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Current-produced magnetic field effects on current collection

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Abstract. Current collection by an infinitely long, conducting cylinder in a magnetized plasma, taking into account the magnetic field of the collected current, is discussed. A region of closed magnetic surfaces disconnects the cylinder from infinity. Owing to this, the collected current depends on the ratio between this region and the plasma sheath region and, under some conditions, current reduction arises. It is found that the upper bound limit of current collection is reduced due to this change of magnetic field topology. The effect can be substantial even if the orbit-limited model of current collection is valid. This model is used to find the reduction of the total current collected by a cylinder (e.g., a bare tether). It is shown that this effect strongly depends on plasma density. The results are applied to a tether system in the ionosphere, and it is found that current reduction can be significant for long tethers in typical dayside ionospheric conditions.

1. Introduction

Current collection by electrostatic probes and electrodes in space plasmas is a widely discussed question [see *Laframboise and Sonmor*, 1990, 1993]. A related problem, current collection from space plasma in the situations arising with electrodynamic tether experiments, has also been the subject of analysis in numerous papers and conferences [e.g., *Raitt et al.*, 1990]. In particular, different methods of current collection have been studied. Current collection by a conductive spherical subsatellite was used in the TSS-1 and TSS-1R missions. The results of these flights are discussed in conference proceedings [see, e.g., *Guidoni et al.*, 1995] and journal articles [see, e.g., *Stone and Bonifazi*, 1998]. Current may also be collected using plasma contactors of different types [see, e.g., *Szuszczewicz*, 1990]. Of special interest is current collection using an uninsulated tether as the anode [*Sanmartin et al.*, 1993]. Current collection depends on a number of factors such as plasma characteristics (density and temperature), absorption properties of the electrode, spacecraft velocity, the ratio between insulated and uninsulated parts of the tether, electrode design (shape), and in particular strength and orientation of the ambient magnetic field in the plasma [*Chen*, 1965; *Swift and Schwar*, 1969].

The magnetic field acting on the plasma surrounding a tether system has two components: (1) the field inherent to the Earth or celestial body and (2) the field produced by current in the tether. The influence of the geomagnetic field on current collection depends on characteristic parameters of the tether-plasma system. *Szuszczewicz and Takacs* [1979] and *Szuszczewicz* [1990] characterized the conditions under which the effects of the geomagnetic field are important as a function of the ratio between the wire radius, electron gyroradius and the characteristic scale of the disturbed plasma (i.e., the plasma sheath). They found that magnetic effects are important, even when the wire radius is small compared to the electron gyroradius, if the latter is less than the thickness of the plasma sheath. This conclusion is based on the calculations by *Laframboise and Rubinstein* [1976], *Bettinger and Walker* [1965], *Miller* [1972], and experimental results collected by a pulsed plasma probe flown on a scientific rocket payload (NASA 18.170).

As far as we know, the question about the role of the magnetic field created by the current through the wire has not been considered in previous treatments. The goal to achieve ever-increasing currents has been a trait of the development of tether systems. As a result, the current-induced magnetic field can be larger than the Earth's magnetic field in a region comparable to other characteristic lengths of the system, such as the Debye radius or even the electron gyroradius. The current magnetic field can also change other characteristic lengths, in particular, the electron gyroradius, which alters the critical ratio between this and the wire radius. Perhaps more important than these effects, however, is that the current-induced magnetic field completely changes the magnetic field topology around the wire.

The goal of this paper is to analyze the role of the current-induced magnetic field in different regimes of current collection and the possible influence of this field on tether system design. A self-consistent calculation of the potential distribution and current collection that takes into account the current magnetic field is intrinsically more complicated than in the case when this field is neglected. This study only examines the upper bound limit of current collection and the orbit-limited current collected by a tether system in the presence of current-induced field. It should be noted that this concept of "magnetic insulation" is used to advantage in other space-borne applications, such as suppression of flashover during pulsed power [e.g., *Korzekwa et al.*, 1989; *Lehr et al.*, 1992], but in this case it is a detriment to the desired goal of maximum current collection. In section 2, the upper bound limit of current collection is discussed, adopting the standard approach developed for this problem [*Parker and Murphy*, 1967; *Laframboise and Parker*, 1973; *Laframboise and Rubinstein*, 1976; *Rubinstein and Laframboise,* 1982]. In section 3, the role of the current-induced magnetic field on the effectiveness of the tether

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system in the orbit-limited model is analyzed. In section 4, the assumptions used in the analyses, in particular those related to the potential distribution, and the role of some tether system parameters in current collection are discussed.

