

**Lutz Kilian**  
**Fall 2008**

## **Economics 407: Financial Econometrics**

Lecture: Monday/Wednesday 4:00AM-5:30PM in Lorch 173.  
Office hours: Monday 5:30PM-6:30PM in Lorch 309.  
First Day of Class: Wednesday, September 3.  
Last Day of Class: Monday, December 8.  
Email: lkilian@umich.edu.

### **Requirements:**

The course covers topics in time series analysis with an emphasis on applications rather than statistical theory. The aim of the course is to equip students with a working knowledge of important econometric techniques used in macroeconomics, international finance, and financial economics. Substantial emphasis will be placed on the development of programming skills in MATLAB (a matrix algebra program). Regression analysis will be conducted primarily in matrix notation.

This is not a finance course; I will not teach you how to succeed as a trader or how to make money. Rather the emphasis is on understanding and learning how to apply the econometric tools used by academics and practitioners working in these areas. The course will be helpful for anyone interested in pursuing a graduate degree in a quantitative field, but equally helpful for students interested in working at research institutions or financial institutions. Rather than focusing narrowly on the application of econometric tools in finance, I will try to convey a deeper understanding of the most important tools used in applied time series analysis, their proper use and their limitations.

Students taking 407 are expected to have completed 405. They must have completed or take concurrently 406 (or equivalently 503). I will take for granted a thorough understanding of the material taught in 405. Unlike in 406, my focus will be on time series analysis, making 406 and 407 somewhat complementary. In addition, I will take for granted a good understanding of matrix algebra and calculus. If you are not familiar with matrix algebra, I recommend that you delay taking 407 until you have completed a course in matrix algebra. 435 is not required for 407, but may be helpful. The same goes for 402, 442, and related macroeconomic courses.

### **Grading:**

There will be no midterm or final exams in this course. Course grades for Economics 407 will be based on a course paper (33%) and regular homework assignments (66%). This may not sound like much, but this course is quite work-intensive and will involve long hours in the computer lab. You should anticipate that this course is likely to be the most time-demanding course you will experience as an undergraduate. If you do not have the time to give this course your full attention, you may want to take the course at some other time.

The problem set questions will be made available on ctools. They typically will consist of programming exercises in the matrix algebra software MATLAB. They may be prepared in groups of up to three students, but must be handed in individually. Please indicate the other group members, as applicable, and include all of the code along with your written interpretation of the results. Problem sets will be graded on a scale of  $\sqrt{+}$ ,  $\sqrt{}$ ,  $\sqrt{-}$ , and fail. There will be no extensions.

All problem sets for this course must be coded in MATLAB. There are no exceptions. One of the aims of this course is to make you proficient in MATLAB programming, so you can tackle new challenges on your own. MATLAB is used extensively among practitioners and among researchers and is indispensable for your career whether you plan to go to Wall Street, the Federal Reserve Board or a research institution. It might not be the only software you will have to master, but it will be the most useful and versatile software. MATLAB is available on UNIX through ITD (<http://www.itd.umich.edu>). It is also available at various locations on campus. Please contact ITD as soon as possible to set up a UNIX account and/or to find out where you will have access to MATLAB. For plotting results, only a PC will work.

The term project involves identifying an econometric technique and applying it to financial or macroeconomic time series. You will write MATLAB code implementing this technique. The code should be well documented and accompanied by a readme-file with instructions, by a description of how this technique works and what it accomplishes, and an empirical application to actual data. The empirical application may replicate some findings in the literature, but it has to be of substantive interest. The empirical analysis should be concisely written and clearly spell out the question of interest and the findings. You may also substitute a methodological question for the empirical application. All topics are subject to my approval. The papers are due by December 17 at 8:00AM without fail. Please drop them off at my office.

The course paper should not exceed 10 pages in length. The format of the papers should adhere to the standards required for submission to an academic journal (including a separate title page with an abstract summarizing the paper; a complete list of references; a list of data sources). The presentation should be explicit enough for a classmate to be able to replicate all results. Data sources must be documented and modeling choices should be defended. You should clearly explain what the research question is, why the question is interesting, and what you have learned.

You may find it useful to consult my homepage for examples of the format of unpublished papers. A short, but polished paper is vastly preferred to a longer, but shoddy one. Papers must not be co-authored.

