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Nonlinear drift-kinetic equation in the presence of a circularly polarized wave

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Abstract

This paper presents a theoretical description of plasma motion in the presence of a circularly polarized wave of arbitrary frequency with its wave vector directed along the ambient magnetic field. Both the equations of motion for a single particle and the nonlinear kinetic equation in the drift approximation are given. The nonstationarity and inhomogeneity of the plasma-wave system are taken into account. The distribution function is expressed with the help of initial, non-averaged variables, which is a transformation that greatly simplifies the calculation of the moments. The time-dependent part of the ponderomotive force is discussed, and it is shown that this component can be on the order of the other terms in the particle motion equation. The examples of ion acceleration in the auroral zone and the effects of lightning discharge are discussed to show that this theoretical study has applications to near-Earth space plasmas.
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1. Introduction

The nonlinear effect of electromagnetic waves on plasmas is a subject of interest in laboratory experiments as well as in space physics. Wave–particle interactions described by the ponderomotive force, which arises in such situations, are responsible for different nonlinear processes that can be important for plasma redistribution, particle energization, and acceleration in space plasmas (Ghildyal and Kalra, 1998; Guglielmi et al., 1996; Shukla et al., 1996). An understanding of these processes is also essential for the problems of plasma heating and confinement in the laboratory (D'ippolito and Myra, 1985). Numerous publications have been devoted to the calculation of this force (see, for instance, Lee (2000) and the literature cited there). In spite of this, a number of aspects of the theory of the ponderomotive force are, up to now, the subject of discussion. In general, the ponderomotive force contains terms proportional to the time derivatives, the so-called Abraham force. In a recently published paper, Lee (2000) has stated that their calculation of the Abraham force (as well as in the paper of Lee and Parks (1983)) differs from that obtained by Barash and Karpman (1983), and that the reason for such a difference is unclear. These calculations are based on the two-fluid equation for a collisionless warm plasma and the corresponding electrodynamic energy relations in moving media.

A kinetic description of the ponderomotive effect leads to the nonlinear kinetic equations (Aamodt and Vella, 1977; Cary and Kaufman, 1981; D'ippolito and Myra, 1985; Ye and Kaufman, 1992; Chiu et al., 2000; Sugama, 2000). Depending on the degree of the wave–particle coherence, the analysis is either performed for a quasi-monochromatic wave (Cary and Kaufman, 1981) or a quasilinear approach is applied (Aamodt and Vella, 1977; Chiu et al., 2000). The problem was analyzed in general (Cary and Kaufman, 1981; Chiu et al., 2000; Sugama, 2000) as well as for a number of specific applications, especially related to magnetically confined plasmas. Nonlinear kinetic equations have been found for plasmas with low-frequency waves (conventional gyrokinetic equations, hereinafter GKEs) and high-frequency waves (the kinetic equation in oscillation-centered coordinates (Chiu et al., 2000; Grebogi and Littlejohn, 1984) or GKE (Sugama, 2000)).

Chen and Tsai (1983) first presented linear GKEs for electromagnetic disturbances with arbitrary frequency. The nonlinear problem for this case has been analyzed with

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emphasis on the ponderomotive effects (Cary and Kaufman, 1981; Grebogi and Littlejohn, 1984) as well as on the general gyrokinetic plasma description and GKE-specific applications (D'ippolito and Myra, 1985; Sugama, 2000). The ordering assumed in these papers, at least where the calculations are carried out to the point of application, corresponds to the nonlinear gyrokinetic ordering (Frieman and Chen, 1982). In particular, it is assumed that the amplitudes of the electromagnetic potentials obey the same length and time scales as the background magnetic field. Such ordering was first adopted for problems involving the bulk mode in experimental nuclear fusion devices. In other applications (for example, in the magnetosphere), the electromagnetic fields (not potentials) can obey the scale of the background magnetic field.

In space plasmas, as well as in some laboratory devices, a circularly polarized wave propagating parallel to the external magnetic field is often a valid description of the real situation (D'ippolito and Myra, 1985; Ye and Kaufman, 1992; Grebogi and Littlejohn, 1984). An example of such waves in space are whistler mode waves in the vicinity of the magnetopause observed on board the AMPTE-UKS satellite, propagating parallel or almost parallel to the ambient magnetic field (Sazhin et al., 1991). Another example is the observation of strong ion cyclotron waves with a wave vector almost parallel to the magnetic field (less than 7°) observed during Galileo's December encounter with Io (Crary and Bagenal, 2000). The frequency of such waves can be comparable to the particles' gyrofrequency and the wave amplitudes can vary over a wide spatiotemporal scale.