2. Upper Bound Limit of Current Collection

In this section we will analyze the general magnetic field around the current-driving cylinder in an external magnetic field and calculate the upper bound limit of current collection under some assumptions regarding the potential distribution. It is assumed below that the fields and plasma are uniform along the cylindrical wire.

A convenient coordinate system is with the wire current **I** along the z axis, and the external magnetic field $\mathbf{B}_0$ in the y-z plane (see Figure 1). In this system the total magnetic field **B** can be written as

$$\mathbf{B}=\mathbf{e}_r B_o \cos\alpha \sin\varphi + \mathbf{e}_\varphi \left[B_o \cos\alpha \cos\varphi + B_c \right] + \mathbf{e}_z B_o \sin\alpha \,, \qquad B_c = \frac{2I}{cr} \,, \tag{1}$$

where c is the velocity of light and polar coordinates r, φ in the x-y plane are introduced. The projections of the magnetic lines for such a field in the x-y plane are described by

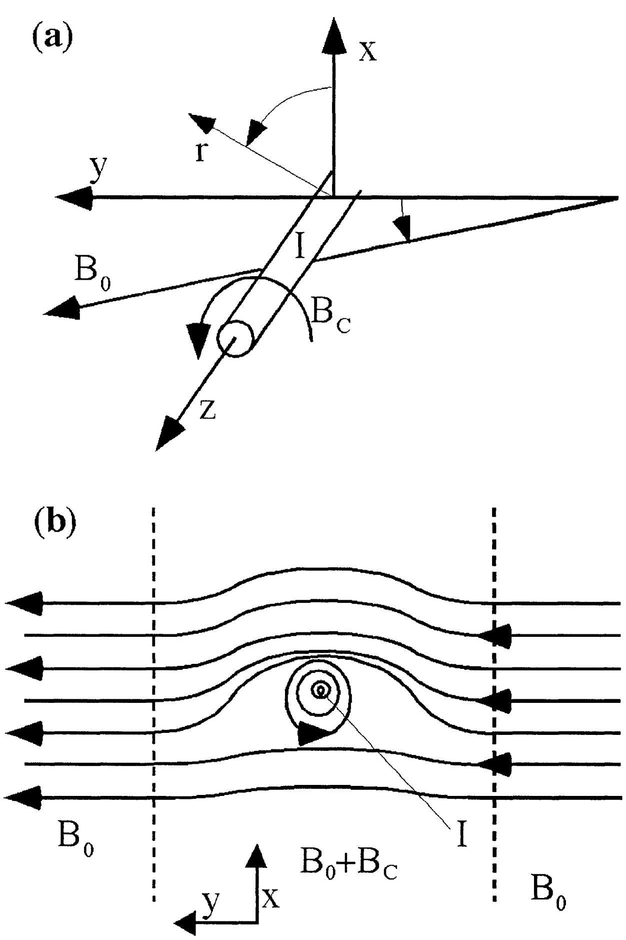

Figure 1. (a) Schematic of the relationship of the coordinate system to the current, ambient magnetic field, and induced magnetic field. (b) Schematic of the magnetic field topology around the wire, showing that the field is significantly modified by B_c only in a region near the wire (α=0).