## **Readings:**

Upon reviewing possible choices for textbooks, I discovered that no book adequately covers the material I have in mind. Hence, I will draw selectively on various sources, depending on the topic. The discussion of the vector autoregressive model will follow chapters 1-7 of Lütkepohl (2005, also available as a paperback) and hence is required. This book is helpful when it comes to coding the vector autoregressive model because it provides detailed instructions and numerical examples. It also contains a useful review of matrix algebra in the appendix.

Lütkepohl, H. (2005), *New Introduction to Multiple Time Series Analysis*, New York: Springer.

An earlier version of this book (also available as a paperback) will do just as well for this course and may be less expensive:

Lütkepohl, H. (1<sup>st</sup> ed. 1991 or 2<sup>nd</sup> ed. 1993), *Introduction to Multiple Time Series Analysis*, New York: Springer.

Finally, for those of you on a tight budget, the book is available online in pdf-format from *mirlyn*.

I will make extensive use of my lecture notes in class. A pdf copy of my lecture notes will be posted on ctools. You may download or print the notes from ctools. I will also ask Dollarbill (on Church near the intersection with South University) to prepare bound copies for your convenience. You will be expected to bring those lecture notes to class.

The list below contains additional textbooks and monographs that you may find useful for this class. I will not follow any one book closely.

### **Financial Econometrics:**

Brooks, C. (2008), *Introductory Econometrics for Finance*. 2<sup>nd</sup> ed., Cambridge: Cambridge University Press.

Christoffersen, P.F. (2003), *Elements of Financial Risk Management*. Amsterdam: Academic Press.

Tsay, R.S. (2005), *Analysis of Financial Time Series*. 2<sup>nd</sup> ed., New York: Wiley.

Wang, P. (2007), *Financial Econometrics*, London: Routledge.

### **Time Series Econometrics:**

Enders, W. (1995), *Applied Econometric Time Series*, New York: Wiley.

Hamilton, J.D. (1994), *Time Series Analysis*, Princeton, NJ: Princeton University Press.

### **Unit Roots and Cointegration:**

Maddala, G.S., and I.-M. Kim (1998), *Unit Roots, Cointegration, and Structural Change*, Cambridge, U.K.: Cambridge University Press.

**Forecasting:**

Diebold, F.X. (2006), *Elements of Forecasting*, Cincinnati, 4<sup>th</sup> ed., South-Western College Publishing.

**Applications in International Finance:**

Mark, N.C. (2001), *International Macroeconomics and Finance*, Blackwell Publishers.

Sarno, L., and M.P. Taylor (2002), *The Economics of Exchange Rates*, Cambridge University Press.

**Econometrics Background:**

Judge, G. G., R.C. Hill, W.E. Griffiths, H. Lütkepohl, and T.-C. Lee (1988), *Introduction to the Theory and Practice of Econometrics*, 2<sup>nd</sup> ed., New York: Wiley.

Kennedy, P. (2008), *A Guide to Econometrics*, 6<sup>th</sup> ed., Blackwell Publishers.

Stock, J.H., and M.W. Watson (2003), *Introduction to Econometrics*, Boston, MA: Addison-Wesley.