This paper considers the specific case of a circularly polarized wave propagating along the magnetic field line with electromagnetic field characteristic scales of the order of the background magnetic field. The paper is organized as follows. Starting with the characteristics of the kinetic equation, the drift equations for the motion of a single particle are found. Then the drift-kinetic equation is formulated and the distribution function needed for moment calculations is presented. Examples of the application of these results are presented in the Conclusion. The averaging procedure and the calculation of the distribution function are outlined in the appendix.

2. Problem formulation

Consider a plasma immersed in an external magnetic field $\vec{B}(s,x)$ with the s-coordinate along this field and the x-axis along the principal normal to the line of force. The plasma is then subjected to a circularly polarized wave

$$E(t,s,x)(\vec{e}_x \pm \mathrm{i}\vec{e}_y)\exp\left(-\mathrm{i}\int \omega(t,s,x)\,\mathrm{d}t + \mathrm{i}\int k(t,s,x)\,\mathrm{d}s\right) + \text{c.c.},$$

where the wave vector k is directed along the external magnetic field. It is assumed below that the initial plasma distribution is independent of the y-coordinate. The magnetic field and the wave amplitude spatiotemporal variations are assumed to be weak on the scale of the wave frequency, wavelength, particle Larmor radius, and the radius of particle drift rotation in the wave field (Khazanov et al., 2000). In the collisionless limit, the kinetic equation can be written as (Hastie et al., 1967)

$$\frac{\partial f}{\partial t} + \vec{v}\frac{\partial f}{\partial \vec{r}} + F_\varepsilon \frac{\partial f}{\partial \varepsilon} + F_\mu \frac{\partial f}{\partial \mu} + F_\varphi \frac{\partial f}{\partial \varphi} = 0. \tag{1}$$

Here, the kinetic energy $\varepsilon = (m/2)(v_s^2 + v_\perp^2)$, magnetic moment $\mu = mv_\perp^2/2B$, and φ, the azimuthal angle about the external magnetic field direction, are introduced in velocity space. In these variables, the characteristic equations of (1) can be presented as

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v_\perp \cos\varphi, \qquad \frac{\mathrm{d}y}{\mathrm{d}t} = v_\perp \sin\varphi, \qquad \frac{\mathrm{d}s}{\mathrm{d}t} = v_s,$$

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = ev_\perp E \mathrm{e}^{\mathrm{i}(\phi+\varphi)}$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}t} = -v_\perp\rho_1\cos\varphi\left(\mu + \frac{mv_s^2}{B}\right) - \rho_2 v_s \mu\cos 2\varphi + e\,\frac{v_\perp \mathrm{e}^{\mathrm{i}\phi}}{B} \times \left[\mathrm{e}^{\mathrm{i}\varphi}\frac{\tilde{\omega} + \mathrm{i}v_s\hat{L}_\mathrm{E}}{\omega} + \frac{\rho_2 v_s}{\omega}\cos\varphi\right]E + \text{c.c.}, \tag{2}$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = -\omega_B + \frac{\rho_1 v_s^2}{v_\perp}\sin\varphi + \frac{\rho_2 v_s}{2}\sin 2\varphi + \mathrm{i}\,\frac{e}{m}\frac{\mathrm{e}^{\mathrm{i}\phi}}{v_\perp} \times \left[\mathrm{e}^{\mathrm{i}\varphi}\frac{\tilde{\omega} + \mathrm{i}v_s\hat{L}_\mathrm{E}}{\omega} - \frac{\rho_2 v_s}{\omega}\sin\varphi + \mathrm{i}\,\frac{v_\perp}{\omega}\frac{\partial}{\partial x}\right]E + \text{c.c.},$$

where $\omega_B = eB/mc$, $\phi = -\int\tilde{\omega}\,\mathrm{d}t$, $\tilde{\omega} = \omega - kv_s$, $\hat{L}_\mathrm{E} = (\partial/\partial t)(k/\omega) + \partial/\partial s$, and the magnetic field line curvature is determined by the coefficients $\rho_1 = (1/B)(\partial B/\partial x)$, $\rho_2 = (1/B)(\partial B/\partial s)$. Eqs. (2) are written for the right-hand polarized mode, $E_y = \mathrm{i}E_x$. For left-hand polarization, the corresponding equations can be found changing E, E^* to E^*, E and (ω, k) to $(-\omega, -k)$, respectively. In Maxwell's equation, the wave magnetic field in the Lorentz force is replaced with the electric one,

