

# Guided plasmaspheric hiss interactions with superthermal electrons

## 1. Resonance curves and timescales

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**Abstract.** Under the proper conditions, guided plasmaspheric hiss is shown to be more efficient than Coulomb collisions at scattering electrons in the superthermal energy range of 50 to 500 eV. Broadband, whistler mode hiss becomes guided by plasma density gradients, intensifying the wave energy densities and focusing the wave normal angles. These waves are shown to interact through Cherenkov (Landau) resonance with electrons below 500 eV, and the presented equatorial plane timescales for pitch angle, energy, and mixed diffusion are shown to be faster than Coulomb collision timescales for typical values at the inner edge of the plasmopause and in detached plasma regions. In the latter case, energy diffusion timescales of less than 100 s for small pitch angle electrons between 250 and 500 eV indicate that these waves have the potential to dramatically change the distribution function.

### Introduction

Right-hand polarized electromagnetic waves between the proton and electron cyclotron frequencies (whistler mode waves) have long been known to interact with high-energy ( $E > 10$  keV) electrons [Dungey, 1963] and lower-energy (at least down to 200 eV) electrons [Liemohn and Scarf, 1964] in the inner magnetosphere. Observations indicate that a particular class of whistler mode waves, broadband hiss, is almost continuously present in the plasmasphere [Thorne *et al.*, 1973; Smith *et al.*, 1974]. Expressions were obtained for the wave amplitude of this plasmaspheric hiss as a function of density in the outer plasmasphere [Chan *et al.*, 1974] and in detached plasma regions [Chan and Holzer, 1976]. Hiss usually occurs at very large wave normal angles [Sonwalker and Inan, 1988; Storey *et al.*, 1991], causing it to predominantly interact with ring current and radiation belt electrons ( $E > 10$  keV) [Dungey, 1963; Liemohn, 1967; Church and Thorne, 1983]. These oblique wave normal angles prevent the bulk of plasmaspheric hiss from ever resonating with electrons in the superthermal energy range (1–500 eV). However, intense hiss is associated with density gradients [Kozyra *et al.*, 1987], and these density structures duct the waves into smaller wave normal angles, making them field-aligned (guided). Hayakawa *et al.* [1986a] calculated wave normal angles of less than  $25^\circ$  in detached plasma regions near the equator and, soon after, showed observations of field-aligned hiss near the plasmopause [Hayakawa *et al.*, 1986b]. These small wave normal angles allow these waves to resonate with lower-energy electrons, down into the superthermal energy range.

Guided plasmaspheric hiss is very similar to whistler waves generated by atmospheric lightning, because the waves share the same frequency range, polarization, and wave normal angles. Liemohn and Scarf [1964] analyzed nose whistlers and determined that the electron energies resonating near the upper cutoff of the whistler frequencies ranged from 0.2 to 2 keV, predicting a smooth distribution of  $E^{-1}$  needed for the interaction. This group then determined that a spectrum with sharp edges ( $E^{-m}$ ,  $m \geq 5$ ) that is otherwise flat ( $E^{-1}$ ) is necessary for whistler absorption [Liemohn, 1965]. These results indicate that superthermal electrons can interact with field-aligned whistler mode waves, and Thorne and Horne [1994] demonstrated that Cherenkov (Landau) resonance with superthermal electrons is the primary loss mechanism for magnetospherically reflected whistlers.

A quantitative relation for this interaction mechanism was formalized by Kennel and Petschek [1966] into what is known as quasi-linear diffusion theory. They derived the coefficients necessary to calculate the influence of a given wave spectrum with a plasma species using the kinetic quasi-linear diffusion equation, confirming the necessity of sharp gradients predicted by Liemohn [1965]. Kennel and Petschek [1966] demonstrated the need to build up the wave energy density to a substantial level to have a significant interaction. This, fortuitously, is the case with guided plasmaspheric hiss [Smith *et al.*, 1974; Kozyra *et al.*, 1987].

