Lumpy Investment, Lumpy Inventories

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Abstract

The link between the physical micro environment (frictions and heterogeneity) and the macroeconomic dynamics in general equilibrium macro models is influenced by the details of how exactly general equilibrium closes such a model. We make this general observation concrete in the context of the recent literature on how important nonconvex capital adjustment costs are for aggregate investment dynamics. Specifically, we introduce inventories into a two-sector lumpy investment model. We find that with inventories nonconvex capital adjustment costs dampen and propagate the reaction of investment to shocks: the initial response of fixed capital investment to productivity shocks is 50% higher with frictionless adjustment than with the calibrated capital adjustment frictions, once inventories are introduced. The reason for this result is that with two means of transferring consumption into the future, fixed capital and inventories, the tight link between aggregate saving and fixed capital investment is broken. In contrast, in the case the literature has focussed on with only one type of capital good to save and invest in, fixed capital investment dynamics are more tightly linked to consumption dynamics, which, in turn, are determined by the Euler equation of a representative household, which holds regardless of whether fixed capital investment is costly or not.

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Keywords: general equilibrium, lumpy investment, inventories, heterogeneous firms, two-sector model.

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1 Introduction

Researchers have now explored an ever more detailed and complex set of microeconomic frictions and heterogeneities in macroeconomic models. It has thus become an important question for macroeconomists, who on the one hand strive to build well-microfounded models, but are also, on the other hand, concerned about tractability and complexity of their models, how microeconomic frictions and heterogeneity affect macroeconomic dynamics. Caplin and Spulber (1987) present a striking example where any degree of nominal price stickiness at the micro level is consistent with the same aggregate outcome, money neutrality. In such a case, macroeconomic researchers arguably need not bother with the details of the microfoundation.

Conceptually, typical macroeconomic general equilibrium models can be split into a decision theoretic part where economic agents make often complex and dynamic decisions, which are, potentially, subject to a host of microeconomic frictions, e.g., physical adjustment frictions, informational frictions, etc. The second part of these models then consists of a formulation of aggregate resource and consistency constraints that will lead to the coordination of the individual decisions through prices (e.g., in Walrasian models) or aggregate quantities (e.g., in Non-Walrasian models, like search-and-matching models).

In this paper we argue that the answer to the question of how the microfoundations of decisions affect macroeconomic outcomes may depend on modeling choices in the second part, i.e., the details of how exactly general equilibrium closes a given physical environment, a perhaps obvious, but nevertheless underappreciated point. In other words, we will show – in a concrete, realistic and quantitative example – that there can be a cross effect between the general equilibrium part of a macroeconomic model and the mapping from microfoundations of decisions to macroeconomic outcomes.

Our example can simultaneously claim both realism with respect to a large body of microevidence (e.g., Doms and Dunne (1998) and Cooper and Haltiwanger (2006)) and also a certain notoriety in the literature: the debate about the aggregate importance of nonconvex capital adjustment costs. In a seminal paper, Caballero and Engel (1999) argue that nonconvex capital adjustment costs not only are powerful smoothers of aggregate investment, but also help explain certain nonlinearities in aggregate investment fluctuations. These results were produced in a macroeconomic model with essentially no general equilibrium elements, i.e., in a model with only a decision theoretic part that was aggregated by simple summation. In a series of papers, Thomas (2002), Khan and Thomas (2003) and Khan and Thomas (2008) argue, however, that once a general equilibrium part is added to the physical environment in Caballero and Engel (1999) not only do aggregate nonlinearities vanish, but also nonconvex capital adjustment costs have essentially no ability to smooth aggregate investment dynamics over and above what is done by general equilibrium price movements. Models with nonconvex capital adjustment costs thus deliver lumpy investment patterns at the micro level, but feature business cycle statistics that are identical to standard RBC models, once real wages and real interest rates adjust to clear markets.1

1Veracierto (2002) makes a similar argument for kinked, but convex adjustment cost functions. House (2008) and Miao and Wang (2011) provide other sets of conditions on preferences, technology and the adjustment cost distribution under which fixed adjustment costs are neutral for business cycles dynamics. On the other side of the debate are Gourio and Kashyap (2007) and Bachmann et al. (2013), who argue that these irrelevance results are a matter of degree, specific to the calibration strategy used, and inconsistent.
This somewhat striking irrelevance result can be understood from the first order conditions of the representative household, which are the same in a frictionless and a lumpy investment model, where the adjustment friction is on the firm side. With a representative household, the intratemporal and intertemporal first order conditions govern the optimal paths of consumption and labor supply, which in turn govern the optimal paths of output/income and saving in the short run. Thus, the households in a lumpy investment model would like to follow the same consumption path as in the frictionless model. The question is, whether they are able to do so when adjusting the capital stock is costly. The answer turns out to be yes, as long as the economy can substitute between the extensive and intensive margins of investment (see Gourio and Kashyap (2007) and, ultimately, Caplin and Spulber (1987) for this insight). To be concrete, after a positive aggregate productivity shock, the economy uses investment to increase consumption in the future. In a frictionless model this is entirely done through the intensive margin of investment: every firm invests a little more. With nonconvex capital adjustment costs this is no longer optimal, instead a few firms invest a lot. The desired amount of delayed consumption is concentrated into a few firms which really need to invest, and the same aggregate saving/investment path as in a frictionless model results.

This intuition rests on the assumption that the economy provides only one means of transferring consumption into the future, fixed capital. This is the familiar dual role of fixed capital in standard models: factor of production on the one hand and the only means of saving on the other, which in turn implies the familiar equality between saving and (fixed capital) investment. Thus for the economy as a whole investment and consumption dynamics are tightly linked. However, it is important to realize that this is only one particular way to introduce general equilibrium in a lumpy investment physical environment. There are others conceivable, and in reality an economy may delay consumption through multiple channels. We show that once we introduce multiple channels of investment and thus break the tight link between aggregate consumption and aggregate fixed capital investment, nonconvex adjustment costs and their magnitude matter much more for fixed capital investment dynamics in the sense that part of the partial equilibrium argument that they can act as smoothers of investment is restored. As has been mentioned above, this paper is very much about a cross-derivative from how the aggregate resource constraint is formulated to the ability of nonconvex adjustment costs to impact aggregate dynamics.\footnote{To be clear, this is not a paper about aggregate investment nonlinearities, but rather about the ability of nonconvex adjustment costs to achieve what adjustment costs more generally are supposed to do: smooth, i.e., dampen and propagate, investment responses to shocks.}

The key intuition for this result is the substitution between different investment channels. Viewed from a social planners’ perspective\footnote{We use in this paper a decentralized equilibrium model, where prices guarantee the social planners’ optimal allocations, but for the intuition a social planners’ perspective is useful.}, introducing more investment channels offers more margins to smooth households’ consumption, in addition to the extensive/intensive margin choice in fixed capital investment: if adjusting fixed capital is costly, the social planner can use other investment channels to optimally spread consumption over time. As a result, investment in fixed capital will be more sensitive to the level of frictions in capital adjustment.