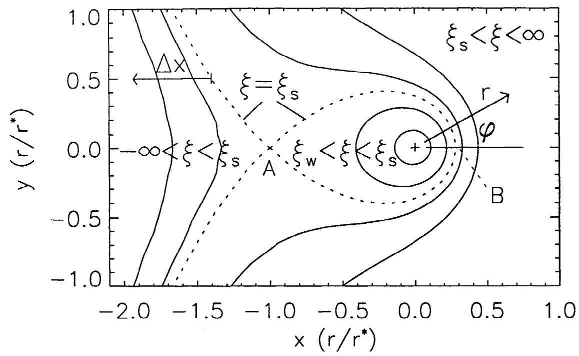

Figure 2. Magnetic field lines in the x-y plane near the tether. The dotted line is the separatrix. Also shown are the values of ξ in each region delineated by the separatrix. The points A and B on the separatrix have the x coordinates $-r^*$ and $0.3\,r^*$, respectively.

$$\xi = r^* \ln\frac{r}{r^*} + r\cos\alpha\cos\varphi = \text{const} \qquad r^* = \frac{2I}{cB_o} \tag{2}$$

and are presented in Figure 2 in normalized units. A separatrix, marked as the dotted line $\xi_s = -r^*(1+\ln\cos\alpha)$, is the x-y plane cross section of the cylinder that divides the closed magnetic surfaces around the wire and the unclosed surfaces farther out. Therefore the magnetic field lines near the wire are disconnected from infinity in the x-y plane. Note that ξ is not a single-valued function of the coordinates; it can be the same for the field lines inside the separatrix and outside of it. The magnetic field potential for **B** can be written as

$$A_\varphi = \frac{rB_o}{2}\sin\alpha \,,$$

$$A_z = -\left(rB_o \cos\alpha\cos\varphi + \frac{2I}{c}\ln r \right) = -B_o \xi \,, \tag{3}$$

where A_φ describes the magnetic field component parallel to the wire (the z component of the external magnetic field) and A_z describes the perpendicular one.

The Lagrangian of a particle immersed in both the magnetic field (3) and an electrostatic field of the wire ϕ is

$$L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\varphi}^2 + \dot{z}^2 \right) + \frac{e}{c}\left(\dot{z}A_z + r\dot{\varphi}A_\varphi \right) - e\phi \,. \tag{4}$$

Owing to the translational symmetry of the problem, L does not depend on z, so $\partial L/\partial \dot{z}$=const. Assuming the collected particles are electrons, this yields

$$P_z = m\dot{z} + \frac{e}{c}A_z = m\left(v_{zo} + \Omega\xi_o \right) = \text{const} \,, \tag{5}$$

where $\Omega = |e|B_0/mc$. Therefore both the z component of the particle generalized momentum in (5) and the particle energy, given by

$$E = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\varphi}^2 + \dot{z}^2 \right) + e\phi = \text{const} \tag{6}$$

are conserved.

The upper bound limit of current collected by a wire can be obtained if the flow of particles on the wire surface is calculated according to the conservation laws under the assumption that the particles do not face potential barriers along their trajectories. If we neglect the end effects of the wire, then the particles reaching the wire should start from the region connected to infinity outside the separatrix, should reach the separatrix, and then the wire. If the potential drops abruptly and turns to zero inside the separatrix, the collected current is restricted by the particles' thermal diffusion flux through the separatrix. For the case when the plasma sheath is larger than the separatrix, the approach developed by *Rubinstein and Laframboise* [1978], to obtain the boundary equations for a "magnetic bottle" within which the particle can reach the wire, can be used. For an electrostatic potential at infinity $\phi_0=0$, (5) and (6) give

$$E=(m/2)\left\{v_\perp^2+\left[v_{zo}+\Omega(\xi_o-\xi)\right]^2\right\}+e\phi \, . \tag{7}$$

Then the particles starting at infinity from the field lines with ξ_o are able reach the point (x,y) on the field line ξ if ξ_0 satisfies the inequalities

$$\xi_-\le\xi_o\le\xi_+ \, , \tag{8}$$

where

$$\xi_\mp=\xi-(1/\Omega)\left(v_{zo}\pm\sqrt{(2/m)(E-e\phi)}\right)$$

and the electrostatic potential ϕ should be taken at the point (x,y). The inequalities in (8) follow from (7) under the condition $v_{\perp w}^2=0$ (an assumption that maximizes the collected current) and also taking into account that the energy at infinity is positive. These inequalities should be supplemented by the condition that the electrons are starting from ξ_0 outside the separatrix. We will assume that inside the separatrix, along the closed magnetic field lines, the electrostatic potential is constant. This assumption will be discussed in section 4. Specifying the field line ξ and the point (x,y), the region of current collection can be found.

