Purchasing Power Parity and the Taylor Rule

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Abstract

If a central bank of a country sets the interest rate according to the Taylor rule, the rule imposes restrictions on the country’s interest rates and inflation rates. Standard exchange rate models with uncovered interest parity and long-run purchasing power parity contain information about how interest rates, inflation rates, and exchange rates are related. This paper develops a system method that combines the Taylor rule and a standard exchange rate model in order to estimate the half-life of the real exchange rate. We use a median unbiased estimator for the system method with nonparametric bootstrap confidence intervals, and compare the results from the system method with those from a single equation method that is typically used in the literature. We apply the method to estimate half lives of exchange rates of 18 developed countries against the U.S. dollar. Most of the estimates from the single equation method fall in the range of 3 to 5 years with wide confidence intervals that extend to the positive infinity. In contrast, the system method yields median unbiased estimates that are typically substantially shorter than 3 years with much sharper confidence intervals most of which range from 1 to 5 years.

Keywords: Purchasing Power Parity, Taylor Rule, Half-Life of PPP Deviations, Median Unbiased Estimator, Grid-t Confidence Interval

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1 Introduction

Reviewing the literature on Purchasing Power Parity (PPP), Rogoff (1996) found a remarkable consensus on 3 to 5 year half-life estimates of deviations from PPP by single equation methods. This consensus may at first seem to support the reliability of these estimates, but Kilian and Zha (2002) and Murray and Papell (2002) showed that these estimators had very large standard errors. Murray and Papell conclude that single equation methods provide virtually no information regarding the size of the half-lives.

Rogoff (1996) also has described the “PPP puzzle” as the question of how one might reconcile highly volatile short-run movements of real exchange rates with an extremely slow convergence rate to PPP. Imbs et al. (2005) point out that sectoral heterogeneity in convergence rates can cause upward bias in half-life estimates, and claim that the bias explains the PPP puzzle. While it is possible that the bias can solve the PPP puzzle under certain conditions, it is also possible that the bias is negligible under other conditions. For example, the bias would be quantitatively negligible if all the real exchange rates in each sector are stationary, and if error terms are symmetrically distributed. Indeed, from their simulation study with the same data set, Chen and Engel (2005) report no noticeable evidence of the aggregation bias. Broda and Weinstein (2007) and Crucini and Shintani (2007) also report negligible aggregation bias from very comprehensive micro data sets. So we believe that the PPP puzzle still remains unanswered.

In this paper, we develop a system method that combines the Taylor rule and a standard exchange rate model in order to estimate the half-life of the real exchange rate. Several recent papers have provided empirical evidence in favor of exchange rate models with Taylor rules (see Mark 2005, Engel and West 2006, Molodtsova and Papell 2007, and Molodtsova, Nikolsko-Rzhevskyy and Papell 2007). Therefore, a system method using an exchange rate model with the Taylor rule is a promising way to try to improve on single equation methods to estimate the half-lives.

Because standard asymptotic theory usually does not provide adequate approximations for estimation of half-lives of real exchange rates, we use nonparametric bootstrap to construct confidence intervals. Median unbiased estimates based on bootstrap are reported.

We apply the system method to estimate half lives of exchange rates of 18 developed countries against the U.S. dollar. Most of the estimates from the single equation method fall in the range of 3 to 5 years with wide confidence intervals that extend to the positive infinity. In contrast, the system method yields median unbiased estimates that are typically substantially shorter than 3 years with
much sharper confidence intervals most of which range from 1 to 5 years.

In the recent papers of two-country exchange rate models with Taylor rules cited above, the authors assume that Taylor rules are adopted by the central banks of the two countries. Because Taylor rules may not be used by some countries, we only assume that the Taylor rule is used by the home country, and remain agnostic about the monetary policy rule in the foreign country. None of these papers with Taylor rules estimates half-lives of real exchange rates.

Kim and Ogaki (2004), Kim (2005), and Kim, Ogaki, and Yang (2007) use system methods to estimate half-lives of real exchange rates. However, they use conventional monetary models without Taylor rules based on money demand functions. Another important difference from the present paper is that their inferences are based on asymptotic theory.

The rest of the paper is organized as follows. Section 2 describes our baseline model. We construct a system of stochastic difference equations for the exchange rate and inflation, explicitly incorporating a forward looking Taylor rule into the system. Section 3 explains our estimation methods. In Section 4, we report our empirical results. Section 5 concludes.

2 The Model

2.1 Gradual Adjustment Equation

We start with a simple univariate stochastic process of real exchange rates. Let $p_t$ be the log domestic price level, $p^*_t$ be the log foreign price level, and $e_t$ be the log nominal exchange rate as the price of one unit of the foreign currency in terms of the home currency. And we denote $s_t$ as the log of the real exchange rate, $p^*_t + e_t - p_t$.

We assume that PPP holds in the long-run. Putting it differently, we assume that there exists a cointegrating vector $[1 - 1 - 1]'$ for a vector $[p_t, p^*_t, e_t]'$, where $p_t$, $p^*_t$, and $e_t$ are difference stationary processes. Under this assumption, the real exchange rate can be represented as the following stationary univariate autoregressive process of degree one.

$$s_{t+1} = d + \alpha s_t + \varepsilon_{t+1},$$  \hspace{1cm} (1)

where $\alpha$ is a positive persistence parameter that is less than one.

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Note that this is a so-called Dickey-Fuller estimation model. One may estimate half-lives by an Augmented Dickey-Fuller estimation model in order to avoid possible serial correlation problems. However, as shown in Murray and Papell (2002), half-life estimates from both models were roughly similar. So it seems that AR(1) specification is not a bad approximation.
It is straightforward to show that the equation (1) could be implied by the following error correction model of real exchange rates by Mussa (1982) with a known cointegrating relation described earlier.

\[ \Delta p_{t+1} = b [\mu - (p_t - p^*_t - e_t)] + E_t \Delta p^*_t + E_t \Delta e_{t+1}, \]  

(2)

where \( \mu = E(p_t - p^*_t - e_t) \), \( b = 1 - \alpha \), \( d = -(1 - \alpha) \mu \), \( \varepsilon_{t+1} = \varepsilon_{1t+1} + \varepsilon_{2t+1} = (E_t \Delta e_{t+1} - \Delta e_{t+1}) + (E_t \Delta p^*_t - \Delta p^*_t) \), and \( E_t \varepsilon_{t+1} = 0 \). \( E(\cdot) \) denotes the unconditional expectation operator while \( E_t(\cdot) \) is the conditional expectation operator on \( I_t \), the economic agent’s information set at time \( t \). Note that \( b \) is the convergence rate \((= 1 - \alpha)\), which is a positive constant less than unity by construction.

2.2 The Taylor Rule Model

We assume that the uncovered interest parity holds. That is,

\[ E_t \Delta e_{t+1} = i_t - i^*_t, \]  

(3)

where \( i_t \) and \( i^*_t \) are domestic and foreign interest rates, respectively.

The central bank in the home country is assumed to continuously set its optimal target interest rate \((i^T_t)\) by the following forward looking Taylor Rule:\(^2\)

\[ i^T_t = \eta + \gamma_\pi E_t \Delta p_{t+1} + \gamma_\delta x_t, \]

where \( \eta \) is a constant that includes a certain long-run equilibrium real interest rate along with a target inflation rate\(^3\), and \( \gamma_\pi \) and \( \gamma_\delta \) are the long-run Taylor Rule coefficients on expected future inflation\(^4\) \((E_t \Delta p_{t+1})\) and current output deviations\(^5\) \((x_t)\), respectively. We also assume that the central bank attempts to smooth the interest rate by the following rule.

\[ i_t = (1 - \rho) i^T_t + \rho i_{t-1}, \]

\(^2\)We remain agnostic about the policy rule of the foreign central bank, because the Taylor rule may not be employed in some countries.