Further advancements in wave-particle interactions were developed by Lyons *et al.* [1971, 1972] and Lyons [1974a, b, c], who derived straightforward quasi-linear diffusion coefficients for ion cyclotron and whistler mode wave interactions and applied them to proton and electron precipitation. It was the coefficients from Lyons [1974b] that Kozyra *et al.* [1994, 1995] recently adapted to calculate the interaction of energetic protons resonating with guided plasmaspheric hiss. These coefficients are also suitable for application to superthermal electron interactions.

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**Table 1.** Parameters for Example Cases

	Description	$L$ Shell	$B_{eq}$ , G	$N_{eq}$ , $\text{cm}^{-3}$	$E_c$ , eV	$\Omega_e$ , $\text{s}^{-1}$
Case 1	inner edge of the plasmopause	3.5	$7.2 \times 10^{-3}$	800	1600	$1.3 \times 10^5$
Case 2	detached plasma region	5.0	$2.5 \times 10^{-3}$	50	3100	$4.4 \times 10^4$

Variables are defined as follows:  $B_{eq}$ , geomagnetic field at the equatorial plane;  $N_{eq}$ , plasma density at the equatorial plane;  $E_c$ , characteristic energy; and  $\Omega_e$ , the electron gyrofrequency.

There is observational evidence for this interaction between plasma waves and superthermal electrons. It was shown by *Johnstone et al.* [1993] that low-energy electrons ( $E < 10$  keV) can and do interact with whistler mode waves. The CRRES observations they presented demonstrated that in the inner magnetosphere electrons down to at least 100 eV are pitch angle scattered into distributions that follow the characteristic curves they derived for interactions with whistler mode waves. Also, it is believed that ion cyclotron waves could influence superthermal electrons, as *Erlanson et al.* [1993] observed an enhancement in the superthermal electron fluxes in regions of increased ion cyclotron wave activity on DE 2.

This paper uses a version of the quasi-linear diffusion coefficient calculation used by *Kozyra et al.* [1994, 1995], updated and modified for superthermal electron interactions, to calculate resonant energy curves and diffusion coefficients. These modifications include correcting a coding error and a few of the graphical results by *Kozyra et al.* are presently being reexamined. Timescales based on these calculations are presented and compared with Coulomb scattering timescales. Of particular interest are spatial regions with thermal plasma density gradients, which act to focus the wave normal angles and intensify the amplitudes, as discussed above. Therefore this analysis will use typical parameters at the equatorial plane for the inner edge of the plasmopause and detached plasma regions, cases 1 and 2, respectively, in Table 1. These results can be used as a basis for incorporating wave-particle interactions into kinetic superthermal electron calculations, such as the field-aligned model of *Khazanov and Liemohn* [1995] and the bounce-averaged global model of *Khazanov et al.* [1996].

## Resonant Energy Curves

An important equation in the calculation of wave-particle interactions is the resonance condition,

$$\omega - k_{\parallel} v_{\parallel} + n\Omega_e = 0 \quad (1)$$

where  $\omega$  is the wave frequency;  $k_{\parallel}$  and  $v_{\parallel}$  are the wave number and particle velocity, respectively, parallel to the magnetic field  $B$ ;  $n$  is a signed integer; and  $\Omega_e = |eB/cm_e|$ .

From cold plasma theory for whistler waves with  $(\omega_{pe}/\Omega_e)^2 \gg \omega/\Omega_e$  [see *Stix*, 1992], (1) can be rewritten in terms of the resonant particle energy,  $E_{\parallel, res}$ , normalized to a characteristic energy  $E_c = B^2/8\pi N$ ,

$$\frac{E_{\parallel, res}}{E_c} = M\Psi \frac{(1+n\Omega_e/\omega)^2}{\cos^2 \theta (1+M)} \quad (2)$$