\footnote{with some nonlinear aspects of the time series of the aggregate investment rate in the U.S.}
To be concrete: we investigate the implications of multiple investment vehicles for the "neutrality question" in a quantitative DSGE model. Building on Khan and Thomas (2003) and Khan and Thomas (2007), we study a two-sector setting with an intermediate goods sector and a final goods sector. The final goods sector has the opportunity to store, at a cost, the output from the intermediate goods sector as inventories. The incentive to hold inventories is generated by fixed ordering costs for shipments from the intermediate goods to the final goods sector. The intermediate goods sector uses fixed capital as a production factor, whose adjustment is subject to nonconvex costs. We choose inventories as the second capital type because, 1) it is a highly cyclical component in the national accounts and, 2) it is a natural means to buffer consumption against temporary shocks. Methodologically, our paper provides the first quantitative analysis of how nonconvex capital adjustment frictions impact aggregate dynamics in the presence of capital good heterogeneity.4

Figure 1 summarizes the point of the paper in a nutshell. It shows the impulse response functions of fixed capital investment to a one standard deviation productivity shock. The nonconvex fixed capital adjustment costs dampen the initial response of fixed capital investment to a productivity shock by 2.99 percentage points in the presence of inventories (‘Model I1’ versus ‘Model I2’). That is, the ‘no capital adjustment costs’-impact response is approximately 50% higher than the one with capital adjustment costs. In contrast, without inventories nonconvex fixed capital adjustment costs dampen the initial response of fixed capital investment to a productivity shock by only 1.91 percentage points (‘Model NI1’ versus ‘Model NI2’). That is, the ‘no capital adjustment costs’-impact response is only 24% higher than the one with capital adjustment costs. This highlights the aforementioned interaction effect or cross-derivative, namely, that the presence of inventories, a second capital good, will quantitatively affect the difference between a frictionless and a frictional model for fixed capital. The difference here is measured in terms of one particular aggregate statistic of interest: the initial response of fixed capital investment to an aggregate productivity shock. In addition, with inventories the response of investment in the model with the baseline level of nonconvex fixed capital adjustment costs is flatter than that in the model without these capital adjustment frictions. This means that with inventories nonconvex capital adjustment costs stretch the propagation of the productivity shock by more than what capital adjustment frictions can do without inventories. We will cast this argument in more quantitative terms below, when we study another important aggregate statistic and how its relation to nonconvex adjustment costs is shaped by the presence of inventories: autocorrelation coefficients of aggregate fixed capital investment.

Figure 1 also shows that inventories dampen the impact response of fixed capital investment at every level of fixed capital adjustment costs. With a positive productivity shock

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4 We push the research frontier in some dimensions, like the two-sector structure and inventories plus lumpy fixed capital investment. A paper related to ours is Fiori (2012), which also features lumpy capital adjustment in a two-sector model, but the focus there is on movements of the relative price of investment, which in our set up is constant by assumption. But we stay admittedly behind the research frontier in some other dimensions for reasons of computational tractability. For instance, unlike Khan and Thomas (2008) and Bachmann et al. (2013) we abstract from persistent idiosyncratic productivity shocks at the firm level. Micro heterogeneity is exclusively generated, just as in Khan and Thomas (2003), Gourio and Kashyap (2007) and Khan and Thomas (2007) by stochastic and ex-post different adjustment cost draws for both intermediate and final goods firms.
Figure 1: Impulse Response Function of Fixed Investment

Notes: This figure shows the impulse response functions of fixed capital investment to a one standard deviation aggregate productivity shock in the intermediate goods sector. ‘Model I1’ has the baseline calibrated nonconvex fixed capital adjustment cost parameter and the baseline calibrated inventory order cost parameter. ‘Model I2’ has zero nonconvex fixed capital adjustment cost and the baseline inventory order cost parameter. ‘Model NI1’ has the baseline calibrated nonconvex fixed capital adjustment cost parameter and zero inventories. ‘Model NI2’ has zero nonconvex fixed capital adjustment cost and zero inventories. The difference between the IRFs of ‘Model I2’ and ‘Model I1’ is the effect of nonconvex fixed capital adjustment costs in the presence inventories. The difference between the IRFs of ‘Model NI2’ and ‘Model NI1’ is the effect of nonconvex fixed capital adjustment costs without inventories. There is no need to recalibrate the fixed capital adjustment cost parameter or the inventory order cost parameter, as our calibration targets, being long-run targets, are not sensitive across model specifications.
the higher demand for consumption transfer into the future can be partially satisfied by inventories, which are now relatively cheap to produce. And this is done the more so, the higher the nonconvex fixed capital adjustment is, i.e., the more costly the usage of fixed capital is: 10.01% impact response versus 9.01% impact response in the frictionless fixed capital adjustment model, yet 8.10% impact response versus 6.02% impact response in the model with the baseline calibrated nonconvex fixed capital adjustment cost parameter.

Another direct implication of our mechanism is, as we will show, that the households’ ability to smooth consumption is enhanced when there are both inventories and fixed capital. In the end, inventories partially offset the hindering effect on consumption smoothing introduced by fixed capital adjustment frictions. As we will show, the impulse response functions of consumption to an aggregate productivity shock from the lumpy investment model and the frictionless adjustment model are very similar when inventories exist. Similarly, the volatility and persistence of aggregate consumption are much less sensitive to fixed capital adjustment frictions in models with inventories.

It is important to reiterate that the particular physical environment we chose – nonconvex capital adjustment costs as the friction and inventories as a way to modify the aggregate resource constraint\(^5\) – are not as important as the general insight here: when aggregate resource constraints and general equilibrium effects are important for aggregate dynamics, the precise details of how these general equilibrium effects are introduced into the physical environment, the precise details of how the model is closed matter. In the words of Caballero (2010): “But instead, the current core approach of macroeconomics preserves many of the original convenience-assumptions from the research on the periphery\(^6\) and then obsesses with closing the model by adding artificial factor supply constraints (note that the emphasis is on the word artificial, not on the word constraints).” This paper provides a quantitative analysis of the effects of closing the model in different ways for a specific, but prominent example. Put differently, unlike Khan and Thomas (2008) and Bachmann et al. (2013), who use the standard formulation for the aggregate resource constraint, this is not mainly a paper about the link between microfrictions and aggregate dynamics per se, but rather a paper about how this link is impacted by the formulation of the general equilibrium part of the model, i.e., a cross effect.

The rest of the paper proceeds as follows. Section 2 outlines the model. Section 3 discusses the calibration and model solution. Section 4 presents the results. Section 5 concludes.

2 The Model

2.1 The Environment

There are three kinds of agents in the economy: final goods producers, intermediate goods producers and households. The final goods producers use the intermediate goods, of which

\(^5\)We conjecture that we could have used other ways of breaking the tight consumption-investment link in the standard model or used other functional form specifications for capital adjustment costs and gained similar insights.

\(^6\)Caballero’s terminology for the first, decision theoretic part of macro models.
they hold inventories in equilibrium, and labor to produce the final goods.\footnote{To be clear on terminology: inventories in this model are not a capital good in the sense that they enter directly a production function, as in some modeling approaches in the literature. Thus, in our model, they lack the dual role of fixed capital. But they are a capital good in the sense that they represent a means of transferring consumption into the future, just like fixed capital. In this sense, we follow the NIPA terminology and denote net inventory changes as investment and the corresponding stock variables as capital.} Final output can be either consumed or invested as fixed capital. The intermediate goods producers combine fixed capital and labor to produce the intermediate goods. Households consume final goods and provide homogeneous labor to both types of producers. They own all the firms. They receive wage and dividend payments from both types of firms and purchase their consumption goods from the final goods producers. All markets are competitive.

2.1.1 The Final Goods Producers

There is a continuum of final goods producers. They use intermediate goods, \(m\), and labor, \(n\), to produce the final output through a production function \(G(m, n)\). The production function is strictly concave and has decreasing returns to scale. Whenever the final goods producers purchase intermediate goods, they face a fixed cost of ordering and delivery, denoted in units of labor, \(\epsilon\). To avoid incurring the fixed cost frequently, the final good producers optimally hold a stock of inventories of the intermediate goods. Denote the inventory level for an individual producer as \(s \in \mathbb{R}_+\).