\(^4\)It may be more reasonable to use real-time data instead of the final release data. However, doing so will introduce another complication as we need to specify the relation between the real-time price index and the consumer price index, which is frequently used in the PPP literature. Hence we leave the use of real-time data for future research.

\(^5\)If we assume that the central bank responds to expected future output deviations rather than current deviations, we can simply modify the model by replacing \( x_t \) with \( E_t x_{t+1} \). However, this does not make any significant difference to our results.
that is, the current actual interest rate is a weighted average of the target interest rate and the previous period's interest rate, where $\rho$ is the smoothing parameter. Then, we can derive the forward looking version Taylor Rule equation with interest rate smoothing policy as follows.

$$i_t = \iota + (1 - \rho)\gamma_e p_{t+1} + (1 - \rho)\gamma_x x_t + \rho i_{t-1}$$ \hfill (4)

Combining (3) and (4), we obtain the following.

$$E_t \Delta e_{t+1} = \iota + (1 - \rho)\gamma_e E_t \Delta p_{t+1} + (1 - \rho)\gamma_x x_t + \rho i_{t-1} - i^*_t$$ \hfill (5)

where $\gamma^e = (1 - \rho)\gamma_e$ and $\gamma^x = (1 - \rho)\gamma_x$ are short-run Taylor Rule coefficients.

Now, let’s rewrite (2) as the following equation in level variables.

$$p_{t+1} = b\mu + E_t e_{t+1} + (1 - b)p_t - (1 - b)e_t + E_t p^*_t + (1 - b)p^*_t$$ \hfill (2')

Taking differences and rearranging it, (2) can be rewritten as follows.

$$\Delta p_{t+1} = E_t \Delta e_{t+1} + \alpha \Delta p_t - \alpha \Delta e_t + [E_t \Delta p^*_t - \alpha \Delta p^*_t + \eta_t] \hfill (6)$$

where $\alpha = 1 - b$ and $\eta_t = \eta_{1,t} + \eta_{2,t} = (e_t - E_{t-1}e_t) + (p^*_t - E_{t-1}p^*_t)$.

From (4), (5), and (6), we construct the following system of stochastic difference equations.

$$\begin{pmatrix} 1 & -1 & 0 \\ -\gamma^e & 1 & 0 \\ -\gamma^x & 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta p_{t+1} \\ E_t \Delta e_{t+1} \\ i_t \end{pmatrix} = \begin{pmatrix} \alpha & -\alpha & 0 \\ 0 & 0 & \rho \\ 0 & 0 & \rho \end{pmatrix} \begin{pmatrix} \Delta p_t \\ \Delta e_t \\ i_{t-1} \end{pmatrix} + \begin{pmatrix} E_t \Delta p^*_t - \alpha \Delta p^*_t + \eta_t \\ \iota + \gamma^e x_t - i^*_t \\ \iota + \gamma^x x_t \end{pmatrix} \hfill (7)$$

For notational simplicity, let’s rewrite (7) in matrix form as follows.

$$A E_t y_{t+1} = B y_t + x_t$$ \hfill (7')
and thus\(^6\)

\[
E_t y_{t+1} = A^{-1} B y_t + A^{-1} x_t
\]

\[
= D y_t + c_t,
\]

where \(D = A^{-1} B\) and \(c_t = A^{-1} x_t\). By eigenvalue decomposition, \(^8\) can be rewritten as follows.

\[
E_t y_{t+1} = V \Lambda V^{-1} y_t + c_t,
\]

where \(D = V \Lambda V^{-1}\) and

\[
V = \begin{bmatrix}
1 & 1 & 1 \\
\frac{\alpha \gamma^2}{\alpha - \rho} & 1 & 1 \\
\frac{\alpha \gamma^2}{\alpha - \rho} & 1 & 0
\end{bmatrix}, \quad \Lambda = \begin{bmatrix}
\alpha & 0 & 0 \\
0 & \frac{\rho}{1 - \gamma \pi} & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

Premultiplying \(^9\) by \(V^{-1}\) and redefining variables,

\[
E_t z_{t+1} = \Lambda z_t + h_t,
\]

where \(z_t = V^{-1} y_t\) and \(h_t = V^{-1} c_t\).

Note that, among non-zero eigenvalues in \(\Lambda\), \(\alpha\) is between 0 and 1 by definition, while \(\frac{\rho}{1 - \gamma \pi} (= \frac{\rho}{1 - (1 - \rho) \gamma \pi})\) is greater than unity as long as \(1 < \gamma \pi < \frac{1}{1 - \rho}\). Therefore, if the long-run inflation coefficient \(\gamma \pi\) is strictly greater than one\(^7\), the system of stochastic difference equations \(^7\) has a saddle path equilibrium, where rationally expected future fundamental variables enter in the exchange rate and inflation dynamics. On the contrary, if \(\gamma \pi\) is strictly less than unity, which might be true in the pre-Volker era in the US, the system would have a purely backward looking solution, where the solution would be determined by past fundamental variables and any martingale difference sequences.

Assuming \(\gamma \pi\) is strictly greater than one, we can show that the solution to \(^7\) satisfies the following

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\(^6\) It is straightforward to show that \(A\) is nonsingular, and thus has a well-defined inverse.

\(^7\) The condition \(\gamma \pi < \frac{1}{1 - \rho}\) is easily met for all sample periods we consider in this paper.
relation (see Appendix for the derivation).

\[
\Delta e_{t+1} = \dot{i} + \frac{\alpha \gamma_\pi}{\alpha - \rho} \Delta p_{t+1} - \frac{\alpha \gamma_\pi}{\alpha - \rho} \Delta p^*_t + \frac{\alpha \gamma_\pi}{\alpha - \rho} \dot{i}_t^* + \gamma_\pi \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_\pi}{\rho} \right)^j E_t f_{t+j+1} + \omega_{t+1},
\]

(11)

where,

\[
\dot{i} = \frac{\alpha \gamma_\pi}{\alpha - \rho} - (\alpha - \rho) \left( \frac{\gamma_\pi}{\alpha - (1 - \rho)} \right) t,
\]

\[
E_t f_{t+j} = - \left[ E_t \dot{i}_{t+j} - E_t \Delta p^*_{t+1+j} \right] + \frac{\gamma_\pi}{\gamma_\pi} E_t x_{t+j} = -E_t \dot{i}_{t+j} + \frac{\gamma_\pi}{\gamma_\pi} E_t x_{t+j},
\]

\[
\omega_{t+1} = \frac{\gamma_\pi}{\alpha - \rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_\pi}{\rho} \right)^j \left( E_{t+1} f_{t+j+1} - E_t f_{t+j+1} \right)
\]

\[
+ \frac{\gamma_\pi}{\alpha - \rho} \eta_{t+1} - \frac{\alpha \gamma_\pi}{\alpha - \rho} \dot{i}_{t+1} + \frac{\gamma_\pi}{\alpha - \rho} \dot{i}_{t+1}^* \eta_{t+1},
\]

and,

\[
E_t \omega_{t+1} = 0
\]

Or, (11) can be rewritten with full parameter specification as follows.

\[
\Delta e_{t+1} = \dot{i} + \frac{\alpha \gamma_\pi(1 - \rho)}{\alpha - \rho} \Delta p_{t+1} - \frac{\alpha \gamma_\pi(1 - \rho)}{\alpha - \rho} \Delta p^*_t + \frac{\alpha \gamma_\pi(1 - \rho) - (\alpha - \rho)}{\alpha - \rho} \dot{i}_t^* + \gamma_\pi \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_\pi(1 - \rho)}{\rho} \right)^j E_t f_{t+j+1} + \omega_{t+1},
\]

(11')

Here, $f_t$ is a proxy variable that summarizes the fundamental variables such as foreign real interest rates ($r_t^*$) and domestic output deviations.