Two aspects of the magnetic bottle are changed in our case compared to the case of a negligible current-induced magnetic field. The first is that the region of current collection is shifted along the x axis (Figure 2) and the shift is different for the left-hand and right-hand borders of the region. The second is that the magnetic potential component A_z from (3) and therefore the z component of the particle generalized momentum from (5) has an extremum at the point where the particle intersects the separatrix if the particles come from the side of weaker magnetic field (left-hand side in Figure 2). For a particle from infinity to be collected by the wire it should cross this separatrix (that is, should also be collected by the separatrix). Therefore the minimal region of current collection from these two, the separatrix and the wire, should be used to calculate the current. For both regions the restricting magnetic field lines from the side of the weaker magnetic field can be found from the left-hand side inequality in (8). It should be set to

$$\xi_-\equiv\xi_-^W \, , \quad \xi=\xi_{w,l}=r^*\ln\left(r_w/r^*\right)-r_w\cos\alpha \, , \quad \phi=\phi_w \, ;$$

$$\xi_-\equiv\xi_-^S \, , \quad \xi=\xi_s=-r^*(1+\ln\cos\alpha) \, , \quad \phi=\phi_s$$

for the wire and the separatrix respectively. It turns out that the magnetic field line ξ_-^w is located farther from the wire than the line ξ_-^s, for any particle energy and electrostatic potential on the separatrix. Therefore the region of current collection from the side of the weaker magnetic field is determined by the collection on the separatrix and restricted by the magnetic field line

$$\xi_-^S=\xi_s-(1/\Omega)\left(v_{zo}+\sqrt{(2/m)(E-e\phi_s)}\right)\le\xi_o \, . \tag{9}$$

The region of field lines contributing to the current from the side where the magnetic field is larger (right-hand side in Figure 2) can be found from the right inequality in (8). Substituting

$$\xi_+\equiv\xi_+^W \, , \quad \xi=\xi_{w,r}=r^*\ln\left(r_w/r^*\right)+r_w\cos\alpha \, , \quad \phi=\phi_w \, ;$$

$$\xi_+\equiv\xi_+^S \, , \quad \xi=\xi_s=-r^*(1+\ln\cos\alpha) \, , \quad \phi=\phi_s$$

for the wire and separatrix cases, respectively, the magnetic field line restricting current collection from the side of larger magnetic field for the wire can be found. To find the upper bound limit of current collection, we must assume that all particles able to reach the wire are able to reach the separatrix. (This is impossible from the side of weaker magnetic field, as was found above.) Therefore, we will assume that the potential distribution is such that the region of current collection for the separatrix is not less than for the wire, at least from this side. This condition, $\xi_+^w\le\xi_+^s$, i.e.,

$$\sqrt{(2/m)(E-e\phi_w)}-\sqrt{(2/m)(E-e\phi_s)}\le-\Omega\left(\xi_{w,r}-\xi_s\right) \tag{10}$$

weakly depends on the particle energy if the wire potential is large compared to the particles' thermal energy. We will restrict the analyses below to this case, and because of this, the region of current collection from the side of the wire with larger magnetic field for all particles is restricted further by

$$\xi_o\le\xi_+^W=\xi_{w,r}+\frac{1}{\Omega}\left[\sqrt{2/m(E-e\phi_w)}+v_{oz}\right]. \tag{11}$$