The final goods producers differ in their fixed cost parameter for ordering, \(\epsilon \in [0, \epsilon]\). In each period, this parameter is drawn independently for every firm from a time invariant distribution \(H(\epsilon)\). At the beginning of the period, a typical final firm starts with its stock of inventories, \(s\), inherited from the previous period. It also learns its fixed cost parameter, \(\epsilon\). The firm decides whether to order intermediate goods. If the firm does so, it pays the fixed cost and chooses a new inventory level. Otherwise, the firm enters the production phase with the inherited intermediate goods inventory level \(s\). We denote the quantity of adjustment by \(x_m\). The inventory stock ready for production is \(s_1 = s + x_m\), with \(x_m = 0\) if the firm does not adjust.

After the inventory decision the firm determines its labor input, \(n\), and the intermediate goods input, \(m \in [0, s_1]\), for current production. Intermediate goods are used up in production. The remaining stock of intermediate goods, \(s' = s_1 - m \geq 0\), is the starting stock of inventories for the next period. Stored inventories incur a unit cost of \(\sigma\), denoted in units of final output. Inventory holding costs capture the idea that the storage technology that is used to partially circumvent the costly shipping technology is not free. Inventories require storage places, management and can lead to destruction of intermediate goods. The inventory management of the final good firms balances the trade-offs between costly shipping and costly storing optimally. In the end, the output of a typical final firm is \(y = G(m, n) - \sigma s'\).

2.1.2 Intermediate Goods Producers

There is a continuum of intermediate goods producers. They are subject to an aggregate productivity shock, which is the sole source of aggregate uncertainty.\footnote{As pointed out in Khan and Thomas (2007), placing aggregate productivity in the intermediate sector is necessary in this physical environment to generate a countercyclical relative price of intermediate goods, a} Let \(z\) denote the
aggregate productivity level. It follows a Markov chain, \( z \in \{ z_1, \ldots, z_N \} \), where \( P(z' = z_j | z = z_i) = \pi_{ij} \geq 0 \) and \( \sum_{j=1}^{N_z} \pi_{ij} = 1 \) for all \( i \).

Each firm produces with fixed capital and labor. Whenever the firm decides to adjust its capital stock, it has to pay a fixed cost, denoted in units of labor. In each period, the cost of adjusting capital is drawn independently for every firm from a time invariant distribution \( I(\zeta) \). A typical intermediate good producer is identified by its capital stock, \( k \), and its cost of adjusting capital, \( \zeta \in [0, \bar{\zeta}] \).

At the beginning of each period, the firm learns aggregate productivity, \( z \), and its idiosyncratic cost of adjusting capital, \( \zeta \). It starts with a fixed capital stock, \( k \), inherited from the previous period. First, it decides about the labor input, \( l \). It combines \( l \) and \( k \) according to a production function \( zF(k, l) \). The \( F(\cdot) \) function is strictly concave and has decreasing returns to scale.\(^9\) After production, the firm chooses whether to adjust its capital stock. It can pay a fixed cost to adjust its capital stock by investing \( i \). In this case, the new capital stock for the next period in efficiency units is \( k' = [(1 - \delta)k + i]/\gamma \), where \( \delta \) is the depreciation rate and \( \gamma \) is the steady state growth rate of the economy. Alternatively, the firm can avoid the adjustment cost and start the next period with the depreciated capital stock \( k' = (1 - \delta)k/\gamma \).

2.1.3 Households

We assume a continuum of identical households who value consumption and leisure. They have access to a complete set of state-contingent claims. Households own all the firms. They provide labor to the firms and receive wage and dividend payments.

The households have the following felicity function:

\[
u(c, n^h) = \log c - A^h n^h,
\]

where \( n^h \) is the total hours devoted to market work.

2.2 Competitive Equilibrium

2.2.1 Aggregate State Variables

In addition to \( z \), the aggregate productivity level, two endogenously determined distributions are aggregate state variables in this model: the distribution of the firm-specific inventory stocks, \( \mu(S) \), and the distribution of firm-specific fixed capital stocks, \( \lambda(K) \). Both \( S \) and \( K \) are subsets of a Borel algebra over \( \mathbb{R}_+ \).

The aggregate state variables are summarized as \((z, A)\), where \( A = (\mu, \lambda) \). The distribution of \( \mu \) evolves according to a law of motion \( \mu' = \Gamma_\mu(z, A) \), and similarly, the distribution of \( \lambda \) evolves according to \( \lambda' = \Gamma_\lambda(z, A) \).

\(^8\) As Miao and Wang (2011) show, fixed adjustment costs cannot be expected to have a large impact with constant return to scale. We follow the majority of the literature, e.g., Bachmann et al. (2013), Bloom (2009), Gourio and Kashyap (2007) as well as Cooper and Haltiwanger (2006), and use a decreasing returns to scale assumption.

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The final good is the numeraire. Workers are paid $\omega(z, A)$ per unit of labor input. The intermediate goods are traded at $q(z, A)$ per unit.

### 2.2.2 Problem of the Household

The households receive a total dividend payment $D(z, A)$ and labor income $n^h(z, A)\omega(z, A)$ from the firms. In each period the households determine how much to work and how much to consume. All we need from the household problem is an intertemporal and an intratemporal first order condition.

We can express the dynamic programming problems for both types of firms with the marginal utility of consumption as the pricing kernel:

$$p(z, A) = \frac{1}{c(z, A)}.$$ 

Then every firm weighs its current profit by this pricing kernel and discounts its future expected earnings by $\beta$. This changes the unit of the firm’s problems in both sectors to utils but leaves the policy functions unchanged.

The first-order conditions also imply that the real wage is given by:

$$\omega(z, A) = \frac{A^h}{p(z, A)}.$$

### 2.2.3 Problem of Final Goods Producers

Let $V_0$ be the value, in utils, of a final goods producer at the beginning of a period after the inventory adjustment cost parameter is realized and before any inventory adjustment and production decisions. Let $V_1$ be the expected value function after the adjustment decision but before the production decision. Given the aggregate laws of motion $\Gamma_\mu$ and $\Gamma_\lambda$, the firm’s problem is characterized by the following three equations. For expositional ease, the arguments for functions other than the value functions are omitted.

$$V_0(s, \epsilon; z, A) = pq s + \max\left\{ -p\omega \epsilon + V_a(z, A), -pq s + V_1(s; z, A) \right\},$$

$$V_a(z, A) = \max_{s_1 > 0} \{-pq s_1 + V_1(s_1; z, A)\},$$

and:

$$V_1(s_1; z, A) = \max_{n \geq 0, s_1 \geq s' \geq 0} \left\{ p[G(s_1 - s', n) - \sigma s' - \omega n] + \beta E_z \left[ \int_0^\tau V_0(s', \epsilon; z', A')d(H(\epsilon)) \right] \right\}.$$ 

The expectation is taken over $z'$, next period’s aggregate productivity.

Equation (1) describes the binary inventory adjustment decision of the firm. The firm adjusts if the value of entering the production phase with the optimally adjusted inventory
level, described by $V_a(\cdot)$ in equation (2), minus the cost of adjustment, exceeds the value of directly entering the production phase with the inherited inventory level, $V_1(s; z, A)$.

The solution to equation (1) amounts to a cut-off rule in $\epsilon$. The firm adjusts if:

$$-p\omega \epsilon + V_a(z, A) \geq -pqs + V_1(s; z, A).$$

Therefore the cut-off value is:

$$\bar{\epsilon}(s; z, A) = \frac{V_a(z, A) - V_1(s; z, A) + pqs}{p\omega}.$$

Given the support of the adjustment cost distribution, this cut-off value is modified to:

$$\epsilon^* = \max(0, \min(\tau, \bar{\epsilon})).$$

The firm adjusts if its draw is smaller than or equal to $\epsilon^*(s; z, A)$.