Note that if $\gamma_\pi$ is strictly less than unity, the restriction in (11) may not be valid, since the system would have a backward looking equilibrium rather than a saddle path equilibrium. Put it differently, exchange rate dynamics critically depends on the size of $\gamma_\pi$. As mentioned in the introduction, however, we have some supporting empirical evidence for such a requirement for the existence of a saddle path.

\[\begin{array}{ll}
\text{If the system has a purely backward looking solution, the conventional structural Vector Autoregressive (SVAR) estimation method may apply.}
\end{array}\]
equilibrium, at least for the post-Volker era. So we believe that our specification would remain valid for our purpose in this paper.

3 Estimation Methods

We discuss two estimation strategies here: a conventional univariate equation approach and the GMM system method (Kim, Ogaki, and Yang, 2007).

3.1 Univariate Equation Approach

A univariate approach utilizes the equations [1] or [2]. For instance, the persistence parameter \( \alpha \) in [1] can be consistently estimated by the conventional least squares method under the maintained cointegrating relation assumption. Once we obtain the point estimate of \( \alpha \), the half-life of the real exchange rate can be calculated by \( \ln(\frac{5}{\ln(\alpha}) \). Similarly, the regression equation for the convergence parameter \( b \) can be constructed from [2] as follows.

\[
\Delta p_{t+1} = b [\mu - (p_t - p_t^* - e_t)] + \Delta p_{t+1}^* + \Delta e_{t+1} + \varepsilon_{t+1},
\]

where \( \varepsilon_{t+1} = \varepsilon_{1t+1} + \varepsilon_{2t+1} = (E_t \Delta e_{t+1} - \Delta e_{t+1}) + (E_t \Delta p_{t+1}^* - \Delta p_{t+1}^*) \) and \( E_t \varepsilon_{t+1} = 0. \)

3.2 GMM System Method

Our second estimation strategy combines the equation [11] with [1]. The estimation of the equation [11] is a challenging task, however, since it has an infinite sum of rationally expected discounted future fundamental variables. Following Hansen and Sargent (1980, 1982), we linearly project \( E_t(\cdot) \) onto \( \Omega_t \), the econometrician’s information set at time \( t \), which is a subset of \( I_t \). Denoting \( \hat{E}_t(\cdot) \) as such a linear projection operator onto \( \Omega_t \), we can rewrite [11] as follows.

\[
\Delta \varepsilon_{t+1} = \hat{i} + \frac{\alpha \gamma^s_{\pi}}{\alpha - \rho} \Delta p_{t+1} - \frac{\alpha \gamma^s_{\pi}}{\alpha - \rho} \Delta p_{t+1}^* + \frac{\alpha \gamma^s_{\pi} - (\alpha - \rho)}{\alpha - \rho} \hat{\iota}_t
\]

\[+ \frac{\gamma^s_{\pi}(\alpha \gamma^s_{\pi} - (\alpha - \rho))}{(\alpha - \rho) \rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma^s_{\pi}}{\rho} \right)^j \hat{E}_t f_{t+j+1} + \hat{\xi}_{t+1},
\]

where

\[
\hat{\xi}_{t+1} = \omega_{t+1} + \frac{\gamma^s_{\pi}(\alpha \gamma^s_{\pi} - (\alpha - \rho))}{(\alpha - \rho) \rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma^s_{\pi}}{\rho} \right)^j (E_t f_{t+j+1} - \hat{E}_t f_{t+j+1}).
\]
and

$$\hat{E}_t \xi_{t+1} = 0,$$

by the law of iterated projections.

Rather than choosing appropriate instrumental variables that are in \( \Omega_t \), we simply assume \( \Omega_t = \{ f_t, f_{t-1}, f_{t-2}, \cdots \} \). This assumption would be an innocent one under the stationarity assumption of the fundamental variable, \( f_t \), and it can greatly lessen the burden in our GMM estimation by significantly reducing the number of coefficients to be estimated.

Let’s assume, for now, that \( f_t \) be a zero mean covariance stationary, linearly indeterministic stochastic process so that it has the following Wold representation.

$$f_t = c(L) \nu_t, \quad (13)$$

where \( \nu_t = f_t - \hat{E}_{t-1} f_t \) and \( c(L) \) is square summable. Assuming that \( c(L) = 1 + c_1 L + c_2 L^2 + \cdots \) is invertible, (13) can be rewritten as the following autoregressive representation.

$$b(L) f_t = \nu_t, \quad (14)$$

where \( b(L) = c^{-1}(L) = 1 - b_1 L - b_2 L^2 - \cdots \). Linearly projecting \( \sum_{j=0}^{\infty} \left( \frac{1-\gamma^j}{\rho} \right)^j \hat{E}_t f_{t+j+1} \) onto \( \Omega_t \), Hansen and Sargent (1980) show that the following relation holds.

$$\sum_{j=0}^{\infty} \delta^j \hat{E}_t f_{t+j+1} = \psi(L) f_t = \left[ \frac{1 - (\delta^{-1} b(\delta))^{-1} b(L) L^{-1}}{1 - (\delta^{-1} L)^{-1}} \right] f_t, \quad (15)$$

where \( \delta = \frac{1-\gamma^\rho}{\rho} \).

For actual estimation, we assume that \( f_t \) can be represented by a finite order AR(\( r \)) process\(^9\) that is, \( b(L) = 1 - \sum_{j=1}^{r} b_j L^j \), where \( r < \infty \). Then, it can be shown that the coefficients of \( \psi(L) \) can be computed recursively (see Sargent 1987) as follows.

$$\psi_0 = (1 - \delta b_1 - \cdots - \delta^r b_r)^{-1} \quad \psi_{r+1} = 0$$

\(^9\)We can use conventional Akaike Information criteria or Bayesian Information criteria in order to choose the degree of such autoregressive processes.
\[ \psi_{j-1} = \delta \psi_j + \delta \psi_0 b_j, \]

where \( j = 1, 2, \cdots, r \). Then, we obtain the following two orthogonality conditions.

\[ \begin{align*}
\Delta e_{t+1} &= \hat{\iota} + \frac{\alpha \gamma^s}{\alpha - \rho} \Delta p_{t+1} + \frac{\alpha \gamma^s}{\alpha - \rho} \Delta p^*_t + \frac{\alpha \gamma^s}{\alpha - \rho} \hat{i}^*_t \\
&\quad + \frac{\gamma^s}{(\alpha - \rho) \rho} (\psi_0 f_t + \psi_1 f_{t-1} + \cdots + \psi_{r-1} f_{t-r+1}) + \xi_{t+1},
\end{align*} \tag{16} \]

\[ f_{t+1} = k + b_1 f_t + b_2 f_{t-1} + \cdots + b_r f_{t-r+1} + \nu_{t+1}, \tag{17} \]

where \( k \) is a constant scalar\(^{10}\) and \( \hat{E} \nu_{t+1} = 0 \).

Finally, the system method (GMM) estimation utilizes all aforementioned orthogonality conditions, \( \Delta e_{t+1} \), \( \Delta e_{t+1} \), and \( \Delta e_{t+1} \). That is, a GMM estimation can be implemented by the following \( 2(p + 2) \) orthogonality conditions.

\[ \begin{align*}
\hat{E} x_{1,t} (s_{t+1} - d - \alpha s_t) &= 0 \tag{18} \\
\hat{E} x_{2,t-\tau} \left( \frac{\Delta e_{t+1} - \hat{\iota} - \alpha \gamma^s \hat{\alpha} \Delta p_{t+1} + \alpha \gamma^s \hat{\alpha} \Delta p^*_t + \alpha \gamma^s \hat{\alpha} \Delta p^*_t}{\alpha - \rho} \hat{i}^*_t \right) &= 0 \tag{19} \\
\hat{E} x_{2,t-\tau} (f_{t+1} - k - b_1 f_t - b_2 f_{t-1} - \cdots - b_r f_{t-r+1}) &= 0 \tag{20}
\end{align*} \]

where \( x_{1,t} = (1 \ s_t)' \), \( x_{2,t} = (1 \ f_t)' \), and \( \tau = 0, 1, \cdots, r \).