$$\vec{B}\mathrm{e}^{\mathrm{i}\phi} = \frac{c\mathrm{e}^{\mathrm{i}\phi}}{\mathrm{i}\omega}\left[-\vec{e}_x(\mathrm{i}k + \hat{L}_\mathrm{E})E_y + \vec{e}_y(\mathrm{i}k + \hat{L}_\mathrm{E} + \rho_2)E_x + \vec{e}_s\frac{\partial E_y}{\partial x}\right]$$

and only the first-order terms over large-scale spatiotemporal variations were taken into account. In this expression, the magnetic field perturbation due to the slow time variation of the electric field is described by the terms with the operator $\hat{L}_\mathrm{E}$, which includes the time derivative.

It can be noted from the structure of Eqs. (2) that the second-order terms, i.e. the terms with the second derivatives, will lead to a third-order correction after averaging these equations. The cause is that these terms appear in Eqs. (2) as the real correction to the zeroth-order terms. This is because in the zeroth-order approximation, the wave field has no impact on the average particle energy, drift velocity, and magnetic moment. The same argument is valid for the second-order correction, which simply redefines the zeroth-order terms.

3. Particle drift equations

The drift equations for a particle can be found from Eqs. (2) by averaging over the Larmor rotation and oscillation in the wave electromagnetic field in the usual way (Morozov and Solov'ev, 1966) as it is presented in the appendix. The left-hand side terms in these equations are periodic functions with respect to the arguments φ and ϕ. In fact, they are periodic with respect to the arguments φ and $\varphi+\phi$, where the main term in φ is proportional to the particle gyrofrequency ω_B. The exception is the term proportional to $\partial E/\partial x$ in the last equation of (2). This term affects only the drift velocity along the y-axis, and this velocity after averaging is still proportional to the product of periods $1/\omega_B, 1/(\omega_B+\tilde{\omega})$. As a result, even when the wave period is large (low-frequency waves), this term is still proportional to the squared gyroperiod. The drift equations can be obtained by averaging (2) over these periods with second-order accuracy with respect to the small parameters, $1/\omega_B, 1/(\omega_B+\tilde{\omega})$. These second-order terms should be taken into consideration not only for high-frequency waves but also if the mass dependence of the wave–particle interaction in the limit of low-frequency waves ($|\tilde{\omega}| \ll \omega_B$) is the subject of interest. The third-order terms over these small parameters can be neglected if the restrictions

$$\left|\frac{kv_\perp}{\Omega}\right|, \left|\frac{k^2 v_\perp^2}{\Omega\omega}\right| \ll 1 \tag{3}$$

are satisfied. Here $\Omega = \omega_B + \tilde{\omega}$. The resulting drift equations for the particle are

$$v_x = 0, \quad v_y = \frac{2\rho_1}{m\omega_B}\left(\varepsilon - \frac{\mu B}{2}\right) - \frac{e^2}{4m^2}\frac{\tilde{\omega}}{\Omega\omega_B\omega^2}\frac{\partial}{\partial x}|E_0|^2,$$

$$\mu - \frac{e^2}{2m}\frac{\tilde{\omega}^2}{\omega^2\Omega^2 B}|E_0|^2 = const, \tag{4}$$

$$\frac{d\varepsilon}{dt} = -\frac{e^2}{2m}\frac{v_s}{\omega}\frac{\partial}{\partial s}\frac{\omega_B}{\Omega^2}|E_0|^2 + \frac{e^2}{2m^2}\frac{\mu B}{\Omega^2}\frac{\partial}{\partial t}\frac{k^2}{\omega^2}|E_0|^2 + \frac{4e^2}{m}\frac{E}{\Omega}\left(\frac{\partial}{\partial t}\frac{\tilde{\omega}E}{\Omega\omega} - \frac{1}{\omega}\frac{\partial}{\partial t}\frac{kv_s}{\omega}E\right),$$

where $|E_0|^2 = 4E^2$. Eqs. (4) describe the particle motion in the drift approximation for a nonstationary, inhomogeneous plasma in the range of velocities restricted by inequalities (3). In the framework of these restrictions the particle velocity can be larger than the wave phase velocity.