Equation (2) describes the value of inventory adjustment. The solution to this equation is the optimal target level of inventory, $s^*_1(s, \epsilon; z, A)$. Note that the optimization problem, which is formulated in terms of the stock of inventories, $s$, instead of order flows, does not depend on any firm-specific characteristics. Therefore in any period, all the adjusting firms choose the same inventory target level, $s^*_1(z, A)$.

Equation (1) and (2) jointly determine the production-time inventory level, $s_1$:

$$s_1(s, \epsilon; z, A) = \begin{cases} s^*_1(z, A) & \text{if } \epsilon \leq \epsilon^*(s; z, A) \\ s & \text{if } \epsilon > \epsilon^*(s; z, A) \end{cases}.$$

Equation (3) describes the production phase. The firm finds the optimal inventory level for the next period and the optimal employment level for this period. The decision for next period’s inventory level, $s'$, is equivalent to deciding about the amount of intermediate goods to be used up in current production.

The solution for employment does not depend on the continuation value function. Therefore, given $s'$, it is the analytical solution to:

$$\frac{\partial G(s_1 - s', n^*)}{\partial n} = \omega.$$

The optimal employment and inventory usage decision jointly imply the optimal output level:

$$y^*(s_1; z, A) = G(s_1 - s'^*(s_1; z, A), n^*(s_1; z, A)) - \sigma s'^*(s_1; z, A).$$

### 2.2.4 Problem of the Intermediate Goods Producers

Let $W_0$ be the value, in utils, of the intermediate good producers prior to the realization of the adjustment cost parameter $\zeta$. Let $W_1$ be the value function after the realization of $\zeta$. 


The intermediate good producer’s problem can be summarized by the following equation:

\[
W_1(k, \zeta; z, A) = \max_l \left\{ p \cdot [q \cdot z F(k, l) - l \omega] + \max \{W_i(k; z, A), -p \zeta \omega + W_a(k; z, A)\} \right\},
\]

(4)

where:

\[
W_a(k; z, A) = \max_{k'}\{-\gamma k' - (1 - \delta)k \} p + \beta E_z [W_0((k'; z', A'))],
\]

(5)

\[
W_i(k; z, A) = \beta E_z [W_0((1 - \delta)k/\gamma; z', A')],
\]

(6)

\[
W_0(k; z, A) = \int_0^\zeta W_1(k, \zeta; z, A) d(I(\zeta)).
\]

(7)

The expectation in equation (5) and (6) is taken over \(z'\), next period’s aggregate productivity.

In equation (4), the firm first solves for the optimal employment, given the fixed capital stock. The solution is:

\[
\frac{\partial q_z F(k, l^*)}{\partial l} = \omega.
\]

After the production decision, the firm solves the binary fixed capital adjustment decision. The firm adjusts if the expected value from the optimally adjusted fixed capital stock, given in equation (5), minus the cost of adjustment, exceeds the expected value from the unadjusted fixed capital stock, given in equation (6).

The solution to the adjustment decision follows a cut-off rule for \(\zeta\). The firm adjusts if:

\[-p \omega \zeta + W_a(k; z, A) \geq W_i(k; z, A).\]

Therefore the cut-off value for \(\zeta\) is:

\[
\zeta(k; z, A) = \frac{W_a(k; z, A) - W_i(k; z, A)}{p \omega}.
\]

The restriction from the support of the cost distribution applies, so that

\[
\zeta^* = \max(0, \min(\zeta, \zeta^*)).
\]

The firm adjusts to the target capital stock if its adjustment cost is smaller than or equal to \(\zeta^*(k; z, A)\).

The optimal adjustment target for fixed capital is given by the solution to equation (5). Although the value function depends on the level of individual capital stocks, the resulting policy function, \(k^*\), does not. After the binary adjustment decision, the capital stock for the next period is:

\[
k'(k; z, A) = \begin{cases} k^*(z, A) & \text{if } \zeta \leq \zeta^*(k; z, A) \\ (1 - \delta)k/\gamma & \text{if } \zeta > \zeta^*(k; z, A) \end{cases}.
\]
2.2.5 Recursive Equilibrium

A recursive competitive equilibrium for the economy defined by:

\[ \{ u(c, n^h), \beta, F(k, l), G(m, n), \sigma, \delta, \gamma, H(\epsilon), I(\zeta), z \}, \]

is a set of functions:

\[ \{ V_0, V_1, W_0, W_1, x_m, n, s', k', l, i, c, n^h, p, q, \omega, D, \Gamma_\mu, \Gamma_\lambda \}, \]

such that:

1. Given \( \omega, q, p, \Gamma_\mu \) and \( \Gamma_\lambda \), \( V_0 \) and \( V_1 \) solve the final firm’s problem.
2. Given \( \omega, q, p, \Gamma_\mu \) and \( \Gamma_\lambda \), \( W_0 \) and \( W_1 \) solve the intermediate firm’s problem.
3. Given \( \omega, D \) and \( p, c \) satisfies the household’s first-order conditions.
4. The final goods market clears:
   \[
   c(z, A) = \int_S \int_0^{\tau} y(s, \epsilon; z, A)d(H(\epsilon))d(\mu(s))
   \]
   \[
   - \int_K \int_0^{\zeta} i(k, \zeta; z, A)d(I(\zeta))d(\lambda(k)).
   \]
5. The intermediate goods market clears:
   \[
   \int_S \int_0^{\tau} x_m(s, \epsilon; z, A)d(H(\epsilon))d(\mu(s)) =
   \int_K \int_0^{\zeta} zF(k, n(k, \zeta; z, A))d(I(\zeta))d(\lambda(k)).
   \]
6. The labor market clears:
   \[
   n^h(z, A) = \int_S \int_0^{\tau} (n(s; z, A) + \epsilon \cdot 1(x_m(s, \epsilon; z, A) \neq 0)) d(H(\epsilon))d(\mu(s))
   \]
   \[
   + \int_K \int_0^{\zeta} (l(k, n(k; z, A)) + \zeta \cdot 1(i(k, \zeta; z, A) \neq 0))d(I(\zeta))d(\lambda(k)).
   \]
7. The laws of motion for aggregate state variables are consistent with individual decisions and the stochastic processes governing \( z \):
   (a) \( \Gamma_\mu(z, A) \) defined by \( s'(s, \epsilon; z, A) \) and \( H(\epsilon) \);
   (b) \( \Gamma_\lambda(z, A) \) defined by \( k'(k, \zeta; z, A) \) and \( I(\zeta) \).
### 2.2.6 Some Terminology

Final Sales (FS), is defined as the total output of the final goods sector. Intermediate goods demand, X, is the total amount of intermediate goods purchased by the final goods sector. Intermediate goods usage, M, is the total amount of intermediate goods used up in production by the final goods sector. The difference between the two evaluated at the relative price of intermediate goods is Net Inventory Investment (NII):

\[ \text{NII} = \frac{q}{\text{price of intermediate goods}} \times (X - M). \]

Finally, Gross Domestic Product (GDP) in this physical environment is defined as the sum of final sales and net inventory investment:

\[ \text{GDP} = \text{FS} + \text{NII}. \]

### 3 Calibration and Computation

#### 3.1 Baseline Parameters

The model period is a quarter. We choose the following functional forms for the production functions:

\[
F(k, l) = k^{\theta_k} l^{\theta_l}, \\
G(m, n) = m^{\theta_m} n^{\theta_n}. 
\]

We discretize the productivity process \( z \) into \( N_z = 11 \) points following Tauchen (1986). The underlying continuous productivity process follows an AR(1) in logarithms with autocorrelation \( \rho_z = 0.956 \) and an innovation process with standard deviation \( \sigma_z = 0.015 \).