### 3.3 Median Unbiased Estimator and Grid-t Confidence Intervals

We correct for the bias in our \( \alpha \) estimates by a GMM version of the grid-t method proposed by Hansen (1999) for the least squares estimator. It is straightforward to generate pseudo samples for the orthogonality condition \( \Delta e_{t+1} \) by the conventional residual-based bootstrapping. However, there are some complications in obtaining samples directly from \( \Delta e_{t+1} \) and \( \Delta e_{t+1} \), since \( p^*_t \) is treated as a forcing variable in our model. We deal with this problem as follows.

In order to generate pseudo samples for the orthogonality conditions \( \Delta e_{t+1} \) and \( \Delta e_{t+1} \), we denote \( \tilde{p}_t \)

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\(^{10}\) Recall that Hansen and Sargent (1980) assume a zero-mean covariance stationary process. If the variable of interest has a non-zero unconditional mean, we can either demean it prior to the estimation or include a constant but leave its coefficient unconstrained. West (1989) showed that the further efficiency gain can be obtained by imposing additional restrictions on the deterministic term. However, the imposition of such an additional restriction is quite burdensome, so we simply add a constant here.

\(^{11}\) In actual estimations, we normalized \((16)\) by multiplying \((\alpha - \rho)\) to each side in order to reduce nonlinearity.

\(^{12}\) \( \rho \) does not necessarily coincide with \( r \).

\(^{13}\) In actual estimations, we use the aforementioned normalization again.
as the relative price index $p_t - p_t^*$. Then, (2') and (16) can be rewritten as follows.

$$
\Delta \tilde{p}_{t+1} = \tilde{\mu} - b(\tilde{p}_t - e_t) + \Delta e_{t+1} + \varepsilon_{t+1}
$$

$$
\Delta e_{t+1} = i + \frac{\alpha \gamma^s}{\alpha - \rho} \Delta \tilde{p}_{t+1} + \frac{\alpha \gamma^s - (\alpha - \rho) i_t^*}{\alpha - \rho} \\
\frac{\gamma^s}{\alpha - \rho} \frac{(\alpha \gamma^s - (\alpha - \rho))}{(\alpha - \rho) \rho} (\psi_0 f_t + \cdots + \psi_{r-1} f_{t-r+1}) + \xi_{t+1}
$$

Or, in matrix form,

$$
\begin{bmatrix}
\Delta \tilde{p}_{t+1} \\
\Delta e_{t+1}
\end{bmatrix}
= C + S^{-1} \begin{bmatrix}
-(1 - \alpha) & 0 \\
0 & 1
\end{bmatrix} \begin{bmatrix}
\tilde{p}_t - e_t \\
\varepsilon_{t+1}
\end{bmatrix}
+ S^{-1} \begin{bmatrix}
0 \\
\frac{\alpha \gamma^s - (\alpha - \rho) i_t^*}{\alpha - \rho} \\
\frac{\gamma^s}{\alpha - \rho} \frac{(\alpha \gamma^s - (\alpha - \rho))}{(\alpha - \rho) \rho} (\psi_0 f_t + \cdots + \psi_{r-1} f_{t-r+1})
\end{bmatrix}
+ S^{-1} \begin{bmatrix}
\xi_{t+1}
\end{bmatrix},
$$

where $C$ is a vector of constants and $S$ is $\begin{bmatrix} 1 & -\frac{\alpha \gamma^s}{\alpha - \rho} \end{bmatrix}$.

Then, treating each grid point $\alpha \in [\alpha_{\min}, \alpha_{\max}]$ as a true value, we can generate pseudo samples of $\Delta \tilde{p}_{t+1}$ and $\Delta e_{t+1}$ by the conventional bootstrapping. The level variables $\tilde{p}_t$ and $e_t$ are obtained by numerical integration. It should be noted that all other parameters are treated as nuisance parameters ($\eta$). Following Hansen (1999), we define the grid-t statistic at each grid point $\alpha \in [\alpha_{\min}, \alpha_{\max}]$ as follows.

$$
t_n(\alpha) = \frac{\hat{\alpha}_{GMM} - \alpha}{se(\hat{\alpha}_{GMM})},
$$

where $se(\hat{\alpha}_{GMM})$ denotes the robust GMM standard error at the GMM estimate $\hat{\alpha}_{GMM}$. Implementing GMM estimations for $B$ bootstrap iterations at each of $N$ grid point of $\alpha$, we obtain the $(\beta$ quantile) grid-t bootstrap quantile functions, $q_{n,\beta}(\alpha) = q_{n,\beta}(\alpha, \eta(\alpha))$. Note that each function is evaluated at each grid point $\alpha$ rather than at the point estimate.

Finally, we define the 95% grid-t confidence interval as follows.

$$
\{ \alpha \in R : q_{n,0.025}(\alpha) \leq t_n(\alpha) \leq q_{n,0.975}(\alpha) \},
$$

---

14The historical data were used for the initial values and the foreign interest rate $i_t^*$. 
16If they are evaluated at the point estimate, the quantile functions correspond to the Efron and Tibshirani’s (1993) bootstrap-t quantile functions.
and the median unbiased estimator is,

\[ \alpha_{MUE} = \alpha \in \mathbb{R}, \text{ s.t. } t_n(\alpha) = q_{n,50\%}(\alpha) \] (24)

4 Empirical Results

This section reports estimates of the persistence parameter \( \alpha \) (or convergence rate parameter \( b \)) and their implied half-lives from the aforementioned two estimation strategies.

We use CPIs to construct real exchange rates with the US$ as a base currency. We consider 19 industrialized countries\(^{17}\) that provide 18 real exchange rates. For interest rates, we use quarterly money market interest rates that are short-term interbank call rates rather than conventional short-term treasury bill rates, since we incorporate the Taylor Rule in the model where a central bank sets its target short-term market rate. For output deviations, we consider two different measures of output gaps, quadratically detrended real GDP gap (see Clarida, Galí, and Gertler 1998\(^{18}\)) and unemployment rate gaps (see Boivin 2006\(^{19}\)). The data frequency is quarterly and from the IFS CD-ROM. The sample period is from 1979:III to 1998:IV for Eurozone countries, and from 1979:III to 2003:IV for the rest of the countries.

The reason that our sample period starts from 1979:III is based on empirical evidence on the US Taylor Rule. As discussed in Section II, the inflation and exchange rate dynamics may greatly depend on the size of the central bank’s reaction coefficient to future inflation. We showed that the rationally expected future fundamental variables appear in the exchange rate and inflation dynamics only when the long-run inflation coefficient \( \gamma_\pi \) is strictly greater than unity. Clarida, Galí, and Gertler (1998, 2000) provide important empirical evidence for the existence of a structural break in the US Taylor Rule. Put it differently, they show that \( \gamma_\pi \) was strictly less than one during the pre-Volker era, while it became strictly greater than unity in the post-Volker era.

We implement similar GMM estimations for (4) as in Clarida, Galí, and Gertler (2000)\(^{20}\) with longer sample period and report the results in Table 1 (see the note on Table 1 for detailed explana-

\(^{17}\)Among 23 industrialized countries classified by IMF, we dropped Greece, Iceland, and Ireland due to lack of reasonable number of observations. Luxembourg was not included because it has a currency union with Belgium.

\(^{18}\)We also tried same analysis with the cyclical components of real GDP series from the HP-filter with 1600 of smoothing parameter. The results were quantitatively similar.