The following calculation of the current is in fact the same as in the paper of *Rubinstein and Laframboise* [1978]. Also, it is assumed that far from the wire, $y_0\to-\infty$, the particle orbits are helical and the particles move along the external magnetic field lines. We will assume that the particle velocity distribution is Maxwellian and the plasma is collisionless. Then the current per unit wire length due to the particles collected from an interval Δx_o is given by

$$J=4\pi e\int_0^{\frac{\pi}{2}}\sin\theta d\theta\int\Delta x_o(E)f(v)v\cos\theta\cos\alpha v^2dv \, , \tag{12}$$

where

$$f(v)=n_o\left(\frac{m}{2\pi T}\right)^{3/2}\exp\left(-\frac{mv^2}{2T}\right).$$

Here a spherical coordinate system with polar axes along the magnetic field line is introduced for the velocity space description at infinity. The term $v\cos\theta\cos\alpha$ is the particle velocity component in the plane transverse to the wire, averaged over a gyroperiod. The domain of integration for θ selects the particles flowing toward the wire. The region of current collection $\Delta x_o(E)$ can be calculated using (9) and (11). Far away from the wire, i.e., in the region where the magnetic field of the current is negligibly small compared to the external magnetic field, the interval $\Delta x_o(E)$ is simply related to the corresponding $\Delta\xi_o(E)$ region as given by (9) and (11) (see Figure 2). In these inequalities the terms ξ_-^s, ξ_+^w are numbers labeling the magnetic field lines restricting the region of current collection for a particle with fixed energy and initial velocity v_{zo}. Equating ξ_-^s, ξ_+^w from (9) and (11) to $\xi_o(x_{o1},y_o)$ and $\xi_o(x_{o2},y_o)$, the region of current collection $\Delta x_o(E)=x_{o2}-x_{o1}$ can be calculated. It is easy to see from (2), written for ξ_0,

$$\xi_0=r^*\ln\left[\frac{\sqrt{x_0^2+y_0^2}}{r^*}\right]+x_0\cos\alpha=\text{const} \tag{13}$$

that along the field line for large x_o, y_o in the zeroth-order approximation, $x_o\sim-\ln y_o$. The width of the region of current collection along the x axis can then be found from

$$\xi_+^W-\xi_-^S=r^*\ln\left[\sqrt{\frac{x_{o2}^2+y_o^2}{x_{o1}^2+y_o^2}}\right]+(x_{o2}-x_{o1})\cos\alpha\,.$$

In the limit of $y_0\to\infty$,

$$\Delta x_o=\frac{\xi_+^W-\xi_-^S}{\cos\alpha} \tag{14}$$

with ξ_-^s, ξ_+^w from (9) and (11). As a result, we can present the region of current collection as

$$\Delta x_o(E)=\frac{1}{\cos\alpha}\left[(\xi_{w,r}-\xi_s)+\frac{1}{\Omega}\left(\sqrt{(2/m)(E-e\phi_w)}+\sqrt{(2/m)(E-e\phi_s)}\right)\right]. \tag{15}$$

The same is valid for $y_0\to-\infty$.

With the help of (12) and (15) the current calculation is straightforward, and the normalized current, $i=J/J_o=J/(r_wn_o(2\pi T/m)^{1/2})$, is given by

$$i=(\xi_{w,r}-\xi_s)/r_w\pi+(\beta\pi^{3/2})^{-1}\left[3\left(\sqrt{\chi_w}+\sqrt{\chi_s}\right)+\sqrt{\frac{\pi}{4}}(3-2\chi_s)\left(\text{erfc}\sqrt{\chi_s}\right)\exp\chi_s+\sqrt{\frac{\pi}{4}}(3-2\chi_w)(\text{erfc}\sqrt{\chi_w})\exp\chi_w\right], \tag{16}$$