We set the subjective discount factor, \( \beta = 0.984 \), the depreciation rate \( \delta = 0.017 \), and the steady state growth factor \( \gamma = 1.004 \). \( A^h \) is calibrated so that the aggregate labor input equals 0.33. \( \theta_m = 0.499 \) is calibrated to match the share of intermediate inputs in final output. We set \( \theta_k = 0.25 \) and \( \theta_l = 0.5 \), the values used in Bloom (2009), which amounts to a capital elasticity of the firms’ revenue function of 0.5\(^{10} \). We calibrate \( \theta_n \) to match an aggregate labor share of 0.64. These parameters are summarized in Table 1:

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( A^h )</th>
<th>( \theta_m )</th>
<th>( \theta_n )</th>
<th>( \theta_k )</th>
<th>( \theta_l )</th>
<th>( \rho_z )</th>
<th>( \sigma_z )</th>
<th>( \delta )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.984</td>
<td>2.128</td>
<td>0.499</td>
<td>0.367</td>
<td>0.250</td>
<td>0.500</td>
<td>0.956</td>
<td>0.015</td>
<td>0.017</td>
<td>1.004</td>
</tr>
</tbody>
</table>

Notes: \( \beta \) is the subjective discount factor of the households; \( A^h \) is the preference parameter for leisure; \( \theta_m \) is the material share in the final good production function; \( \theta_n \) is the labor share in the final good production function; \( \theta_k \) is the capital share in the intermediate good production function; \( \theta_l \) is the labor share in the intermediate good production function; \( \rho_z \) is the auto-correlation for the aggregate productivity process; \( \sigma_z \) is the standard deviation for aggregate productivity innovations; \( \delta \) is the depreciation rate; \( \gamma \) is the steady state growth rate.

\(^{10}\)Cooper and Haltiwanger (2006), using LRD manufacturing data, estimate this parameter to be 0.592; Hennessy and Whited (2005), using Compustat data, find 0.551.
3.2 Inventory and Adjustment Cost Parameters

We assume that the inventory adjustment costs are uniformly distributed on $[0, \tau]$. $\tau$ is set so that the average inventory-to-sales ratio in the model equals 0.8185, the average of the real private non-farm inventory-to-sales ratio in the United States between 1960:1 and 2006:4. The unit cost of holding inventories, $\sigma$, is chosen so that the annual storage cost for all inventories is 12% of aggregate final output in value (see Richardson (1995) for details). These two targets jointly determine $\tau = 0.3900$ and $\sigma = 0.0127$.

We assume that $I(\zeta)$ is uniform between $[0, \zeta]$. The upper bound of the distribution is chosen so that the fraction of lumpy investors, defined as the firms whose gross investment rate is larger than 20% in a given year, is 18%. This calibration target is taken from Cooper and Haltiwanger (2006)'s analysis of manufacturing firms in the Longitudinal Research Database (LRD). This yields $\zeta = 0.1841$.

3.3 Numerical Solution

The inherent non-linearity of the model suggests global numerical solution methods. We use value function iterations from equation (1) to equation (3) to solve the problem of the final good producers. We use value function iterations from equation (4) to equation (7) to solve the intermediate firm’s problem. Howard policy function accelerations are used to speed up convergence.

Our model gives rise to two endogenous distributions as state variables. We adopt the methods in Krusell and Smith (1997), Krusell and Smith (1998), Khan and Thomas (2003) as well as Khan and Thomas (2008) to compute the equilibrium. Denote the $I$th moment of distribution $\mu(S)$ and $\lambda(K)$ as $\mu_I(S)$ and $\lambda_I(K)$ respectively. We approximate each distribution function with its first moment. We find that a log-linear form for the $\Gamma(\cdot)$ functions approximates the law of motion rather well in terms of forecasting accuracy:

$$\Gamma_\mu(z, \lambda_1, \mu_1) = \log \mu_1' = \alpha_\mu + \beta_\mu \log(\lambda_1) + \gamma_\mu \log(\mu_1) + \psi_\mu \log(z),$$

$$\Gamma_\lambda(z, \lambda_1, \mu_1) = \log \lambda_1' = \alpha_\lambda + \beta_\lambda \log(\lambda_1) + \gamma_\lambda \log(\mu_1) + \psi_\lambda \log(z).$$

We adopt similar rules for the pricing kernel and the relative price of intermediate goods:

$$\log p = \alpha_p + \beta_p \log(\lambda_1) + \gamma_p \log(\mu_1) + \psi_p \log(z),$$

$$\log q = \alpha_q + \beta_q \log(\lambda_1) + \gamma_q \log(\mu_1) + \psi_q \log(z).$$

11 It should be clear that the exact numbers for $\tau$ and $\zeta$ have little direct economic meaning and cannot be compared to other calibrations for these parameters in the literature. They are essentially free parameters to hit observable calibration targets (which are what is common across papers), such as the inventory-to-sales ratio and the fraction of firms that are lumpy investors. They will also lead to additional interpretable economic statistics like the average adjustment cost paid conditional on adjustment that we display below in Table 2. The precise values of these parameters are sensitive to the entire model environment and its calibration.

12 We have experimented with other functional forms for the forecasting rules such as adding interaction terms between aggregate productivity and the capital and inventory moments. This did not lead to significant improvements in goodness-of-fit and often jeopardized numerical stability. Our specifications perform very well as measured by the $R^2$ of the equilibrium OLS regressions, which exceeds 0.9996 in all specifications.
where $\lambda_1$ is the first moment of the capital stock distribution, and $\mu_1$ is the first moment of the inventory stock distribution.

Given an initial guess for $\{\alpha_{(\cdot)}, \beta_{(\cdot)}, \gamma_{(\cdot)}, \psi_{(\cdot)}\}$, we solve the value functions as described above. Then we simulate the model without imposing the pricing rules in equations (10) and (11). In each model simulation period we search for a pair of prices, $(p, q)$ such that all the firms optimize and all the markets clear under the forecasting rules in equation (8) and (9). To improve numerical accuracy, we use the value functions to re-solve all the optimization problems period by period and for every guess of $(p, q)$. Given the market clearing prices, we update the capital and inventory stock distributions and proceed into the next period.

At the end of the simulation, we update the parameters $\{\alpha_{(\cdot)}, \beta_{(\cdot)}, \gamma_{(\cdot)}, \psi_{(\cdot)}\}$ using the simulated time series for the approximating moments and the market clearing prices. Then we repeat the algorithm with the updated parameters. Upon convergence of the parameters, we check the accuracy of the $\Gamma(\cdot)$ functions by the $R^2$ in the regression stage.

4 Results

We study the influence of nonconvex fixed capital adjustment costs on aggregate dynamics in our model by numerical simulation. We analyze four models that share all parameters other than $\tau$ and $\zeta$. ‘Model I1’ and ‘Model I2’ have the calibrated baseline equilibrium inventory holdings with $\bar{\tau} = 0.39$. ‘Model I1’ has calibrated fixed capital adjustment cost given by $\bar{\zeta} = 0.1841$, while ‘Model I2’ features a frictionless technology for adjusting the fixed capital stock. We also simulate two models without inventories, ‘Model NI1’ and ‘Model NI2’. In these models, we set $\bar{\tau} = 0$ to eliminate equilibrium inventory holdings. ‘Model NI1’ has the same level of $\bar{\zeta}$ as ‘Model I1’, while ‘Model NI2’ does not feature any frictions in adjusting the fixed capital stock. The parameter specifications for the four models are summarized in Table 2. We do not recalibrate $\bar{\zeta}$ in ‘Model NI1’ as the calibration targets are largely insensitive to the changes in equilibrium inventory levels, as shown in the fourth column of Table 2. To understand how the presence of inventories interacts with the effects of nonconvex fixed adjustment costs, we study the cross differences. That is, we contrast the differences between ‘Model I1’ and ‘Model I2’ with the differences between ‘Model NI1’ and ‘Model NI2’.