\(^{19}\)The unemployment gap is defined as a 5 year backward moving average subtracted by the current unemployment rate. This specification makes its sign consistent with that of the conventional output gap.

\(^{20}\)They used GDP deflator inflation along with the CBO output gaps (and HP detrended gaps).

\(^{21}\)Unlike them, we assume that the Fed targets current output gap rather than future deviations. However, this doesn’t make any significant changes to our results. And we include one lag of interest rate rather than two lags for simplicity.
tion). We use two output gap measures for three different sub-samples. Most coefficients were highly significant and specification tests by $J$-test were not rejected. More importantly, our requirement for the existence of a saddle path equilibrium met for the post-Volker era rather than the pre-Volker era. Therefore, we may conclude that this provides some empirical justification for the choice of our sample period.

Insert Table 1 Here

Our GMM estimates and the conventional 95% bootstrap confidence intervals are reported in Table 2. We also report our GMM version median unbiased estimates and the 95% grid-$t$ confidence intervals in Table 3. We implemented estimations using both gap measures, but report the full estimates with unemployment gaps in order to save space. We chose $N = 30$ and $B = 500$ totaling 15,000 GMM simulations for each exchange rate. We chose $p = r = 8$ by the conventional Bayesian Information Criteria, and standard errors were adjusted using the QS kernel estimator with automatic bandwidth selection in order to deal with unknown serial correlation problems. For comparison, we report the corresponding estimates by the least squares in Tables 4 and 5.

One interesting finding is that the system method provides much shorter half-life estimates compared with ones from the single equation method (see Tables 2 and 4). The median half-life estimates was 2.59 years from the univariate estimations. However, we obtained the 0.90-year median half-life from the system method. This finding remains valid even when we adjust for the median bias using the grid-$t$ bootstrap. The median value of the GMM median unbiased estimates was still below 1 year, 0.94 year, while the least squares method produced the 3.42-year median half-life when we correct for the bias. Interestingly, our estimates are roughly consistent with the average half-life estimates from the micro-data evidence by Crucini and Shintani (2007).

Regarding efficiency, we obtained substantial efficiency gains from the system method over the single equation method. Murray and Papell (2002) report a version of the grid-$\alpha$ confidence intervals.

\footnote{The results with quadratically detrended real GDP gaps were quantitatively similar.}

\footnote{For the OECD countries, their baseline half-life estimates for traded good prices were 1.5 years, while 1.58 and 2.00 years for all and non-traded good prices.}

\footnote{Our point estimates are smaller than those of Murray and Papell (2002), but the differences of point estimates between countries are very similar to theirs. The exceptions to this similarity are Japan and the UK, as our point estimates for the countries are much smaller than others. Using the same sample period of Murray and Papell (2002), however, we obtained the $\alpha$ estimates of 0.89 and 0.82 for Japan and the UK, respectively. Therefore, these exceptions seem to have arisen from the difference in the sample periods.}
(Hansen, 1999) of which upper limits of their half-life estimates are infinity for every exchange rates they consider. Based on such results, they conclude that single equation methods may provide virtually no useful information due to wide confidence intervals.

Our grid-t confidence intervals from the single equation method were consistent with such a view (see Table 5). The upper limits are infinity for most real exchange rates. However, when we implement estimations by the system method, the standard errors were reduced significantly, and our 95% GMM version grid-t confidence intervals were very compact. Our results can be also considered as great improvement over Kim, Ogaki, and Yang (2007) who acquired limited success in efficiency gains.

Insert Table 2 Here

Insert Table 3 Here

Insert Table 4 Here

Insert Table 5 Here

5 Conclusion

In this paper, we developed a system method that combines the Taylor rule and a standard exchange rate model, and estimated the half-lives of the real exchange rates of 18 developed countries against the U.S.

We used two types of nonparametric bootstrap methods in order to construct confidence intervals: the standard bootstrap and Hansen’s (1999) grid bootstrap. The standard bootstrap evaluates bootstrap quantiles at the point estimate of the AR(1) coefficient, which implicitly assumes that the bootstrap quantile functions are constant functions. This assumption does not hold for the AR model, and Hansen’s grid bootstrap method that avoids this assumption has better coverage properties. In

\footnote{Their confidence intervals are constructed following Andrews (1993) and Andrews and Chen (1994), which are identical to the Hansen’s (1999) grid-\(\alpha\) confidence intervals if we assume that the errors are drawn from the empirical distribution rather than the i.i.d. normal distribution.}
our applications, we often obtain very different confidence intervals for these two methods. Therefore, the violation of the assumption is deemed quantitatively important.

When we use the grid bootstrap method, most of the (approximately) median unbiased estimates from the single equation method fall in the range of 3 to 5 years with wide confidence intervals that extend to the positive infinity. In contrast, the system method yields median unbiased estimates that are typically substantially shorter than 3 years with much sharper confidence intervals most of which range from 1 to 5 years.

These results indicate that monetary variables from the exchange rate model based on the Taylor rule provide useful information about the half-lives of the real exchange rates. The estimators from the system method are much sharper in the sense that confidence intervals are much narrower than those from a single equation method. Approximate median unbiased estimates of the half-lives are typically about one year, which is much more reasonable than consensus “3-5 years” from single equation methods especially given recent empirical literature on price adjustments such as Bills and Klenow (2005) that find frequent price adjustments.
A Derivation of (11)

Since $A$ in (10) is diagonal, assuming $0 < \alpha < 1$ and $1 < \gamma_\pi < \frac{1}{1-\rho}$, we can solve the system as follows.

$$ z_{1,t} = \sum_{j=0}^{\infty} \alpha^j h_{1,t-j-1} + \sum_{j=0}^{\infty} \alpha^j u_{t-j} $$

(a1)

$$ z_{2,t} = -\sum_{j=0}^{\infty} \left( \frac{1-\gamma_\pi^s}{\rho} \right)^{j+1} E_t h_{2,t+j} $$

(a2)

$$ z_{3,t} = h_{3,t-1} + v_t, $$

(a3)

where $u_t$ and $v_t$ are any martingale difference sequences.

Since $y_t = Vz_t$,

$$ \begin{bmatrix} \Delta p_t \\ \Delta e_t \\ i_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \frac{\alpha \gamma_\pi^s}{\alpha - \rho} & 1 & 1 \\ \frac{\alpha \gamma_\pi^s}{\alpha - \rho} & 1 & 0 \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ z_{3,t} \end{bmatrix} $$

(a4)

From first and second rows of (a4), we get the following.

$$ \Delta e_t = \frac{\alpha \gamma_\pi^s}{\alpha - \rho} \Delta p_t - \frac{\alpha \gamma_\pi^s - (\alpha - \rho)}{\alpha - \rho} z_{2,t} - \frac{\alpha \gamma_\pi^s - (\alpha - \rho)}{\alpha - \rho} z_{3,t} $$

(a5)

Now, we find the analytic solutions for $z_t$. Since $h_t = V^{-1}c_t$,

$$ h_t = \frac{1}{1-\gamma_\pi^s} \begin{bmatrix} -\frac{\alpha - \rho}{\alpha \gamma_\pi^s - (\alpha - \rho)} & -\frac{\alpha - \rho}{\alpha \gamma_\pi^s - (\alpha - \rho)} & 0 \\ \frac{\alpha \gamma_\pi^s}{\alpha \gamma_\pi^s - (\alpha - \rho)} & -\frac{\alpha \gamma_\pi^s}{\alpha \gamma_\pi^s - (\alpha - \rho)} & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} E_t \Delta p_{t+1}^s - \alpha \Delta p_t^s + \eta_t + \gamma_\pi^s i_t^s \\ \gamma_\pi^s (E_t \Delta p_{t+1}^s - \alpha \Delta p_t^s + \eta_t) + \eta_t + \gamma_\pi^s i_t^s - \gamma_\pi^s i_t^s \\ \gamma_\pi^s (E_t \Delta p_{t+1}^s - \alpha \Delta p_t^s + \eta_t) + \gamma_\pi^s i_t^s - \gamma_\pi^s i_t^s \end{bmatrix}, $$

and thus,

$$ h_{1,t} = -\frac{\alpha - \rho}{\alpha \gamma_\pi^s - (\alpha - \rho)} (E_t \Delta p_{t+1}^s - \alpha \Delta p_t^s + \eta_t) $$

