. Gamma Distribution	74
11.2.3. Exponential Distribution	75
11.2.4. Erlang Distribution	76
11.2.5. Chi-Squared Distribution	78
11.2.6. Beta Distribution	79
11.2.7. Normal Distribution	79
11.3. Digression: Moment-Generating Function Technique	90
11.4. The Multivariate Normal Distribution	92
11.4.1. Bivariate Normal Distribution	92
11.4.2. Multivariate Normal Distribution	93
11.4.3. Partitioning Multivariate Normal Distributions	94
11.4.4. The Distribution of Quadratic Forms in Multivariate Normal Random Variables	95
Part 2: Sampling Distributions	
12. Sampling Distributions	98
12.1. Sampling Distribution of the Mean	98
12.2. Exact Finite-Sample Distributions for Gaussian Populations	101
12.2.1. The Chi-Squared Distribution	101
12.2.2. The Student-t Distribution	103
12.2.3. The F-distribution	107
13. Large-Sample Theory	109
13.1. Modes of Convergence	110
13.1.1. Convergence in Distribution	110
13.1.2. Convergence in Probability	115
13.1.3. Convergence in Quadratic Mean	119
13.1.4. Almost Sure Convergence	119
13.2. Logical Relationships between Modes of Convergence	119
13.2.1. Some Useful Theorems on Convergence in Probability	121
13.2.2. Some Useful Theorems on Convergence in Distribution	123
13.2.3. Some Results on Combining Convergence in Distribution and Convergence in Probability	123
13.3. Laws of Large Numbers	124
13.3.1. Why Are LLNs so Important?	125
13.4. Central Limit Theorems	126
13.4.1. Graphical Illustration of Sampling Properties	132

13.4.2. Why Are CLTs so Important?	133
13.4.3. Making the CLT Operational	136

Part 3: Parameter Estimation and Inference

14. Point Estimation	139
14.1. Estimation of Parameters	140
14.2. Finite Sample Properties of Estimators	140
14.3. Asymptotic Properties of Estimators	145
14.4. Estimation Methods	149
14.4.1. Method of Moments	149
14.4.2. Least-Squares Estimation	155
14.4.3. Maximum Likelihood Estimation	158
14.5. Minimum Variance Estimation	164
14.5.1. Scalar Case	164
14.5.2. Vector Case	169
14.6. Asymptotic Properties of the ML Estimator	173
14.6.1. Asymptotic Normality and Asymptotic Efficiency of the MLE	174
14.7. Properties of the Gaussian MLE under Misspecification	176
14.8. The Delta Method	177
15. Interval Estimation	178
15.1. Estimation of Means	179
15.2. Estimation of Variances	181
15.3. Other Estimation Problems	182
15.3.1 Estimation of the Ratio of Two Variances	182
15.3.2. Estimation of the Differences between Means	183
15.3.3. Estimation of Proportions	184
15.3.4. Estimation of Differences between Proportions	184
16. Hypothesis Testing	188
16.1. Review	188
16.1.1. One-Sided and Two-Sided Tests	190
16.1.2. Size versus Power	191
16.1.3. Marginal Significance Level	193
16.1.4. Relationship between Confidence Intervals and Tests	194
16.1.5. Treatment Effects	195
16.2. Theory of Hypothesis Testing	199
16.2.1. Neyman-Pearson Lemma	200
16.2.2. Beyond the Neyman-Pearson Lemma	204
16.2.3. Likelihood Ratio Tests	208
16.3. The Trinity of Tests	213
16.3.1. Wald Test	215
16.3.2. Lagrange Multiplier Test	216
16.3.3. How Do We Choose Between the Three Tests?	218

Part 4: Classical Linear Regression

17. Classical Linear Regression Model: Outline	220
18. The Model and its Assumptions	221
18.1. The Linear Regression Model	222
18.1.1. Notation	222
18.1.2. Assumptions	222
19. Analysis under Full Ideal Conditions Without Normality	224

19.1. Ordinary Least Squares Regression	224
19.1.1. Statistical Properties of $\hat{\beta}$	228
19.1.2. Statistical Properties of $\hat{\sigma}^2$	233
19.1.3. Asymptotic Distribution of $\hat{\beta}$	238
19.1.4. Asymptotic Distribution of $\hat{\sigma}^2$	245
19.1.5. Consistency of the Estimator of the Limiting Variance of $\sqrt{T}(\hat{\beta} - \beta)$	
19.2. Method-of-Moments Estimation in the Linear Regression Model	246
20. Analysis under Full Ideal Conditions including Normality	247
20.1. Finite-Sample Properties of the OLS Estimator	248
20.2. Maximum Likelihood Estimation	250
20.3. Asymptotic Properties of the Maximum Likelihood Estimator	253
20.3.1. Gaussian MLE under FIC including Normality	253
20.3.2. What Happens When the Gaussian MLE is Applied but the True Error Is Not Gaussian?	257
20.4. Inference in the Classical Linear Regression Model under Normality	258
20.4.1. Confidence Intervals	258
20.4.2. Hypothesis Testing	260
20.5. Inference in the Classical Linear Regression Model without Normality	260
20.5.1. Linear Functions of the OLS Estimator	261
20.5.2. Nonlinear Functions of the OLS Estimator	262
20.5.3. LM and LR Tests	262
20.5.4. The Relationship between the F-Test and the Trinity of Tests	264
20.5.5. Some Results for Linear Models with Linear Restrictions	264
20.5.6. A Different Motivation for the F-Test	265
20.5.7. Relationship of R^2 and F-Test	268

Part 5: Problem Sets

Problem Set 1	270
Problem Set 2	276
Problem Set 3	278
Problem Set 4	279
Problem Set 5	282
Problem Set 6	284
Problem Set 7	285
Problem Set 8	291
Problem Set 9	292
Problem Set 10	294
Problem Set 11	297
Problem Set 12	300
Problem Set 13	302
Problem Set 14	304

Part 6: Old Exams

Midterm 1: 2000	307
Midterm 1: 2003	309
Midterm 1: 2004	312
Midterm 1: 2005	314
Midterm 2: 2000	316
Midterm 2: 2003	319

Midterm 2: 2004	321
Midterm 2: 2005	323
Midterm: 2006	326
Midterm: 2007	328
Final: 2000	329
Final: 2003	332
Final: 2004	334
Final: 2005	336
Final: 2006	338
Final: 2007	340
<u>Part 7: Matlab Instructions</u>	
Instructions	343
Sample Code	355