We present four sets of results on those four models. We first compare their unconditional business cycle moments. Second, we study the impulse response functions for fixed capital investment and consumption across the four models. Third, we plot the volatility and persistence for consumption, fixed capital investment and, for the models with inventories, net inventory investment for a wider range of $\bar{\zeta}$. And finally, we analyze the role of general equilibrium price movements in bringing about these results.

\footnote{In theory, zero ordering costs are not inconsistent with positive inventory holdings as the firms might want to hedge against changes in the relative price of intermediate goods. However, in our simulations, given the inventory holding costs, no firm holds a positive level of inventories when $\tau = 0$.}
### Table 2: Model Specifications

<table>
<thead>
<tr>
<th>Model Name</th>
<th>ζ</th>
<th>ϵ</th>
<th>Average Adjustment Cost</th>
<th>Fraction of Lumpy Adjusters</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>0.1841</td>
<td>0.3900</td>
<td>0.9300%</td>
<td>18.00%</td>
<td>Baseline fixed capital adjustment cost with inventory</td>
</tr>
<tr>
<td>I2</td>
<td>0.0000</td>
<td>0.3900</td>
<td>0.0000%</td>
<td>0.000%</td>
<td>Frictionless fixed capital adjustment with inventory</td>
</tr>
<tr>
<td>NI1</td>
<td>0.1841</td>
<td>0.0000</td>
<td>0.8900%</td>
<td>18.18%</td>
<td>Baseline fixed capital adjustment cost without inventory</td>
</tr>
<tr>
<td>NI2</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000%</td>
<td>0.000%</td>
<td>Frictionless fixed capital adjustment without inventory</td>
</tr>
</tbody>
</table>

**Notes:** `Model I1` has the baseline calibrated nonconvex fixed capital adjustment cost parameter and the baseline calibrated inventory order cost parameter. `Model I2` has zero nonconvex fixed capital adjustment cost and the baseline inventory order cost parameter. `Model NI1` has the baseline calibrated nonconvex fixed capital adjustment cost parameter and zero inventories. `Model NI2` has zero nonconvex fixed capital adjustment cost and zero inventories. “Average Adjustment Cost” is the average adjustment cost paid as a fraction of firms’ output, conditional on adjustment. “Fraction of Lumpy Adjusters” is the share of lumpy adjusters, defined as the firms that adjust more than 20% of their initial capital stocks in a given year, in all firms.

### 4.1 Unconditional Business Cycle Analysis

After computing the equilibrium, we simulate the model for 1,000 periods, of which we discard the first 100 to eliminate the influence of initial conditions. Except for net inventory investment and fixed capital investment, all the simulated time series are transformed by natural logarithms and then detrended by an HP filter with smoothing parameter 1600. We detrend fixed capital investment with the HP filter directly and then divide the deviations by the trend. We divide net inventory investment by GDP and then apply the HP filter to this ratio.

### Table 3: Business Cycle Statistics

#### (a) Standard Deviation

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>Consumption</th>
<th>Fixed Investment</th>
<th>NII</th>
<th>Inventory Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model I1</td>
<td>1.4975</td>
<td>0.6416</td>
<td>9.6619</td>
<td>0.3793</td>
<td>1.2204</td>
</tr>
<tr>
<td>Model I2</td>
<td>1.5637</td>
<td>0.6336</td>
<td>11.5762</td>
<td>0.3240</td>
<td>1.1404</td>
</tr>
<tr>
<td>Model NI1</td>
<td>1.4772</td>
<td>0.7624</td>
<td>11.7371</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Model NI2</td>
<td>1.5694</td>
<td>0.7436</td>
<td>13.8684</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Data</td>
<td>1.6530</td>
<td>0.9015</td>
<td>4.8903</td>
<td>0.4220</td>
<td>1.6552</td>
</tr>
</tbody>
</table>

#### (b) First Order Auto-correlation

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>Consumption</th>
<th>Fixed Investment</th>
<th>NII</th>
<th>Inventory Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model I1</td>
<td>0.6833</td>
<td>0.7623</td>
<td>0.7298</td>
<td>0.6137</td>
<td>0.9259</td>
</tr>
<tr>
<td>Model I2</td>
<td>0.6646</td>
<td>0.7632</td>
<td>0.6110</td>
<td>0.6616</td>
<td>0.9379</td>
</tr>
<tr>
<td>Model NI1</td>
<td>0.6889</td>
<td>0.7281</td>
<td>0.6648</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Model NI2</td>
<td>0.6885</td>
<td>0.7739</td>
<td>0.6251</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Data</td>
<td>0.8422</td>
<td>0.8833</td>
<td>0.9006</td>
<td>0.3696</td>
<td>0.8908</td>
</tr>
</tbody>
</table>

**Notes:** “NII” denotes net inventory investment. GDP, consumption, and inventory levels are logged and detrended with an HP filter with a penalty parameter of 1600. We detrend fixed investment with the HP filter and then divide the deviations by the trend. We divide NII by GDP and then detrend this ratio with the HP filter. All the standard deviations reported in Panel (a) are percentage points. Time period for the data moments: 1960:1 - 2006:4.

The business cycle statistics in Panel (a) and (b) of Table 3 show several effects of inventories on aggregate dynamics.\(^{14}\) The first message is that nonconvex fixed capital adjustment technology that is consistent with the micro data able to do what stand-in adjustment technologies do, namely, dampen and propagate aggregate investment.

\(^{14}\)Bachmann et al. (2013) is explicitly about how nonconvex fixed capital adjustment costs shape the implied model investment dynamics in terms of higher moments than standard second moments. They argue that aggregate investment data exhibits conditional heteroskedasticity and that micro nonconvexities are a natural mechanism to explain this. This paper is about micro nonconvexities and their role in shaping standard second moments and impulse response functions. Basically, this paper asks: is the fixed capital adjustment technology that is consistent with the micro data able to do what stand-in adjustment technologies do, namely, dampen and propagate aggregate investment.
adjustment costs matter for aggregate dynamics. Business cycle dynamics differ significantly between ‘Model I1’ and ‘Model I2’. For example, the percentage standard deviation of fixed capital investment decreases from 11.58 in the frictionless ‘Model I2’ to 9.66 in the lumpy investment ‘Model I1’. Persistence of fixed capital investment increases from 0.61 to 0.73. In contrast, consumption volatility and persistence do not vary as much with the fixed capital adjustment cost parameter. Consumption dynamics are largely insulated from variations in capital adjustment frictions in the presence of inventories.\textsuperscript{15}

Regarding the cross differences, the effects of nonconvex fixed capital adjustment costs change significantly in models where inventories are absent. Most notably, the persistence of fixed investment only increases by 0.04 between ‘Model NI2’ and ‘Model NI1’, while it increases by 0.12 between ‘Model I2’ and ‘Model I1’. The unconditional volatility of consumption increases by 0.0188 percentage points between ‘Model NI2’ and ‘Model NI1’ while it only increases by 0.0080 percentage points between ‘Model I2’ and ‘Model I1’.\textsuperscript{16} These results suggest that inventories strengthen the dampening and propagation effect of fixed adjustment costs on fixed capital investments.\textsuperscript{17} At the same time, inventories enhance the households’ ability to smooth consumption, making fixed capital adjustment costs much less effective in affecting consumption volatility.