(a6)

$$ h_{2,t} = \frac{\rho \gamma_\pi^s}{\alpha \gamma_\pi^s - (\alpha - \rho)} (E_t \Delta p_{t+1}^s - \alpha \Delta p_t^s + \eta_t) + \gamma_\pi^s i_t^s - \gamma_\pi^s i_t^s $$

(a7)

$$ h_{3,t} = -i_t^s $$

(a8)
Plugging (a6) into (a1),

\[
z_{1,t} = -\frac{\alpha - \rho}{\alpha \gamma^s - (\alpha - \rho)} \sum_{j=0}^{\infty} \alpha^j \left( \frac{\Delta p_{t+j}^* - \alpha \Delta p_{t+j-1}^* + \eta_{t-1}}{\gamma^s} \right) + \sum_{j=0}^{\infty} \alpha^j u_{t-j} \tag{a9}
\]

\[
= -\frac{\alpha - \rho}{\alpha \gamma^s - (\alpha - \rho)} \Delta p_t^* + \sum_{j=0}^{\infty} \alpha^j u_{t-j} - \frac{\alpha - \rho}{\alpha \gamma^s - (\alpha - \rho)} \sum_{j=0}^{\infty} \alpha^j \eta_{t-j-1}
\]

Plugging (a7) into (a2)\textsuperscript{26},

\[
z_{2,t} = -\frac{\gamma^s}{\alpha \gamma^s - (\alpha - \rho)} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma^s}{\rho} \right)^j \left( E_t \Delta p_{t+j+1}^* - \alpha E_t \Delta p_{t+j}^* + E_t \eta_{t+j} \right)
\]

\[
- \frac{1}{\rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma^s}{\rho} \right)^j \left( t + \gamma^s E_t x_{t+j} - \gamma^s E_t x_{t+j} \right)
\]

\[
= \frac{\alpha \gamma^s}{\alpha \gamma^s - (\alpha - \rho)} \Delta p_t^* - \frac{\gamma^s}{\alpha \gamma^s - (\alpha - \rho)} \eta_t - \frac{t}{\gamma^s - (1 - \rho)}
\]

\[
- \frac{\gamma^s}{\rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma^s}{\rho} \right)^j \left( E_t \Delta p_{t+j+1}^* - \frac{\gamma^s}{\gamma^s - (1 - \rho)} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma^s}{\rho} \right)^j \left( \frac{\gamma^s}{\gamma^s - (1 - \rho)} E_t x_{t+j} - E_t x_{t+j} \right) \right)
\]

Then, denoting \( E_t f_{t+j} \) as \( \left( E_t x_{t+j} - E_t \Delta p_{t+j+1}^* \right) + \frac{\gamma^s}{\rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma^s}{\rho} \right)^j \left( E_t x_{t+j} - E_t x_{t+j} \right) = -E_t x_{t+j}^* + \frac{\gamma^s}{\rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma^s}{\rho} \right)^j E_t x_{t+j}, \)

\[
z_{2,t} = \frac{\alpha \gamma^s}{\alpha \gamma^s - (\alpha - \rho)} \Delta p_t^* - \frac{\gamma^s}{\alpha \gamma^s - (\alpha - \rho)} \eta_t - \frac{t}{\gamma^s - (1 - \rho)}
\]

\[
- \frac{\gamma^s}{\rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma^s}{\rho} \right)^j \left( E_t f_{t+j}^* - \frac{\gamma^s}{\gamma^s - (1 - \rho)} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma^s}{\rho} \right)^j E_t f_{t+j} \right)
\]

Finally, plugging (a8) into (a3),

\[
z_{3,t} = -i_{t-1}^* + v_t \tag{a11}
\]

Now, plugging (a10) and (a11) into (a5),

\[
\Delta e_t = \frac{\alpha \gamma^s}{\alpha - \rho} \Delta p_t - \frac{\alpha \gamma^s}{\alpha - \rho} \Delta p_t^* + \frac{\gamma^s}{\alpha - \rho} \eta_t + \frac{\alpha \gamma^s - (\alpha - \rho)}{(\alpha - \rho)(\gamma^s - (1 - \rho))^t}
\]

\[
+ \frac{\gamma^s (\alpha \gamma^s - (\alpha - \rho))}{(\alpha - \rho) \rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma^s}{\rho} \right)^j E_t f_{t+j}^* + \frac{\alpha \gamma^s - (\alpha - \rho)}{\alpha - \rho} i_{t-1}^* - \frac{\alpha \gamma^s - (\alpha - \rho)}{\alpha - \rho} v_t
\]

Updating (a12) once and applying law of iterated expectations,

\[
\Delta e_{t+1} = \hat{t} + \frac{\alpha \gamma^s}{\alpha - \rho} \Delta p_{t+1} - \frac{\alpha \gamma^s}{\alpha - \rho} \Delta p_{t+1}^* + \frac{\alpha \gamma^s - (\alpha - \rho)}{\alpha - \rho} i^*_t
\]

\[
+ \frac{\gamma^s (\alpha \gamma^s - (\alpha - \rho))}{(\alpha - \rho) \rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma^s}{\rho} \right)^j E_t f_{t+j+1} + \omega_{t+1}, \tag{a13}
\]

\textsuperscript{26}We use the fact \( E_t \eta_{t+j} = 0, j = 1, 2, \ldots \).
where

\[ \tilde{\eta} = \frac{\alpha \gamma_{\pi}^s - (\alpha - \rho)}{(\alpha - \rho)(\gamma_{\pi}^s - (1 - \rho))} \eta_t, \]

\[ \omega_{t+1} = \frac{\gamma_{\pi}^s}{(\alpha - \rho)\rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_{\pi}^s}{\rho} \right)^j \left( E_t f_{t+j+1} - E_t f_{t+j+1} \right) \]

\[ + \frac{\gamma_{\pi}^s}{\alpha - \rho} \eta_{t+1} - \frac{\alpha \gamma_{\pi}^s - (\alpha - \rho)}{\alpha - \rho} \nu_{t+1}, \]

and,

\[ E_t \omega_{t+1} = 0 \]
References


Table 1. GMM Estimation of the US Taylor Rule Estimation

<table>
<thead>
<tr>
<th>Deviation</th>
<th>Sample Period</th>
<th>$\gamma_\pi$ (s.e.)</th>
<th>$\gamma_x$ (s.e.)</th>
<th>$\rho$ (s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP</td>
<td>1959:Q1-2003:Q4</td>
<td>1.466 (0.190)</td>
<td>0.161 (0.054)</td>
<td>0.820 (0.029)</td>
</tr>
<tr>
<td></td>
<td>1959:Q1-1979:Q2</td>
<td>0.605 (0.099)</td>
<td>0.577 (0.183)</td>
<td>0.708 (0.056)</td>
</tr>
<tr>
<td></td>
<td>1979:Q3-2003:Q4</td>
<td>2.517 (0.306)</td>
<td>0.089 (0.218)</td>
<td>0.806 (0.034)</td>
</tr>
<tr>
<td>Unemployment</td>
<td>1959:Q1-2003:Q4</td>
<td>1.507 (0.217)</td>
<td>0.330 (0.079)</td>
<td>0.847 (0.028)</td>
</tr>
<tr>
<td></td>
<td>1959:Q1-1979:Q2</td>
<td>0.880 (0.096)</td>
<td>0.217 (0.072)</td>
<td>0.710 (0.057)</td>
</tr>
<tr>
<td></td>
<td>1979:Q3-2003:Q4</td>
<td>2.435 (0.250)</td>
<td>0.162 (0.078)</td>
<td>0.796 (0.034)</td>
</tr>
</tbody>
</table>