## Table of Contents

### **Part 1: Basic Regression Analysis**

1. Introduction to MATLAB .....	3
1.1. MATLAB as a language .....	3
1.2. Basics .....	3
1.3. Script Files and Function Files .....	4
1.4. File Management Inside MATLAB .....	5
1.5. Variables .....	5
1.6. Loading and Saving Data .....	7
1.7. Mathematical Operators .....	8
1.8. Pausing and Terminating Programs .....	9
1.9. Using Logical Statements and Writing Loops .....	10
1.10. Random Number Generators and Distributions .....	11
1.11. Some Useful Functions for Generating Descriptive Statistics .....	13
1.12. The Basics of Plotting Data in MATLAB .....	13
1.13. Data Sources for Economic Time Series .....	15
1.14. Check Your Data .....	17
1.15. Simple Data Transformations .....	17
2. The Notion of Repeated Sampling .....	18
2.1. The i.i.d. Model .....	18
2.2. Random Number Generators and Seeds .....	19
2.3. Drawing from a Pre-Specified Distribution .....	19
2.4. From Histograms to Kernel Density Estimates .....	21
3. The Basic Linear Regression Model with i.i.d. Errors in Matrix Notation .....	25
3.1. From the i.i.d. Model to the Linear Regression Model .....	25
3.2. The Same Model in Matrix Notation .....	26
3.3. Assumptions .....	27
3.4. Estimating the Regression Parameters by Ordinary Least Squares .....	28
3.4.1. Loss Functions and Curve Fitting .....	28
3.4.2. The OLS Estimator of $\beta$ in the Basic Linear Regression Model .....	31
3.4.3. Regression Fit and Prediction .....	32
3.4.4. Statistical Properties of $\hat{\beta}$ .....	34
3.4.5. Statistical Properties of $\hat{\sigma}^2$ .....	36
3.4.6. Asymptotic Normality of the OLS Estimator .....	37
3.4.7. Regression t-tests, Confidence Intervals and $p$ -Values .....	38
3.4.8. Economic Versus Statistical Significance .....	40
3.5. The MLE of the Basic Linear Regression Model .....	41
3.5.1. The Idea Behind Maximum Likelihood Estimation .....	41
3.5.2. Closed-Form MLE for the Basic Linear Regression Model .....	44
3.5.3. Asymptotic Properties of the MLE .....	47
3.6. Inference on Transformations of Regression Parameters .....	47
3.6.1. The Taylor Series Expansion .....	47
3.6.2. The Delta Method .....	49
3.6.3. The Wald Test .....	49
3.6.4. Testing Restrictions on the Regression Model with Wald and $t$ -Tests .....	50
3.6.5. One-Sided versus Two-Sided Tests .....	52
4. MLE by Numerical Methods in the i.i.d. Case .....	53
4.1. Numerical Optimization in MATLAB .....	53
4.2. Alternative Approaches to Numerical Optimization .....	55

## **Part 2: Univariate Time Series Models**

5. Basic Concepts in Time Series Analysis.....	57
5.1. The Origins of Time Series Econometrics in Business Cycle Theory .....	57
5.1.1. Periodic Cycles? .....	57
5.1.2. Irregular Cycles .....	58
5.2. Stochastic Processes .....	59
5.2.1. Stationarity .....	60
5.2.2. Ergodicity .....	61
5.3. White Noise .....	62
5.4. The Wold Representation Theorem .....	62
6. Approximating the Wold Representation .....	63
6.1. MA(q) Models .....	64
6.2. AR(p) Models .....	65
6.3. ARMA(p,q) Models .....	70
7. Data Transformations .....	71
7.1. Time-Varying Variances .....	72
7.2. Time-Varying Means .....	72
7.2.1. Deterministic Detrending .....	72
7.2.2. Log-Differencing .....	74
7.2.3. The Hodrick-Prescott (HP) Filter .....	76
7.2.4. Other Forms of Detrending .....	77
7.3. Seasonality .....	77
7.3.1. Seasonal Dummies .....	77
7.3.2. Seasonal Differencing .....	78
7.3.3. Other Forms of Seasonal Adjustment .....	78
7.3.4. Seasonality in High-Frequency Financial Data .....	78
7.4. The Danger of Applying the Wrong Transformation to Economic Time Series .....	79
8. Parametric Analysis of Time Series: Estimating AR, MA, and ARMA Processes .....	82
8.1. OLS Estimator and Conditional MLE of AR Models .....	82
8.2. Numerical MLE of MA and ARMA Models .....	85
9. Nonparametric Analysis of Time Series .....	86
10. Measuring Volatility .....	87
10.1. ARCH Models .....	87
10.1. GARCH Models .....	89
10.3. The ARCH-in-Mean Model .....	91
10.4. Other Models of Conditional Heteroskedasticity .....	92
11. Measuring Risk .....	93
11.1. Value at Risk .....	93
11.2. Other Risk Measures .....	94
12. What if the Regression Errors are not i.i.d.? Robust Regression Standard Errors .....	96
12.1. Regression Error Heteroskedastic, but Serially Uncorrelated .....	97
12.2. Regression Error Serially Correlated and Heteroskedastic of Unknown Form .....	98