Because the magnetic field and wave amplitude change only in the x,s-plane, the drift velocity is directed along the y-axis, listed as v_y in Eqs. (4). The first term is the usual drift due to the magnetic field curvature and its gradient. A particle subjected to a circularly polarized wave with a wave vector along the magnetic field line rotates under the influence of this magnetic field and the electric field of the wave. Averaged over these rotations, the motion results in additional drift due to the wave electric field inhomogeneity and leads to the second term in the drift velocity v_y.

It can be noted that the result for the magnetic moment μ in Eqs. (4) immediately follows from the character of the particle motion in the circularly polarized wave. By neglecting the slow drift along the y-axis, the guiding center rotates with a velocity that can be written as (Khazanov et al., 2000)

$$v_\pm \equiv v_x \pm i v_y = \pm i\frac{e}{m}\frac{\tilde{\omega}}{\omega\Omega}E\exp(\pm i\phi).$$

The particle is also involved in Larmor rotation. The total average energy of the perpendicular motion is the sum of the energy of the Larmor rotation and the guiding center rotation $mv_+v_-/2$. The expression for the magnetic moment μ is the same as this last energy of perpendicular rotation in the wave field divided by the external magnetic field.

The second term in the energy equation, in the limit of a stationary medium and neglecting the velocity dependence in Ω, turns out to be the magnetization energy per particle as found by Similon et al. (1986). This term also can be found from a simple consideration for the stationary case. The guiding center trajectory is wound along a cylinder with a radius of $r_\pm = \mp i v_\pm/\tilde{\omega}$ (Khazanov et al., 2000). The rate of change of the magnetic field flux through the surface restricted by such a circle divided by the guiding center rotation period ($2\pi/\omega$) leads to the same magnetization energy. The μ-dependent term in the equation for the energy is the same as the corresponding term in the ponderomotive Hamiltonian found by Grebogi and Littlejohn (1984).

The force component acting on the particle along the external magnetic field, $m\,dv_s/dt$, which frequently appears in different applications for space plasma, can be calculated from Eqs. (4). The part of this force that depends on the inhomogeneity is

$$F^{(s)} = -\mu\frac{\partial B}{\partial s} - \frac{e^2}{2m} \times \left(\frac{1}{\omega}\frac{\partial}{\partial s}\frac{\omega_B}{\Omega^2}|E_0|^2 + B\frac{\partial}{\partial s}\frac{\tilde{\omega}^2}{\Omega^2 B\omega^2}|E_0|^2\right).$$

The μ-dependent term in this expression is the usual mirroring force. The ponderomotive term is the same as found by Aamodt and Vella (1977) in the quasilinear approximation, if ω is constant and the terms kv_s in $\tilde{\omega}, \Omega$ are omitted. That is,

$$F^{(s)} = -\frac{e^2}{2m}\frac{B}{\omega(\omega_B+\omega)}\frac{\partial}{\partial s}\frac{|E_0|^2}{B}.$$

The part of the ponderomotive force in $m\,\mathrm{d}v_s/\mathrm{d}t$ that arises due to nonstationarity is

$$F^{(t)} = -\frac{4e^2}{m}\left(\frac{E}{\omega\Omega}\frac{\partial}{\partial t}\frac{kE}{\omega} - \frac{kE}{\omega\Omega}\frac{\partial}{\partial t}\frac{\tilde{\omega}E}{\omega\Omega} - \frac{v_\perp^2}{8\Omega^2 v_s}\frac{\partial}{\partial t}\frac{k^2|E_0|^2}{\omega^2}\right). \tag{5}$$

Note that $\mathrm{d}v_s/\mathrm{d}t$ for this case describes the total particle acceleration along the magnetic field line because the external magnetic field and the unit vectors are time independent in our consideration. The last term in this expression can be essential for particles with a velocity comparable to the wave phase velocity. The first and the second terms of this force, for a constant wave frequency, ω and omitting the terms kv_s in $\tilde{\omega}, \Omega$, lead to the force

$$F^{(t)} = -\frac{e^2}{m}\left(\frac{|E_0|^2}{\omega^2(\omega+\omega_B)}\frac{\partial k}{\partial t} + \frac{k\omega_B}{2\omega^2(\omega+\omega_B)^2}\frac{\partial}{\partial t}|E_0|^2\right). \tag{5a}$$

For this part of the ponderomotive force, the same result was found by Barash and Karpman (1983) using a macroscopic approach based on the energy relations in a moving media. To compare the results between their approach and the one presented here, the external magnetic field in their calculations should be taken as time independent. The part of the ponderomotive force, $F^{(t)}$, acting along the external magnetic field can be calculated directly starting with (2) (i.e., the non-averaged equations) for the acceleration term $\mathrm{d}v_s/\mathrm{d}t$. The result for the time-dependent part is still the same as that presented by the expressions (5) and (5a). Note that this differs from the results of Lee and Parks (1983) and Lee (2000), who based their results on the equations of motion. Their result follows from the ponderomotive Hamiltonian calculated by Grebogi and Littlejohn (1984). The time-dependent part of the acceleration found as a derivative of the guiding center drift velocity is the same as that calculated from the results of Lee and Parks (1983) and Lee (2000).