Notes: i) Inflations are quarterly changes in log CPI level ($\ln p_t - \ln p_{t-1}$). ii) Quadratically detrended gaps are used for real GDP output deviations. iii) Unemployment gaps are 5 year backward moving average unemployment rates minus current unemployment rates. iv) The set of instruments includes four lags of federal funds rate, inflation, output deviation, long-short interest rate spread, commodity price inflation, and M2 growth rate.
Table 2. GMM Estimates and 95% Bootstrap Confidence Intervals

<table>
<thead>
<tr>
<th>Country</th>
<th>$\hat{\alpha}_{\text{GMM}}$</th>
<th>s.e</th>
<th>CI$_{\text{ET}}$</th>
<th>HL$_{\text{GMM}}$</th>
<th>HL$_{\text{ET}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.869</td>
<td>0.021</td>
<td>[0.795,0.906]</td>
<td>1.234</td>
<td>[0.755,1.758]</td>
</tr>
<tr>
<td>Austria</td>
<td>0.802</td>
<td>0.009</td>
<td>[0.737,0.835]</td>
<td>0.784</td>
<td>[0.568,0.964]</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.813</td>
<td>0.010</td>
<td>[0.751,0.850]</td>
<td>0.839</td>
<td>[0.606,1.067]</td>
</tr>
<tr>
<td>Canada</td>
<td>0.980</td>
<td>0.017</td>
<td>[0.893,0.997]</td>
<td>8.653</td>
<td>[1.531,49.42]</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.904</td>
<td>0.025</td>
<td>[0.828,0.927]</td>
<td>1.715</td>
<td>[0.918,2.286]</td>
</tr>
<tr>
<td>Finland</td>
<td>0.902</td>
<td>0.021</td>
<td>[0.827,0.903]</td>
<td>1.672</td>
<td>[0.912,1.699]</td>
</tr>
<tr>
<td>France</td>
<td>0.798</td>
<td>0.010</td>
<td>[0.727,0.840]</td>
<td>0.767</td>
<td>[0.543,0.994]</td>
</tr>
<tr>
<td>Germany</td>
<td>0.785</td>
<td>0.010</td>
<td>[0.704,0.828]</td>
<td>0.717</td>
<td>[0.493,0.918]</td>
</tr>
<tr>
<td>Italy</td>
<td>0.827</td>
<td>0.011</td>
<td>[0.729,0.865]</td>
<td>0.912</td>
<td>[0.548,1.196]</td>
</tr>
<tr>
<td>Japan</td>
<td>0.757</td>
<td>0.012</td>
<td>[0.714,0.795]</td>
<td>0.622</td>
<td>[0.515,0.754]</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.827</td>
<td>0.016</td>
<td>[0.749,0.860]</td>
<td>0.910</td>
<td>[0.599,1.147]</td>
</tr>
<tr>
<td>New Zealand</td>
<td>0.803</td>
<td>0.010</td>
<td>[0.747,0.834]</td>
<td>0.791</td>
<td>[0.594,0.956]</td>
</tr>
<tr>
<td>Norway</td>
<td>0.847</td>
<td>0.031</td>
<td>[0.791,0.878]</td>
<td>1.043</td>
<td>[0.737,1.337]</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.791</td>
<td>0.006</td>
<td>[0.712,0.834]</td>
<td>0.739</td>
<td>[0.510,0.952]</td>
</tr>
<tr>
<td>Spain</td>
<td>0.883</td>
<td>0.018</td>
<td>[0.801,0.921]</td>
<td>1.391</td>
<td>[0.781,2.114]</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.974</td>
<td>0.030</td>
<td>[0.887,0.987]</td>
<td>6.469</td>
<td>[1.445,13.24]</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.822</td>
<td>0.015</td>
<td>[0.775,0.846]</td>
<td>0.885</td>
<td>[0.680,1.039]</td>
</tr>
<tr>
<td>UK</td>
<td>0.779</td>
<td>0.011</td>
<td>[0.699,0.830]</td>
<td>0.693</td>
<td>[0.484,0.928]</td>
</tr>
<tr>
<td>Median</td>
<td>0.825</td>
<td>-</td>
<td>[0.750,0.855]</td>
<td>0.898</td>
<td>[0.603,1.107]</td>
</tr>
</tbody>
</table>

Notes: i) The US$ is the base currency. ii) Unemployment gaps are used for output deviations. iii) Sample periods are 1979.II-1998.IV (78 observations) for Eurozone countries and are 1979.II-2003.IV (98 observations) for non-Eurozone countries. iv) 95% residual-based bootstrap confidence intervals were obtained from 2.5% and 97.5% quantile estimates from 500 bootstrap replications at the GMM point estimates (Efron and Tibshirani, 1993).
Table 3. GMM Median Unbiased Estimates and 95% Grid- \( t \) Confidence Intervals

<table>
<thead>
<tr>
<th>Country</th>
<th>( \hat{\alpha}_{\text{GMM,MUE}} )</th>
<th>CI( \text{grid-} t )</th>
<th>HL( \text{GMM,MUE} )</th>
<th>HL CI( \text{grid-} t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.883 [0.837,0.940]</td>
<td>1.393 [0.977,2.817]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Austria</td>
<td>0.804 [0.785,0.826]</td>
<td>0.795 [0.717,0.905]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belgium</td>
<td>0.816 [0.794,0.844]</td>
<td>0.853 [0.750,1.020]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>1.000 [0.967,1.000]</td>
<td>( \infty ) [5.109, ( \infty ) ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Denmark</td>
<td>0.937 [0.874,1.000]</td>
<td>2.675 [1.290, ( \infty ) ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finland</td>
<td>0.948 [0.897,1.000]</td>
<td>3.235 [1.587, ( \infty ) ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>0.798 [0.778,0.822]</td>
<td>0.769 [0.689,0.885]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>0.787 [0.767,0.810]</td>
<td>0.722 [0.653,0.824]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>0.832 [0.807,0.864]</td>
<td>0.944 [0.810,1.182]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>0.754 [0.729,0.782]</td>
<td>0.613 [0.549,0.706]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.838 [0.800,0.885]</td>
<td>0.977 [0.777,1.414]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>New Zealand</td>
<td>0.805 [0.786,0.827]</td>
<td>0.799 [0.719,0.914]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Norway</td>
<td>0.874 [0.788,0.969]</td>
<td>1.287 [0.727,5.579]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Portugal</td>
<td>0.792 [0.779,0.806]</td>
<td>0.741 [0.692,0.803]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>0.896 [0.856,0.945]</td>
<td>1.574 [1.117,3.078]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td>1.000 [0.945,1.000]</td>
<td>( \infty ) [3.088, ( \infty ) ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.831 [0.795,0.870]</td>
<td>0.937 [0.755,1.240]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>0.778 [0.755,0.806]</td>
<td>0.690 [0.618,0.803]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>0.832 [0.795,0.867]</td>
<td>0.941 [0.753,1.211]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: i) The US$ is the base currency. ii) Unemployment gaps are used for output deviations. iii) Sample periods are 1979.II-1998.IV (78 observations) for Eurozone countries and are 1979.II-2003.IV (98 observations) for non-Eurozone countries. iv) CI\( \text{grid-} t \) denotes the 95% confidence intervals that were obtained by 500 residual-based bootstrap replications on 30 grid points (Hansen 1999).
Table 4. Univariate Estimates and 95% Bootstrap Confidence Intervals