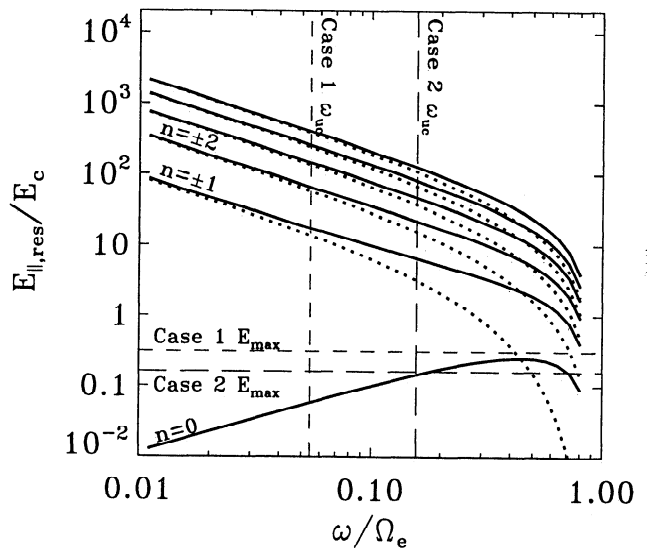
The values of  $E_c$  and  $\Omega_e$  for the two cases to be discussed are given in Table 1. In (2),  $M = m_e/m_p$ ,  $m_p$  is the mass of a proton,  $\theta$  is the wave normal angle, and  $\Psi$  is from the refractive

index and has the form

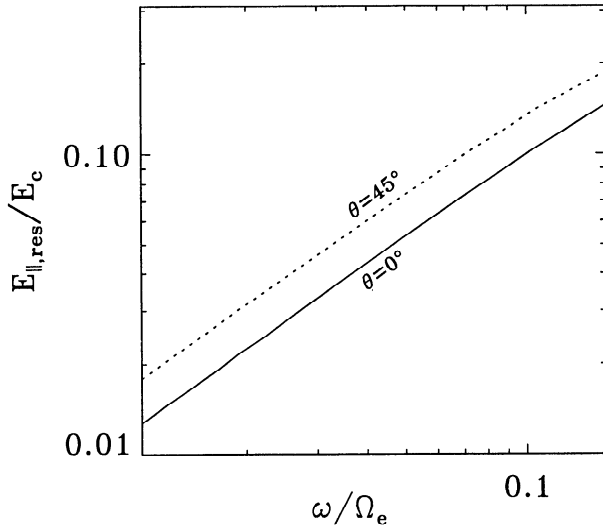
$$\Psi = 1 - \frac{\omega^2}{\Omega_p \Omega_e} - \frac{\sin^2 \theta}{2} + \left[ \frac{\sin^4 \theta}{4} + \left( \frac{\omega}{\Omega_p} \right)^2 (1-M)^2 \cos^2 \theta \right]^{1/2} \quad (3)$$

Equation (2) can be used to determine the energies of electrons that will resonate with whistler mode waves. Figure 1 shows solutions to (2) for  $\theta = 0^\circ$ , with solid curves denoting the case for  $n \geq 0$  and dotted curves representing  $n < 0$ , up to  $n = \pm 5$ . The cyclotron resonances ( $n \neq 0$ ) clearly require a much higher electron parallel energy than the Cherenkov resonance of  $n = 0$ , except near  $\omega/\Omega_e = 1$ . The two horizontal dashed lines indicate  $E_{\parallel, res} = 500$  eV, an upper limit to the superthermal electron energy range, for  $B$  and  $N$  (plasma density) from the two cases in Table 1 (short-dashed line is case 1, long-dashed line is case 2). The two vertical dashed lines show the location of  $f = 1100$  Hz, an upper cutoff for the frequency of plasmaspheric hiss used by *Lyons* [1974b] and *Kozyra et al.* [1994, 1995]. These limits, although they are quite flexible, clearly show that the interaction of superthermal electrons with guided plasmaspheric hiss is obtained through Cherenkov resonance only.