Results for the interaction of an Alfvén wave (Khazanov et al., 2000) with a particle undergoing acceleration along the magnetic field due to some external potential non-electric force follows from Eqs. (4) in the limit of the stationary and homogeneous system when only the particle velocity v_s changes due to this force.

4. Kinetic equation

The averaged characteristic equations in (4) permit one to write the kinetic equation in the drift approximation. Note, that we started with the characteristic Eqs. (2), with the right-hand side oscillating due to the dependence on the angle variables, φ and ϕ. The corresponding system of equations is system (A.1) in the appendix. The averaged Eqs. (4) or (A.3) are independent of these oscillations. We did not include in (4) the averaged equation for the angle variable φ but, as it can be seen from (A.3), the right-hand side of this equation is also angle independent. The kinetic equation (1), with the use of the chain rule, can be rewritten in the new averaged variables ξ_k, α_m (A.6). In our case, ξ_k are the averaged variables $x, y, z, \mu, \varepsilon$ with the time derivatives determined by Eqs. (4), and α_m are two new angle variables instead of ϕ and φ. Because time derivatives of the new variables given in Eqs. (4) and (A.3) are taken independent of the angle variables, the characteristic equations of the transformed kinetic equation (A.6) are also independent of them. As discussed in the appendix, in this case the distribution function is independent on the angle variables ϕ and φ. This leads to a kinetic equation for the distribution function f_0 in the new variables,

$$\frac{\partial f_0}{\partial t} + v_s\frac{\partial f_0}{\partial s} + v_y\frac{\partial f_0}{\partial y} + \frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\frac{\partial f_0}{\partial \varepsilon} = 0. \tag{6}$$

Here v_y, v_s, and $\mathrm{d}\varepsilon/\mathrm{d}t$ are determined by Eqs. (4) and conservation of the magnetic moment is taken into account.

To calculate macroscopic quantities such as plasma density, current, and heat, the distribution function f_0 should be transformed to the old variables (Bernstein and Catto, 1985). The dependence between these two sets of variables is presented by expression (A.2). So, the averaged variables can be expressed as function of the old, non-averaged variables (A.7). Then the distribution function can be expanded in a series and, with the help of (A.8) and (A.7), we have

$$\begin{aligned} f = f_0 &+ \frac{1}{\omega_B}v_\perp\left(\sin\varphi\frac{\partial f_0}{\partial x} - \cos\varphi\frac{\partial f_0}{\partial y}\right) \\ &+ \frac{e^2}{m\omega\Omega}\left\{-\frac{\partial f_0}{\partial\varepsilon}\frac{v_\perp\sin\varphi}{4\omega_B}\frac{\partial}{\partial x}|E_0|^2\right. \\ &- \frac{\partial f_0}{\partial\mu}\frac{1}{\omega}\left[\frac{\tilde{\omega}}{B}\frac{v_\perp\sin\varphi}{4\omega_B}\frac{\partial}{\partial x} + \rho_2\frac{\mu k}{2m}\frac{\sin 2\varphi}{\omega_B-\tilde{\omega}}\right]|E_0|^2 \\ &+ \frac{\mu}{2}|E_0|^2\left[\frac{\omega B}{\Omega}\frac{\partial^2}{\partial\varepsilon^2} + \frac{\tilde{\omega}}{B\omega}\left(\frac{\tilde{\omega}}{\Omega} - \frac{\rho_2 v_s\sin 2\varphi}{\omega_B-\tilde{\omega}}\right)\frac{\partial^2}{\partial\mu^2}\right. \\ &+ \left.\left.\left(\frac{2\tilde{\omega}}{\Omega} - \frac{\rho_2 v_s\sin 2\varphi}{\omega_B-\tilde{\omega}}\right)\frac{\partial^2}{\partial\mu\partial\varepsilon}\right]f_0\right\}, \end{aligned} \tag{7}$$

where v_s and $v_\perp$ are determined by the expressions $\varepsilon = (m/2)(v_s^2+v_\perp^2)$, $\mu = mv_\perp^2/2B$.