<table>
<thead>
<tr>
<th>Country</th>
<th>$\hat{\alpha}_{LS}$</th>
<th>s.e.</th>
<th>CI $_{ET}$</th>
<th>HL $_{LS}$</th>
<th>HL $_{ET}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.935</td>
<td>0.033</td>
<td>[0.752,0.977]</td>
<td>2.572</td>
<td>[0.609,7.534]</td>
</tr>
<tr>
<td>Austria</td>
<td>0.936</td>
<td>0.038</td>
<td>[0.902,0.957]</td>
<td>2.608</td>
<td>[1.686,3.955]</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.918</td>
<td>0.038</td>
<td>[0.894,0.935]</td>
<td>2.038</td>
<td>[1.552,2.597]</td>
</tr>
<tr>
<td>Canada</td>
<td>0.971</td>
<td>0.023</td>
<td>[0.821,0.994]</td>
<td>5.970</td>
<td>[0.877,29.85]</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.929</td>
<td>0.035</td>
<td>[0.885,0.954]</td>
<td>2.351</td>
<td>[1.417,3.660]</td>
</tr>
<tr>
<td>Finland</td>
<td>0.945</td>
<td>0.037</td>
<td>[0.895,0.969]</td>
<td>3.051</td>
<td>[1.564,5.473]</td>
</tr>
<tr>
<td>France</td>
<td>0.918</td>
<td>0.041</td>
<td>[0.863,0.948]</td>
<td>2.015</td>
<td>[1.173,3.225]</td>
</tr>
<tr>
<td>Germany</td>
<td>0.910</td>
<td>0.042</td>
<td>[0.693,0.960]</td>
<td>1.841</td>
<td>[0.473,4.247]</td>
</tr>
<tr>
<td>Italy</td>
<td>0.936</td>
<td>0.039</td>
<td>[0.923,0.943]</td>
<td>2.607</td>
<td>[2.152,2.943]</td>
</tr>
<tr>
<td>Japan</td>
<td>0.947</td>
<td>0.032</td>
<td>[0.930,0.957]</td>
<td>3.188</td>
<td>[2.391,3.981]</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.902</td>
<td>0.043</td>
<td>[0.717,0.955]</td>
<td>1.688</td>
<td>[0.521,3.723]</td>
</tr>
<tr>
<td>New Zealand</td>
<td>0.946</td>
<td>0.017</td>
<td>[0.808,0.979]</td>
<td>3.142</td>
<td>[0.815,8.026]</td>
</tr>
<tr>
<td>Norway</td>
<td>0.922</td>
<td>0.037</td>
<td>[0.887,0.945]</td>
<td>2.142</td>
<td>[1.451,3.074]</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.969</td>
<td>0.029</td>
<td>[0.957,0.978]</td>
<td>5.503</td>
<td>[3.964,7.696]</td>
</tr>
<tr>
<td>Spain</td>
<td>0.954</td>
<td>0.030</td>
<td>[0.942,0.964]</td>
<td>3.704</td>
<td>[2.889,4.686]</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.947</td>
<td>0.028</td>
<td>[0.910,0.968]</td>
<td>3.152</td>
<td>[1.830,5.355]</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.916</td>
<td>0.039</td>
<td>[0.730,0.958]</td>
<td>1.976</td>
<td>[0.552,4.068]</td>
</tr>
<tr>
<td>UK</td>
<td>0.908</td>
<td>0.043</td>
<td>[0.784,0.949]</td>
<td>1.796</td>
<td>[0.711,3.293]</td>
</tr>
<tr>
<td>Median</td>
<td>0.936</td>
<td>-</td>
<td>0.886,0.958</td>
<td>2.590</td>
<td>1.434,4.025</td>
</tr>
</tbody>
</table>

Notes: i) The US$ is the base currency. ii) Sample periods are 1979.II-1998.IV (78 observations) for Eurozone countries and are 1979.II-2003.IV (98 observations) for non-Eurozone countries. iii) 95% residual-based bootstrap confidence intervals were obtained from 2.5% and 97.5% quantile estimates from 500 bootstrap replications at the least squares point estimates (Efron and Tibshirani, 1993).
Table 5. Univariate Median Unbiased Estimates and Grid-t Confidence Intervals

<table>
<thead>
<tr>
<th>Country</th>
<th>$\hat{\alpha}_{LS,MUE}$</th>
<th>CI_{grid-t}</th>
<th>HL_{LS,MUE}</th>
<th>HL CI_{grid-t}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.972</td>
<td>[0.891,1.000]</td>
<td>6.173</td>
<td>[1.494, $\infty$]</td>
</tr>
<tr>
<td>Austria</td>
<td>0.945</td>
<td>[0.866,1.000]</td>
<td>3.087</td>
<td>[1.205, $\infty$]</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.924</td>
<td>[0.847,1.000]</td>
<td>2.203</td>
<td>[1.045, $\infty$]</td>
</tr>
<tr>
<td>Canada</td>
<td>1.000</td>
<td>[0.946,1.000]</td>
<td>$\infty$</td>
<td>[3.122, $\infty$]</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.942</td>
<td>[0.866,1.000]</td>
<td>2.886</td>
<td>[1.200, $\infty$]</td>
</tr>
<tr>
<td>Finland</td>
<td>0.959</td>
<td>[0.883,1.000]</td>
<td>4.107</td>
<td>[1.390, $\infty$]</td>
</tr>
<tr>
<td>France</td>
<td>0.931</td>
<td>[0.847,1.000]</td>
<td>2.432</td>
<td>[1.044, $\infty$]</td>
</tr>
<tr>
<td>Germany</td>
<td>0.950</td>
<td>[0.852,1.000]</td>
<td>3.349</td>
<td>[1.078, $\infty$]</td>
</tr>
<tr>
<td>Italy</td>
<td>0.943</td>
<td>[0.859,1.000]</td>
<td>2.932</td>
<td>[1.138, $\infty$]</td>
</tr>
<tr>
<td>Japan</td>
<td>0.952</td>
<td>[0.886,1.000]</td>
<td>3.511</td>
<td>[1.428, $\infty$]</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.936</td>
<td>[0.839,1.000]</td>
<td>2.619</td>
<td>[0.990, $\infty$]</td>
</tr>
<tr>
<td>New Zealand</td>
<td>0.959</td>
<td>[0.923,0.997]</td>
<td>4.089</td>
<td>[2.174,61.29]</td>
</tr>
<tr>
<td>Norway</td>
<td>0.934</td>
<td>[0.851,1.000]</td>
<td>2.529</td>
<td>[1.073, $\infty$]</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.975</td>
<td>[0.913,1.000]</td>
<td>6.765</td>
<td>[1.904, $\infty$]</td>
</tr>
<tr>
<td>Spain</td>
<td>0.959</td>
<td>[0.898,1.000]</td>
<td>4.129</td>
<td>[1.604, $\infty$]</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.959</td>
<td>[0.891,1.000]</td>
<td>4.089</td>
<td>[1.497, $\infty$]</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.951</td>
<td>[0.862,1.000]</td>
<td>3.481</td>
<td>[1.168, $\infty$]</td>
</tr>
<tr>
<td>UK</td>
<td>0.932</td>
<td>[0.845,1.000]</td>
<td>2.442</td>
<td>[1.028, $\infty$]</td>
</tr>
<tr>
<td>Median</td>
<td>0.951</td>
<td>[0.866,1.000]</td>
<td>3.415</td>
<td>[1.203, $\infty$]</td>
</tr>
</tbody>
</table>

Notes: i) The US$ is the base currency. ii) Sample periods are 1979.II-1998.IV (78 observations) for Eurozone countries and are 1979.II-2003.IV (98 observations) for non-Eurozone countries. iii) CI_{grid-t} denotes the 95% confidence intervals that were obtained by 500 residual-based bootstrap replications on 30 grid points (Hansen, 1999).