where

$$\chi_{s,w}=-e\phi_{s,w}/T \qquad \beta=r_w\Omega/\sqrt{\pi T/2m}$$

$$\text{erfc}x=\frac{2}{\sqrt{\pi}}\left(\int_x^\infty\exp(-t^2)dt\right).$$

In the limit of vanishing current-induced magnetic field, i.e., setting $\xi_s=\xi_{w,l}$ and $\phi_s=\phi_w$, (16) leads to the result obtained by *Rubinstein and Laframboise* [1978]. Taking into account our assumption that $\chi_w\gg1$ and supposing also that $\chi_s\gg1$, (16) can be reduced to

$$i=\frac{r^*}{r_w\pi}\left(1+\frac{r_w\cos\alpha}{r^*}+\ln\frac{r_w\cos\alpha}{r^*}\right)+\frac{2}{\beta\pi^{3/2}}\left(\sqrt{\chi_w}+\sqrt{\chi_s}\right) \tag{17}$$

Expressions (16) and (17) for the collected current depend on the unknown electrostatic potential on the separatrix. The calculation of the potential distribution is a nontrivial problem that can only be addressed numerically. Nevertheless, a reasonable estimation for the upper bound limit of current collection can still be found with the help of the obtained results. The upper bound limit of current collection can be reached only if the electrostatic potential at the separatrix satisfies (10). Substituting the minimal potential from (10), or setting $\phi_s=\phi_w$ in (9), the range of upper bound limit current change can be found. Under conditions used in obtaining (17) we have

$$\frac{4}{\beta\pi^{3/2}}\sqrt{\chi_w}+\frac{2\cos\alpha}{\rho\pi}(1+\rho+\ln\rho)\le i<\frac{4}{\beta\pi^{3/2}}\sqrt{\chi_w}+\frac{\cos\alpha}{\rho\pi}(1+\rho+\ln\rho), \tag{18}$$

where $\rho=r_w\cos\alpha/r^*$.

The left-hand side equality in (18), in the limit $r^*\to0$, corresponds to the result discussed by *Parker and Murphy* [1967] for a spherical probe. The role of the current-induced magnetic field can be estimated from (18) by use of the presentation

$$i=i(B_c=0)\left[1+\frac{2r^*(1+\ln\rho)}{\pi r_w i(B_c=0)}\right]. \tag{19}$$

As can be seen from (10) the term $1+\ln\rho$ in (19) is negative.

It follows from (16)-(19) that the current-induced magnetic field reduces the upper bound limit of current collection. It can be noted that the situation inside the region of closed magnetic surfaces is to some degree similar to the case with the collecting cylinder parallel to the external magnetic field. In both cases the magnetic field lines did not intersect the probe. In the latter case, the collected current is also reduced [*Szuszczewicz and Takacs,* 1979]. As can be seen from (19), the current collected per unit wire length is zero at some point $\tilde{z}$ along the probe if the total current in the probe is large enough. The current collected from the plasma is zero for the part of the wire beyond the point where this maximum current in the probe, $I(\tilde{z})$, is reached. This is because when the current, and therefore the region of the closed magnetic surfaces around the wire, increases to some critical value, the region of current collection (15) collapses to zero for any fixed particle energy. Then, for a Maxwellian particle distribution the collected current (16)-(19) becomes zero. Note that this result is also a consequence of neglecting the probe end current.

The dependence of the current collected per unit wire length on the angle α between the probe and the external magnetic field in expressions (16)-(19) is not evident. These expressions depend on the parameter $r^*=r^*(z)$, which is a function on the total current $I(z)$ collected along the probe (2). This current depends on the potential distribution along the wire and on α. However, it is still possible to obtain the approximate dependence of the maximum total current, $I(\tilde{z})$, which can be collected by the probe, on this angle. With the help of (2) this dependence can be found by equating (19) to zero, yielding

$$\frac{I(\tilde{z})}{\sqrt{\chi(\tilde{z})}}\left(1+\frac{cr_wB_o\cos\alpha}{2I(\tilde{z})}+\ln\frac{cr_wB_o\cos\alpha}{2I(\tilde{z})}\right)=\text{const}\,,$$