The results in Figure 1 are for a wave normal angle of  $0^\circ$ . For larger wave normal angles within the guided hiss regime



**Figure 1.** Normalized resonant parallel energy curves versus wave frequency with  $\theta = 0^\circ$  for  $n = 0$  to  $n = \pm 5$ . Solid curves are  $n \geq 0$  and dotted curves are  $n < 0$ . The two horizontal dashed lines indicate  $E_{\parallel} = 500$  eV for cases 1 (short-dashed line) and 2 (long-dashed line) in Table 1 and the two vertical lines indicate  $f = 1100$  Hz for the same cases.



**Figure 2.** Normalized resonant parallel energy curves versus wave frequency with  $\theta=0^\circ$  and  $\theta=45^\circ$  for  $n=0$ .

the results do not differ greatly. Figure 2 shows the  $n=0$  resonant energy curves for  $\theta=0^\circ$  and  $\theta=45^\circ$  in the energy and frequency ranges of interest. It can be seen that the curves are within 30% of each other, and other resonances ( $n \neq 0$ ) will not become a factor in the diffusion coefficient calculations. The increase is because the larger wave normal angle requires a larger  $v_{\parallel}$  to compensate for the decreased  $k_{\parallel}$  in (1). Note that the resonant energy increases dramatically as the wave becomes more oblique owing to the  $\cos^2\theta$  term in the denominator of (2). Indeed, these cold plasma wave equations are only valid below a cutoff wave normal angle (such as that defined by *Stix* [1992, chapter 1]), usually close to  $90^\circ$  for whistler mode waves.

### Diffusion Timescales

Knowing that there is a possible interaction between these waves and particles, the diffusion coefficients can be calculated to find their relative magnitude compared with other scattering mechanisms. These coefficients influence the particles through the kinetic diffusion equation,

$$\begin{aligned} \frac{\partial f}{\partial t} = & \frac{1}{v^2} \frac{\partial}{\partial v} v^2 \left( D_{vv} \frac{\partial f}{\partial v} + D_{v\alpha} \frac{1}{v} \frac{\partial f}{\partial \alpha} \right) \\ & + \frac{1}{v \sin \alpha} \frac{\partial}{\partial \alpha} \sin \alpha \left( D_{\alpha v} \frac{\partial f}{\partial v} + D_{\alpha\alpha} \frac{1}{v} \frac{\partial f}{\partial \alpha} \right) \end{aligned} \quad (4)$$

shown here in spherical coordinates from *Lyons* [1974b]. Here  $f$  is the particle distribution function,  $v$  and  $\alpha$  are the particle velocity and pitch angle, respectively; and  $D_{\alpha\alpha}$ ,  $D_{\alpha v}$ ,  $D_{v\alpha}$  ( $=D_{v\alpha}$ ), and  $D_{vv}$  are diffusion coefficients calculated from quasi-linear theory. The reader is referred to *Lyons* [1974b] and *Kozyra et al.* [1994] for details on the calculation of these coefficients.

To compute these coefficients, a distribution of the wave energy density with respect to frequency and wave normal angle must be assumed, and these will be taken as Gaussian distributions in  $\omega$  and  $x$  (following *Kozyra et al.* [1994]), where  $x = \tan\theta$ . The frequency distribution is centered on  $f_m = 600$  Hz, with a spread of  $\delta f = 400$  Hz and upper and lower frequency cutoffs of  $f_{lc} = 100$  Hz and  $f_{uc} = 1100$  Hz, where  $\omega = 2\pi f$ . The angle distribution is centered on  $x=0$  with  $\delta x = x_{\max} = 1$ . The plasmaspheric hiss wave energy density is observed to vary from  $10^{-4}$  to  $10^{-7} \gamma^2 \text{ Hz}^{-1}$  [*Smith et al.*, 1974], and a high value of  $5 \times 10^{-5} \gamma^2 \text{ Hz}^{-1}$  is used for the results presented here.