Because the velocity components depend only on the angle φ, ($v_x = v_\perp\sin\varphi$, $v_y = v_\perp\cos\varphi$) and we are interested in the slowly changing part of the macroscopic quantities (calculated with the help of these velocity components), the terms in the transformed distribution function dependent on the wave phase ϕ can be omitted.

The elementary volume in the five-dimensional space $x, y, z, \mu, \varepsilon$ can be defined as $B\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\mu\,\mathrm{d}\varepsilon/m\times$

$\sqrt{2m(\varepsilon-\mu B)}$. This volume is invariant under the transformation presented by (A.2) and (A.3) assuming that the conditions in Eq. (3) are satisfied. Then Liouville's theorem, with an accuracy of second-order in the small parameters $1/\omega_B, 1/(\omega_B+\tilde{\omega})$ is satisfied and the moments of the distribution function given in Eq. (7) can be calculated.

5. Conclusion

Ponderomotive forces are essential for the understanding of a wide variety of phenomena in the solar wind and in planetary magnetospheres. Examples of their importance include the cold plasma redistribution toward the Earth's magnetospheric equator and the energization of ionospheric ions. Circularly polarized waves (ion cyclotron waves and whistlers) are widely prevalent throughout space. The wave vector of these waves is often directed along the external magnetic field (or close to it). Under these conditions, a kinetic description of the ponderomotive wave-particle interaction is significantly simplified and can be used for a number of applications.

The role of additional velocity-dependent terms in the time-dependent part of the ponderomotive force can be illustrated by the problem of upward acceleration of ions in the auroral region. As pointed out by Shukla et al. (1996), parallel ion acceleration is strongly dependent on the spatiotemporal variation of the wave electric field energy density. Assuming for simplicity a magnetic moment equal to zero, the expression for an ion's energy change in the field of ion cyclotron wave will have a time-dependent term from (4), and that can be presented as

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}=\frac{4e^2}{m}\frac{E}{\omega_{Bi}-\tilde{\omega}}\left(\frac{\partial}{\partial t}\frac{\tilde{\omega}E}{(\omega_{Bi}-\tilde{\omega})\omega}+\frac{1}{\omega}\frac{\partial}{\partial t}\frac{kv_s}{\omega}E\right).$$

For an altitude of $\sim$ 2000 km, the ion velocity is in the range of about 5 km/s, the wave phase velocity is around 6000 km/s, and the wave frequency $\sim$ 2 Hz (Boehm et al., 2000). Therefore, the additional second term for hydrogen ions is at least of the same order of magnitude as the first and can essentially change the rate of acceleration. Note the dependence of the last term on the velocity and wave vector orientation.

Another problem for which the same additional term can be important is related to the nonlinear ionospheric effects produced by lightning discharge. For example, Yukhimuk and Roussel-Dupre (1997) proposed the parametric decay of a whistler mode wave into a lower hybrid wave and an ultra-low-frequency electromagnetic wave as an explanation of an observed electrostatic wave and magnetic pulse. A complete description of such phenomena includes the energy density evolution of the whistler wave. The whistler wave amplification is also of interest for some other problems related to lightning produced whistlers. According to observations (Kelley et al., 1985) the whistler burst duration is 10–20 ms. They describe a case where the whistler electric field changes 2.4 times over the range from 300 to 650 km away from the lightning discharge. The whistler, electron gyro- and plasma-frequencies are $\omega=3\times10^4\ \mathrm{s}^{-1}$, $\omega_{Be}=2\times10^5\ \mathrm{s}^{-1}$, $\omega_{pe}=4\times10^7\ \mathrm{s}^{-1}$. Assuming the magnetic moment is equal to zero, it follows from expressions (4) that the main term in the parallel ponderomotive force is the term with the time derivative. The whistler phase velocity is about 550 km/s and both components of this term can be of the same order.

Results obtained for a single particle (Eqs. (4)) can be useful for a particle-in-cell simulation. The kinetic equation (6) along with the distribution in Eq. (7) permit the analysis of the interaction of a multicomponent plasma with a circularly polarized wave, taking into account the system nonstationarity and inhomogeneity, and allow for the calculation of the macroscopic plasma parameters. The results are found under the assumption that the wave field varies on the same scale as the background magnetic field.