where $\tilde{z}$ depends on α and the constant is an α-independent term. As far as this equality holds for an arbitrary angle, the current $I(\tilde{z})$ should be proportional to $\cos\alpha$. This result is exactly the same current dependence on angle between the probe and the external magnetic field that was found by *Laframboise and Rubinstein* [1976] in the limit when the probe radius is large compared to the electron gyroradius. The reasons why the results are the same are plain. As it can be seen from the model that we used in calculating (19), the "fine" structure of the magnetic field (the difference between the magnetic fields to the left- and right-hand sides of the wire) was neglected owing to the use of (10) instead of the potential on the separatrix, ϕ_s. It was also assumed that if the particle is able to reach the separatrix, the probe collects it. Under these assumptions the region of closed magnetic surfaces plays the role of the effective probe size. We also used the distribution of the guiding centers to describe the particles. Therefore our result should be the same in this approximation as in the named paper.

Finally, we should like to note that our results are not valid in the limit when the component of the external magnetic field perpendicular to the probe vanishes. In this case the restriction of particle angular momentum conservation should be added to the conditions (5) and (6) for the calculation of the region of current collection.

3. Effect of a Bare Tether Magnetic Field on Current Collection

It follows from the above analysis that because of the change in the magnetic field topology the current-induced magnetic field can lead to an additional restriction on the maximum current that can be collected by a tether with fixed length and therefore on the device's effectiveness. When the electric force is large compared to the magnetic force, the orbit-limited current can be used for the calculation of the current collection. Even in this case, however, in spite of the smallness of the magnetic force, the collected current can depend on the current-induced magnetic field. If the potential on the wire is screened to such an extent that the plasma sheath is inside the separatrix, then the current collection will be restricted by the diffusion rate across the magnetic field lines. We will discuss the restriction on the total current collected by a long wire due to the current-induced magnetic field assuming that the current is orbit-limited, i.e., the best possible case.

Let us consider an ideally conducting bare wire with a positive biased segment length L along the z axis. The potential $V(z)$ along the wire can be presented as

$$V(z)=E_m(L-z), \tag{20}$$

where E_m is the motional electric field projection along the wire. Then the orbit-limited current [*Sanmartin et al.*, 1993]

$$I(z)=\frac{4}{3}en_o r_w\sqrt{\frac{2eE_m}{m}}L^{1.5}\left[1-(1-z/L)^{1.5}\right] \tag{21}$$

and therefore also the parameter r^* (given in (2)) and the region disconnected from infinity are both growing functions with respect to z.

To estimate the radius of the plasma sheath, $R_{sh}(z)$, we will use the expression obtained by *Szuszczewicz and Takacs* [1979] and *Szuszczewicz* [1990]. With the help of (20), this quantity is found to be

$$R_{sh}(z)=\lambda_D\left[2.50-1.54\exp\left(\frac{-0.32r_w}{\lambda_D}\right)\right]\sqrt{\frac{eE_mL}{T}\left(1-\frac{z}{L}\right)}, \tag{22}$$

where λ_D is the electron Debye length. Note that the plasma sheath radius is a diminishing function of z. From this the critical point along the wire $\tilde{z}$, at which begins a region where the plasma sheath is totally inside the separatrix, can be found as the solution of

$$0.3r^*(\tilde{z})-r_L^e=R_{sh}(\tilde{z}) \tag{23}$$

where

$$r_L^e=\frac{1}{4\Omega}\sqrt{\frac{\pi T}{2m}}$$

is the electron Larmor radius. We diminished the region r^* to $0.3\,r^*$ because this is the smallest distance from the wire to the separatrix. The region of closed surfaces is also diminished on the Larmor radius calculated for the same distance, where the magnetic field is approximately four times larger than the external one. The external magnetic field is taken perpendicular to the wire. In this case the region of closed magnetic surfaces is smaller. The portion of wire with $z>\tilde{z}$ is disconnected from infinity in the x-y plane and therefore there is no current collected in this region. The fraction of wire collecting the current then can be found from (2), (21), (22), and (23) as a function of L