As for net inventory investment and the level of inventories, we see that they behave exactly the opposite way from fixed capital investment, when the latter is subject to adjustment frictions. Their volatility rises and their persistence falls, when capital adjustment frictions are introduced. This is due to the substitution towards inventories as a means of consumption smoothing, as fixed capital becomes more costly to use.

### 4.2 Conditional Business Cycle Analysis - Impulse Response Functions

The first two panels of Figure 2 show the impulse response functions of aggregate fixed capital investment and consumption to a positive productivity shock in the intermediate goods sector. We simulate a shock process that starts with one standard deviation above the median level of productivity, $z = 1$, and falls back to unity at the rate of $\rho_z = 0.956$.\textsuperscript{18}

\textsuperscript{15}The excessively high fixed investment volatility, as shown in the third column of Panel (a), is a common property of two-sector models where fixed capital is only used in intermediate goods production. Khan and Thomas (2007) find similar results. As fixed adjustment cost works to dampen investment volatility, this might point to our calibration of $\zeta$ being conservative, especially in light of the insights of Bachmann et al. (2013), who argue that focusing only on the fraction of lumpy investment episodes when calibrating nonconvex adjustment costs might lead to a downward biased estimate.

\textsuperscript{16}Notice that the unconditional volatility of fixed investment drops roughly by the same amount in the cases with inventories and without. However, the unconditional volatility numbers are of course a combination of the increase in persistence and a decrease in conditional volatility, as shown in the impulse response function in Figure 1, and in the case of fixed investment both effects happen to roughly offset each other. In general, unconditional volatility can sometimes hide the interaction between conditional volatility and persistence, which is why we focus on the latter two in what follows.

\textsuperscript{17}Note that already without inventories we have that nonconvex fixed capital adjustment costs matter somewhat for aggregate dynamics as, in line with the recent evidence in Bloom (2009) and Cooper and Haltiwanger (2006), our implied revenue elasticity of capital is closer to the calibration in Gourio and Kashyap (2007), where the substitution between the extensive and intensive margin of fixed capital investment is more difficult.
Figure 2: Impulse Response Functions

Notes: This figure shows the impulse response functions of fixed capital investment, consumption, net inventory investment (NII) over GDP and the relative price to a one standard deviation aggregate productivity shock in the intermediate goods sector. ‘Model I1’ has the baseline calibrated nonconvex fixed capital adjustment cost parameter and the baseline calibrated inventory level. ‘Model I2’ has zero nonconvex fixed capital adjustment cost and the baseline calibrated inventory level. ‘Model NI1’ has the baseline calibrated nonconvex fixed capital adjustment cost parameter and zero inventories. ‘Model NI2’ has zero nonconvex fixed capital adjustment cost and zero inventories. The impulse response of net inventory investment over GDP is reported in absolute values, instead of percentage points, as the steady state value of net inventory investment is zero.

Fixed Capital Investment  Panel(a) of Figure 2 presents the four impulse response functions for fixed capital investment. Comparing the models with $\zeta = 0.1841$ against the models with $\zeta = 0$ at the same level of inventories, we can see that nonconvex fixed capital adjustment costs dampen the initial responses both with and without inventories. However, at different levels of inventories, capital adjustment costs dampen these responses to a different degree. Without inventories, the initial response is dampened by 1.91 percentage points. In contrast, the initial response is dampened by 2.99 percentage points in models with inventories. Inventories also increase shock propagation. Comparing the impulse response function of ‘Model I1’ with that of ‘Model NI1’ without inventories, we see that the impulse response function in the model with inventories is flatter.
Both the extra dampening effect and the increased propagation of the shocks come from the key mechanism in our model: the substitution between fixed capital investment and inventory investment as a means of consumption smoothing. When adjusting fixed capital is costly, the economy switches to inventories. As a result, fixed capital investments do not need to respond to productivity shocks as much as when inventories are absent. The responses are also more protracted because firms tend to wait for lower adjustment cost draws to invest.

The flip side of the substitution between the two investment means can be observed in Panel(c) of Figure 2, which shows the impulse response functions of net inventory investment (over GDP). As expected, the response of net inventory investment is stronger when adjusting fixed capital investment is costly. In ‘Model I1’, the impact response is roughly 0.003, while in ‘Model I2’ it is only 0.002.\textsuperscript{18}

The same mechanism can also explain the other cross effect, namely, how lumpy fixed capital investment changes the effect of inventories on aggregate investment dynamics. For both levels of fixed capital capital adjustment costs, inventories dampen the positive response of fixed capital investment to a positive productivity shock, as the latter is no longer used as much to ensure consumption smoothing. This switching away from fixed capital investment as a means of transferring consumption into the future is stronger, the more costly it is to use, i.e., when fixed capital adjustment frictions are present. This explains why inventories dampen the initial response of fixed capital investment by somewhat over 2 percentage points with fixed capital adjustment frictions, but only by 1 percentage point, when fixed capital can be freely adjusted.

Consumption Another implication from the above mechanism is that consumers’ ability to smooth consumption is enhanced by inventories. We illustrate this with the impulse response functions for consumption in Panel(b) of Figure 2.

First, the impact response from the models with inventories is below those from the models without inventories, for every level of fixed capital adjustment costs. Secondly, the smoothing effectiveness of inventories is so good that consumers despite the presence of capital adjustment costs can almost exactly recreate their frictionless consumption path. Nonconvex fixed capital adjustment costs barely change the response of consumption after the initial impact, when there are inventories. In contrast, without inventories nonconvex fixed capital adjustment costs do interfere with consumption smoothing.

We interpret these response functions as evidence that inventories provide an effective smoothing device for the consumers. As a result, consumption dynamics are less volatile when productivity shocks hit and capital adjustment frictions are less relevant for consumption dynamics in the presence of inventories.

4.3 Conditional Volatility and Persistence as a Function of Capital Adjustment Costs

In this section we illustrate the substitution mechanism between the two investment goods from a slightly different angle. We now simulate our model under our calibrated inventory

\textsuperscript{18}The impulse responses for NII are reported in absolute changes as a fraction of GDP, not in percentage changes relative to the steady state. This is because the steady state value for NII is zero.
Figure 3: Conditional Volatility and Persistence of Fixed Capital Investment

Notes: This figure shows the impact response to an aggregate technology shock and the first-order autocorrelation coefficient of fixed capital investment for models with \( \bar{\zeta} \in [0, 0.4] \). The x-axis for both panels shows the upper bound of the capital adjustment cost distribution, \( \bar{\zeta} \). In Panel (a), the y-axis shows the first element of the IRF of fixed capital investment to a one-standard deviation aggregate technology shock in percentage points. In Panel (b), the y-axis shows the first-order autocorrelation of fixed capital investment. For Panel (b) we detrend fixed capital investment with the HP(1600) filter and then divide the deviations by the trend.

We study how the conditional volatility, i.e., the impact response in the impulse-response function, and the persistence of fixed capital investment, consumption and net inventory investment change over this range of fixed capital adjustment costs. Panel (a) of Figure 3 presents the conditional volatility of fixed capital investment over said \( \bar{\zeta} \)-range for both the inventory model and the “No Inventory” model. Independently of the level of inventories, higher capital adjustment costs dampen the impact response of fixed capital investment to aggregate shocks, and they do this in a more pronounced way in the model with inventories. The interaction between inventories and nonconvex capital adjustment costs is also apparent in the behavior of the persistence of fixed capital investment in Panel (b) of Figure 3. With inventories, persistence increases from 0.61 to 0.74 when \( \bar{\zeta} \) changes from 0 to 0.4. In contrast, without inventories persistence only increases from 0.62 to 0.67 over the same range of \( \bar{\zeta} \). The agents rely less on fixed capital investment when inventories are available. As a result, the fluctuations in fixed capital investments are dampened and stretched. It is important to emphasize again that the central message of the paper lies in the different slopes of the two lines in both panels of Figure 3, which is precisely a graphical representation of the nontrivial cross effect between general equilibrium modeling and the impact of adjustment costs for fixed capital on aggregate statistics - conditional volatility and persistence.