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Appendix.

Because the method of averaging for the approximate solution of differential equations is well known and widely used in plasma physics, we will only briefly outline the approach and present the results for the second-order approximation. Below, we are following the analysis of the system of first-order differential equations with multiple periodic coefficients presented by Morozov and Solov'ev (1966).

The characteristic equations (2) of the kinetic equation (1) can be presented as

$$\frac{\mathrm{d}x_k}{\mathrm{d}t}=f_k(x_i,t,\theta_m),$$

$$\frac{\mathrm{d}\theta_n}{\mathrm{d}t}=\frac{1}{\varepsilon}\,\omega_n(x_i,t,)+A_n(x_i,t,\theta_m) \tag{A.1}$$

with functions f_k, A_n, which are periodic with respect to the arguments θ_m, and ε is a small parameter. Eqs. (2) can be presented in this form introducing the angle $\phi=-\int\tilde{\omega}\,\mathrm{d}t$. We seek a substitution of variables like this,

$$x_k=\xi_k+\sum_{r=1}\varepsilon^r g_k^r(\xi_i,t,\alpha_m),$$

$$\theta_n=\alpha_n+\sum_{s=1}\varepsilon^s q_n^s(\xi_i,t,\alpha_m), \tag{A.2}$$

such that the new variables ξ_k, α_m satisfy equations which do not contain the phase θ_m:

$$\frac{\mathrm{d}\xi_k}{\mathrm{d}t} = \varphi_{ok}(\xi_i, t) + \sum_{r=1} \varepsilon^r \varphi_k^r(\xi_i, t),$$

$$\frac{\mathrm{d}\alpha_n}{\mathrm{d}t} = \frac{1}{\varepsilon}\,\omega_n(\xi_i, t) + \sum_{s=0} \varepsilon^s \Omega_n^s(\xi_i, t). \tag{A.3}$$

First, expressions (A.2) are substituted into (A.1), and then the time derivatives in terms like $(\partial g_k^r)/(\partial \xi_i)\,(\mathrm{d}\xi_i)/(\mathrm{d}t)$ are replaced by Eqs. (A.3). The right-hand side of the Eqs. (A.1) can be expanded in a Taylor series over the small parameter ε. Equating terms of the same order in ε, we can find equations which relate the functions $f_k(\xi_i, t, \alpha_m)$, $A_n(\xi_i, t, \alpha_m)$ and the new functions φ_k, Ω_n, g_k and q_n. For example, the zeroth-order equalities are

$$\varphi_k^0(\xi_i, t) + \frac{\mathrm{d}g_k^1}{\mathrm{d}\alpha_m}\,\omega_m = f_k,\ \ \Omega_n^0 + \frac{\mathrm{d}q_n^1}{\mathrm{d}\alpha_m}\,\omega_m = A_n + \frac{\partial\omega_n}{\partial\xi_i}\,g_i^1.$$

The periodic functions $f_k(\xi_i, t, \alpha_m)$, $A_n(\xi_i, t, \alpha_m)$ can be written as a sum of harmonics $C_{o,o}(\xi_i, t) + \sum_{v,\mu} C_{v,\mu}(\xi_i, t)$ $\exp\mathrm{i}(v\alpha_1 + \mu\alpha_2 + \cdots)$. Requiring that g_k and q_n be periodic functions with no constant terms, the equation of given order in ε can be split in two: an angle-dependent part and an angle-independent part. The latter part is found by integration over the period and yields $\varphi_k^0(\xi_i, t) = \bar{f}_k$, $\Omega_n^0 = \bar{A}_n$. Subtracting the result from the initial equation and integrating over α_m yields $g_k^1 = f_k/i\omega_n \equiv \overbrace{f_k}$, $q_n^1 = \overbrace{A_n} + (\partial\omega_n/\partial\xi_k)\,\overbrace{g_k^1}$. More details of these calculations can be found in Morozov and Solov'ev (1966), presented there with an accuracy to the first order in ε. The solution of the obtained system of equations for the unknown variables ξ_k can be presented as

$$\frac{\mathrm{d}\xi_k}{\mathrm{d}t} = \overline{f_k} + \overline{\sum_{r=1}^{r=2} \frac{\partial f_k}{\partial \xi_n}\, g_n^r + \frac{\partial f_k}{\partial \alpha_m}\, q_m^r}$$
$$+\frac{1}{2}\left(\overline{\frac{\partial^2 f_k}{\partial\xi_n\partial\xi_m}\, g_n^1 g_m^1 + 2\,\frac{\partial^2 f_k}{\partial\xi_n\partial\alpha_m}\, g_n^1 q_m^1 + \frac{\partial^2 f_k}{\partial\alpha_n\partial\alpha_m}\, q_n^1 q_m^1}\right), \tag{A.4}$$