## **Part 3: Multivariate Time Series Models**

13. Estimating Reduced-Form Vector Autoregressions .....	102
13.1. From Structural to Reduced-Form Models .....	102
13.2. Multivariate LS Estimation of VAR Models .....	104
13.3. Cross-Sectional Aggregation of Time Series Models .....	107
14. AR and VAR Lag Order Selection .....	107

14.1. Tests for Serial Correlation .....	108
14.2. Information Criteria .....	109
15. Structural VAR Models: Lessons from the Money-Income Causality Debate .....	111
15.1. Granger Causality Tests for Covariance Stationary VAR Models .....	112
15.2. Granger Causality, Predeterminedness, and Exogeneity .....	114
15.2.1. Basic Concepts .....	114
15.2.2. (Granger) Causality in Stock Markets .....	116
15.2.3. Exogeneity Testing: The Case of Oil Prices .....	118
15.3. Responses to Unanticipated Changes in Money Growth .....	119
15.3.1. The Narrative Approach to Monetary Policy .....	120
15.3.2. News Shocks .....	121
15.3.3. VAR Shocks .....	122
15.4. Structural VAR Examples .....	125
15.5. VAR Impulse Responses .....	127
15.6. VAR Variance Decompositions .....	130
15.7 VAR Historical Decompositions .....	133

#### **Part 4: Unit Roots, Spurious Regressions and Cointegration**

16. Testing the Unit Root Hypothesis .....	136
16.1. The Dickey-Fuller (DF) Test .....	137
16.2. The Augmented Dickey-Fuller (ADF) Test .....	138
16.3. Other Unit Root Tests .....	140
17. Spurious Regressions .....	141
18. Cointegration .....	143
18.1. Cointegration Tests .....	144
18.2. Implications of Cointegration for VAR Models .....	145
18.3. Pitfalls in Interpreting VEC Model Estimates .....	146

#### **Part 5: Forecasting**

19. Univariate Forecasting .....	149
19.1. The Bias-Variance Trade-Off .....	150
19.2. The Role of Trends .....	151
19.3. The Role of Forecast Uncertainty .....	151
19.4. Forecasting Model Selection .....	152
19.4.1. Recursive Pseudo Forecasts .....	153
19.4.2. Rolling Pseudo Forecasts .....	154
19.5. Real-Time Data versus Ex-Post Revised Data .....	155
19.6. Forecast Efficiency Tests .....	155
20. Univariate Forecasting with Large Cross-Sections .....	157
20.1. Shrinkage Methods .....	157
20.2. Model Averaging .....	159
20.3. Approximate Factor Models .....	160
21. Predictability Tests .....	162
22. Pseudo Out-of-Sample Tests of Equal Predictive Accuracy .....	162
23. Direction-of-Change Tests .....	164
24. Data Mining .....	164
24.2. What is Data Mining .....	164
24.2. Cures for Data Mining .....	165

## **Part 6: Bootstrapping**

25. Bootstrapping Time Series Models .....	169
25.1 What is Bootstrapping .....	169
25.1.1. Motivation .....	169
25.1.2. The Bootstrap Analogy: An Illustrative Example .....	170
25.2. A Primer on Bootstrap Techniques for Linear Regression Models .....	174
25.2.1. Bootstrapping i.i.d. Observations .....	174
25.2.2. Bootstrapping in the Fixed Regressor Model with i.i.d. Innovations .....	176
25.2.3. Bootstrapping in the Random Regressor Model with i.i.d. Innovations ...	177
25.2.4. Bootstrapping in the Dynamic Regressor Model with i.i.d. Innovations ...	179
25.2.5. Bootstrapping if the Assumption of i.i.d. Innovations is Violated .....	180
Heteroskedasticity in the Innovations .....	180
Serial Correlation in the Innovations .....	181
25.3. Uses of the Bootstrap Approximation .....	185
25.3.1. A Selective Review of Two-Sided Bootstrap Confidence Intervals .....	186
25.3.2. Which Interval Should We Use? .....	188
25.3.3. Bootstrap Confidence Intervals for VAR Impulse Responses .....	188
25.3.4. Bootstrap Approximations and Near Unit Roots .....	190
25.3.5. Bootstrap Confidence Intervals in the Presence of Unit Roots .....	192
25.3.6. Bootstrap Critical Values for ADF Tests .....	193
<b>Appendix: Examples of Ideas for Paper Topics .....</b>	<b>196</b>