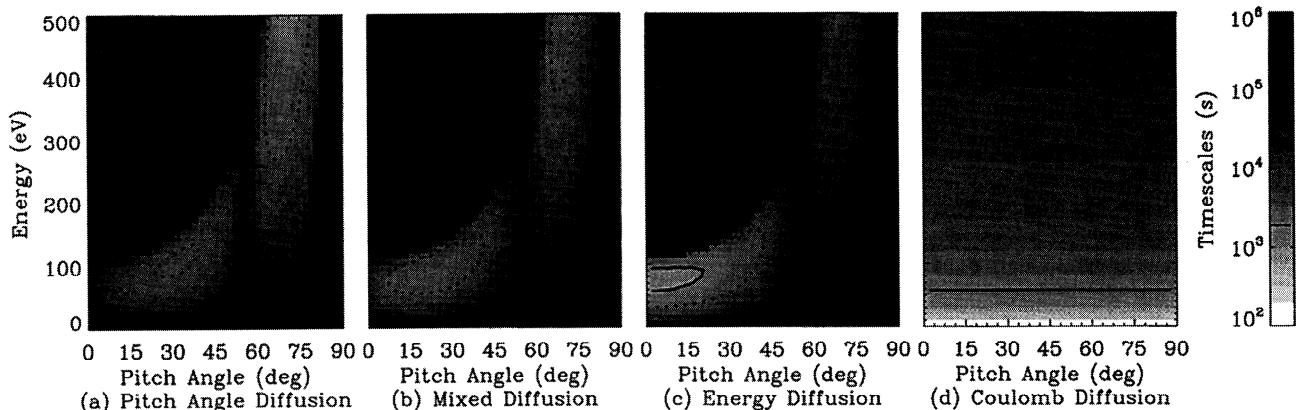
A rough estimate for wave-particle interaction diffusion timescales can be written as

$$\tau_{\alpha\alpha} = \frac{v^2}{D_{\alpha\alpha}} \quad \tau_{\alpha v} = \frac{v^2}{D_{\alpha v}} \quad \tau_{vv} = \frac{v^2}{D_{vv}} \quad (5)$$

This quantity can be compared to the diffusion timescale for Coulomb collisions,

$$\tau_{cc} = \frac{v^2}{D_{cc}} = \frac{\beta^4 v^3}{2AN} \quad (6)$$

where  $\beta = 1.7 \times 10^{-8} \text{ eV}^{1/2} \text{ s cm}^{-1}$  and  $A = 2\pi e^4 \ln \Lambda = 2.6 \times 10^{-12} \text{ eV}^2 \text{ cm}^2$ , with  $\ln \Lambda$  being the Coulomb logarithm. A comparison of these four timescales is shown in Figures 3 and 4 for cases 1 and 2, respectively, with the wave assumptions above. A solid line reference contour is shown at  $\tau = 30$  min in Figures 3a-3d and 4a-4d, and a dotted line reference contour is shown at  $\tau = 3$  hours. It should be kept in mind that the definition of these timescales assumes nothing about the shape of the electron distribution function and that gradients in  $f$  and  $D$  will



**Figure 3.** (a) Pitch angle, (b) mixed, (c) energy, and (d) Coulomb diffusion timescales from (5) and (6) for case 1. The dotted contour indicates  $\tau = 3$  hours, and the solid contour indicates  $\tau = 30$  min.