where the bar notes that the oscillating terms are removed from these expressions and

$$g_k^1 = \overbrace{f_k},\quad q_n^1 = \overbrace{A_n} + \frac{\partial\omega_n}{\partial\xi_k}\,\overbrace{g_k^1},\quad \Omega_n^0 = \bar{A}_n,$$

$$g_k^2 = -\overbrace{\frac{\partial g_k^1}{\partial t}} - \bar{f}_i\,\overbrace{\frac{\partial g_k^1}{\partial \xi_i}} - \frac{\Omega_n^0}{\omega_n}\, g_k^1 + \overbrace{\frac{\partial f_k}{\partial \xi_i}\, g_i^1} + \overbrace{\frac{\partial f_k}{\partial \alpha_n}\, q_n^1},$$

$$q_n^2 = -\overbrace{\frac{\partial q_n^1}{\partial t}} - \bar{f}_i\,\overbrace{\frac{\partial q_n^1}{\partial \xi_i}} - \frac{\Omega_m^0}{\omega_m}\, q_n^1 + \frac{\partial\omega_n}{\partial\xi_i}\,\overbrace{g_i^2}$$
$$+\overbrace{\frac{\partial A_n}{\partial \xi_k}\, g_k^1} + \overbrace{\frac{\partial A_n}{\partial \alpha_m}\, q_m^1}. \tag{A.5}$$

The operation $\overbrace{z}$ applied to trigonometric functions in a multiple periodic system means

$$\overbrace{\exp\mathrm{i}(v\alpha_1 + \mu\alpha_2 + \cdots)} = \frac{\exp\mathrm{i}(v\alpha_1 + \mu\alpha_2 + \cdots)}{\mathrm{i}(v\omega_1 + \mu\omega_2 + \cdots)}.$$

It is assumed that there is no resonance in the system, i.e., $v\omega_1 + \mu\omega_2 + \cdots \neq 0$.

Kinetic equation (1), with the use of the chain rule, can now be rewritten in the new variables ξ_k, α_m

$$\frac{\partial f}{\partial t} + \frac{\mathrm{d}\xi_k}{\mathrm{d}t}\frac{\partial f}{\partial \xi_k} + \frac{\mathrm{d}\alpha_m}{\mathrm{d}t}\frac{\partial f}{\partial \alpha_m} = 0 \tag{A.6}$$

with $\mathrm{d}\xi_k/\mathrm{d}t, \mathrm{d}\alpha_m/\mathrm{d}t$ from the Eq. (A.3). As a result, the coefficients in the transformed kinetic equation are independent on the angle variables α_m. It can be proved that the distribution function f satisfying this equation is also angle independent (Qin et al., 1998). Hence Eq. (A.6) is angle independent and reduces to the first two terms, with the coefficients $\mathrm{d}\xi_k/\mathrm{d}t$ calculated with the help of expressions (A.4).

The solution of Eq. (A.6) can be transformed (Bernstein and Catto, 1985) to the old variables x_k, θ_m. With accuracy to the second-order in ε, the inverse transformation to (A.2) is

$$\xi_k = x_k - \varepsilon g_{1k} + \varepsilon^2\left(-g_{2k} + g_{1l}\frac{\partial g_{1k}}{\partial x_l} + q_{1m}\frac{\partial g_{1k}}{\partial \theta_m}\right), \tag{A.7}$$

where g_k and q_n are now functions of old variables x_k, θ_m, and t. Then, with the same accuracy, we have

$$f(\xi_k, t) = f(x_k, t) + \frac{\partial f}{\partial x_k}(\xi_k - x_k)$$
$$+\varepsilon^2\frac{\partial^2 f}{\partial x_k\partial x_l}\frac{g_{1k}g_{1l}}{2} \tag{A.8}$$

with $\xi_k - x_k$ from expression (A.7). It is not necessary to distinguish between the initial and the transformed variables in the right-hand side of this expression. This difference is explicitly taken into account in the transformation of the distribution function, with second-order accuracy. It should be noted that only if the terms that are linear with respect to the wave energy are taken into account, then Eqs. (A.4) and (A.8) are significantly simplified.

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