We can directly observe the substitution between different investment channels by contrasting the conditional volatility of fixed capital investment in Figure 3 to the conditional volatility level and the “No Inventory” setup over a wide range of \( \bar{\zeta} \in [0, 0.4] \). The lower bound is frictionless adjustment, whereas the upper bound, 0.4, is approximately twice our baseline \( \bar{\zeta} = 0.1841 \). We study how the conditional volatility, i.e., the impact response in the impulse-response function, and the persistence of fixed capital investment, consumption and net inventory investment change over this range of fixed capital adjustment costs.

\footnote{At \( \bar{\zeta} = 0.4 \) the annual fraction of firms which have lumpy investments is 15.23\%, and the annual average adjustment cost paid conditional on adjustment and measured as a fraction of the firm’s output is 1.66\%.}
Figure 4: Conditional Volatility and Persistence of Net Inventory Investment

![Graphs showing volatility and persistence](image)

Notes: See notes to Figure 3. This figure shows the impact response to an aggregate technology shock and the first-order autocorrelation coefficient of net inventory investment (NII) divided by GDP for models with $\zeta \in [0, 0.4]$.

Volatility of net inventory investment in Panel (a) of Figure 4. As fixed adjustment costs increase, the agents rely more on inventories and less on fixed capital for consumption smoothing. As a result, higher fixed adjustment costs lead to more volatile net inventory investment and less volatile fixed capital investment. Panel (b) of Figure 4 shows the opposite, albeit with a small nonmonotonicity, effect on persistence of net inventory investment.

Also, we can see the implications of the investment substitution mechanism in the dynamics of consumption. Figure 5 shows that with inventories the conditional volatility of consumption is lower for every level of capital adjustment costs. More importantly, as the slopes of the two curves suggest, the rate at which fixed adjustment costs increases conditional consumption volatility is lower when inventories exist. In other words, the same increase in fixed adjustment cost makes conditional consumption volatility move up higher when inventories are absent from the economy, whereas it can barely increase conditional consumption volatility when inventories are present.

The change in consumption persistence reveals the same mechanism, as shown in Panel (b) of Figure 5. The existence of inventories changes the degree to which fixed capital adjustment costs affect consumption persistence. Over the same range of $\zeta$, consumption persistence decreases by much less in the inventory models compared to the “No Inventory” models.

4.4 The Effect of Market Clearing

The results on the effectiveness of fixed capital adjustment costs with or without inventories so far take into account all general equilibrium (GE) effects, i.e., adjustments of real interest rates and real wages, as well as the relative price of intermediate goods. In this section we isolate the effects of these price movements on how inventories impact the (ir)relevance of nonconvex fixed capital adjustment costs.
Figure 5: Conditional Volatility and Persistence of Consumption

Notes: See notes to Figure 3. This figure shows the impact response to an aggregate technology shock and the first-order autocorrelation coefficient of consumption for models with $\zeta \in [0, 0.4]$. For Panel(b) consumption is logged and detrended with an HP filter with a smoothing parameter of 1600.

To this end, we solve three partial equilibrium versions of our model. In the first case, we fix both the pricing kernel, $p$, and the relative price $q$, at their long-run general equilibrium averages and simulate the model. In the second case, we fix the pricing kernel (and thus the real wage) to its long-run general equilibrium average, but allow the relative price to adjust so that the intermediate goods market clears. In the last case we fix the relative price to its long-run general equilibrium average, but allow the pricing kernel (and the real wage) to adjust so that the final goods market clears.

The impulse response functions of fixed capital investment for all three cases are reported next to the full general equilibrium case – Panel (a) – in Figure 6. Panel(b) is the response from the first partial equilibrium case where both prices are fixed. Two messages emerge from this case. First, as is well known in the literature, nonconvex adjustment frictions matter a lot in partial equilibrium: the impact response drops substantively, and propagation arises only when fixed adjustment frictions are introduced. Second, inventories by and large do not change the effect of fixed adjustment frictions, as the differences between Model I1 and I2 are very similar to the differences between Model NI1 and NI2. Put differently, the effect of fixed capital adjustment frictions swamps the differential effect of inventories.

Panel(c) presents the response functions from the models where the pricing kernel is fixed but the relative price is not. The results in these models are very similar to those in the first case where both prices are fixed. Once again, nonconvex adjustment frictions matter a lot, but inventories do not interact with them significantly. Market clearing in the intermediate goods market only leads to slightly dampened fixed investment responses overall, as decreases in the relative price $q$ (see Panel(d) of Figure 2) lead consumption smoothing activities away from fixed capital investment.

In other words, our exercise of comparing differences in differences really becomes only interesting, once real interest rate and real wage movements have been taken into account. The response functions in Panel(d) of Figure 6 come from the models where the pricing
Figure 6: IRF for Fixed Capital Investments in Partial Equilibrium Models

Notes: These are the impulse response functions for fixed capital investments. Panel(a) is the reproduction of Figure 1. Panel(b) is based on models where both the pricing kernel and the relative price are fixed. Panel(c) is based on models where only the pricing kernel is fixed. Panel(d) is based on models where only the relative price is fixed.

Kernel and the real wage move freely to clear the final goods market, yet the relative price of intermediate goods is fixed. These response functions resemble those from the general equilibrium case in that in models with inventories the impact response of fixed investment is 40% higher with frictionless fixed capital adjustment, whereas in models without inventories it is only 26% higher.\(^{20}\) Nevertheless, market clearing in the intermediate goods market

\(^{20}\)The relative impact conditional on the same level of adjustment costs for fixed capital has changed between Panels (a) and (d) of Figure 6. For example, with no fixed capital adjustment costs, fixed capital reacts more to a productivity shock when there are inventories, but the price of intermediate goods is fixed, compared to the case where the price of intermediate goods adjusts downward, where the relative size of the reaction of fixed capital investment is reversed between the inventory and the ‘no inventory’-case. Of course, with frictionless fixed capital adjustment, positive inventory holding costs and a fixed price at which inventories can be stocked up, there is really not much reason to smooth consumption via inventories and thus fixed capital investment reacts more strongly. This changes, when the price of intermediate goods
does play a role in rendering fixed capital adjustment frictions more relevant. Recall that in full general equilibrium the difference in the initial fixed investment response between the frictionless model and the lumpy model was 50% vs. 24%. The decline of the relative price \( q \) after an increase in aggregate productivity further facilitates the shifting of consumption smoothing through building up inventories and away from fixed capital investment. This substitution channel, for a given decline in \( q \), is more valuable in an economy, when fixed capital adjustment is costly.

5 Conclusion

This paper shows that it matters for the aggregate implications of microfrictions how general equilibrium effects are introduced into the physical environment of dynamic stochastic general equilibrium models with these microfrictions. Specifically, we show that how relevant nonconvex fixed capital adjustment costs are for business cycle dynamics depends on how the aggregate resource constraint is modeled, depends on how the model is closed. Future research will explore the general insight in more general frameworks.

Here we develop a dynamic stochastic general equilibrium model to evaluate how the availability of multiple investment channels, here inventories in addition to fixed capital, affects the aggregate implications of nonconvex capital adjustment costs. We find that with more than one ways to invest, capital adjustment costs are more effective in dampening and propagating the response of fixed capital investment to an aggregate productivity shock.

References