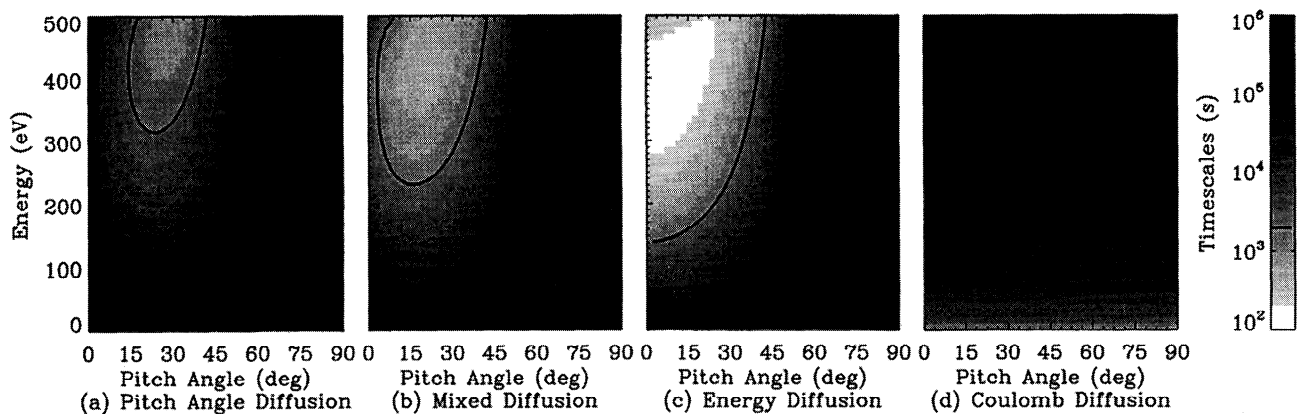


Figure 4. Same as Figure 3, but for case 2.

play a role in the actual scattering timescales. Particularly, steep pitch angle gradients near the edge of the loss cone could drive the diffusion process, reducing the presented timescales. These results, however, show the general magnitude of the timescales for these processes.

For the outer plasmasphere (Figure 3), clearly all three wave interactions are faster than Coulomb collisions for some part of velocity space, particularly below 150 eV at pitch angles less than  $45^\circ$ . There is also a high energy, large pitch angle region where the wave interactions are more efficient than Coulomb collisions, especially pitch angle diffusion. Note that the Coulomb timescale is near 50,000 s at 500 eV, below the 1 day dotted contour level. The sharp increase in the wave diffusion timescales near  $57^\circ$  is due to a term in the diffusion coefficients that makes them drop to zero [see *Kozyra et al.*, 1994, equation (14)].

Figure 4 shows that in a detached plasma region there is an even greater possibility for the wave interactions to dominate over Coulomb collisions in the formation of the superthermal electron distribution function. Scattering by plasmaspheric hiss would influence most of the pitch angle distribution above about 50 eV. Notice that the energy diffusion timescale is faster than 100 s for a portion of velocity space above 250 eV. This interaction is more than 1000 times faster than Coulomb collisions and will have a significant impact of the distribution function, even for low levels of wave energy density.

## Discussion

Superthermal electrons play an important role in the energy budget of the ionosphere and inner magnetosphere, and it is important to understand the processes affecting their transport through the Earth's magnetic field. Of particular interest is the low-energy regime, below 50 eV, where Coulomb collisions with the thermal plasma are efficient and energy deposition occurs. These results presented indicate, for certain plasmaspheric conditions, that it is desirable to include wave-particle interactions with plasmaspheric hiss in superthermal electron transport calculations to obtain a more comprehensive distribution function. Although it was not shown that guided hiss can interact with this low-energy regime, the cascading of electrons to lower energies through collisions with the thermal plasma and atmospheric neutral particles connects the high-energy superthermal electrons to this critical low-energy

portion. Therefore an understanding of the processes that alter the distribution function at all energies is useful.

While most plasmaspheric hiss is oblique to the magnetic field and will only resonate with ring current and radiation belt electrons, hiss can be forced to become field-aligned under certain conditions. Regions of sharp density gradients can act to guide hiss along the field line, intensifying the wave amplitudes and focusing the wave normal angles. The two most obvious occurrences of such gradients are at the plasmapause and in detached plasma regions. This guided plasmaspheric hiss can be more efficient than Coulomb collisions at scattering 50 to 500 eV superthermal electrons.

These diffusion coefficient calculations are presently being incorporated into existing superthermal electron transport models (namely, the model of *Liemohn et al.* [1997]), including not only the pure pitch angle and energy diffusion terms, but also the mixed diffusion terms. This study will also use input wave data from the DE 1 satellite and compare the plasma distribution results with the corresponding superthermal electron data.

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