$$\frac{\tilde{z}}{L}=1-\left[\sqrt[3]{a+\sqrt{a^2+b}}+\sqrt[3]{a-\sqrt{a^2+b}}\right]^2 \tag{24}$$

where

$$a=\frac{1}{2}\left(1-\frac{\beta}{\alpha L^{1.5}}\right),\quad b=(3\alpha L)^{-3},$$

$$\alpha=0.2\frac{\sqrt{2}}{\pi}\frac{r_w\omega_{Pe}^3}{c^2\Omega},\quad \beta=\frac{\omega_{Pe}}{4\Omega}\sqrt{\frac{\pi T}{2eE_m}}$$

and $\omega_{Pe}^2=4\pi ne^2/m$ is the electron plasma frequency. Equation (23) has a solution only if the length of the positive biased segment L is larger than some minimal magnitude. This magnitude is determined by the condition a=0. For smaller L the plasma sheath is larger than the region of closed magnetic field surfaces anywhere along the segment L and $\tilde{z}=L$.

With the help of (24) the total current collected by the positive biased segment L can be calculated by substituting $\tilde{z}$ into (21). The result strongly depends on plasma density as can be seen from (24).

The current collecting part of the wire and the total collected current for an ionospheric altitude of ~ 300 km and some typical plasma densities are presented in Figure 3. The total collected current is normalized by its magnitude calculated neglecting the current-induced magnetic field. This current can be found from expression (21) setting $\tilde{z}=L$. The electron temperature is taken to be 0.1eV with an electric field of E=200 V/km. As it was mentioned, the role of the current-induced magnetic field strongly depends on plasma density. A typical dayside plasma density is $n\sim1.6\times10^6\,\mathrm{cm}^{-3}$. For this condition, there is practically no reduction in effectiveness for tethers with a positively biased length <3 km (assuming a wire radius of 0.2 cm). However, for tethers greater than ~5 km, its effectiveness is greatly reduced by the influence of the current-

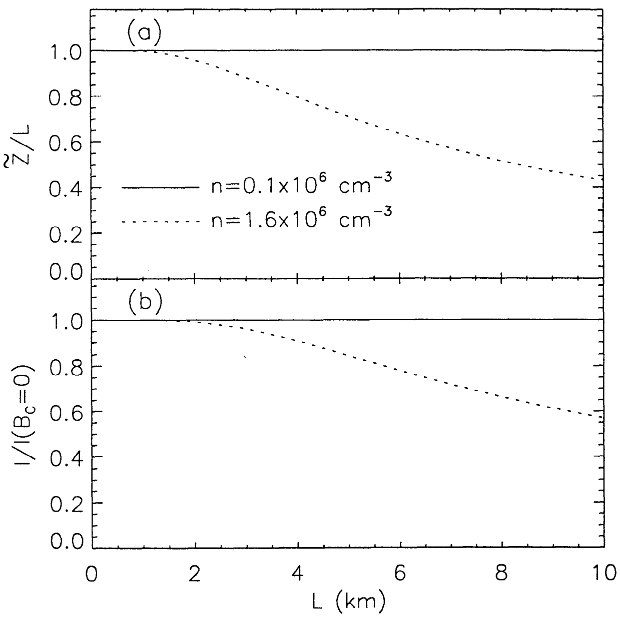

Figure 3. Dependence on the ambient thermal plasma density as a function of L (the length of the positively biased tether segment) for (a) the ratio of the tether length that is collecting current $\tilde{z}$ to L from (24) and (b) the ratio of the total collected current with current-produced effects to the total current neglecting the current-induced magnetic field from (21). A wire radius of 0.2 cm was assumed.

induced magnetic field. For nightside conditions, assuming a typical density of $n \sim 10^5 \text{cm}^{-3}$, the effect is much smaller and can be neglected for a reasonable tether length (that is, $\tilde{z} \cong L$). It should be stressed that the results presented in Figure 3 describe the minimum possible impact of the current-induced magnetic field on current collection. This will be discussed below. It can be seen from the presented results that even this minimal effect of the current-induced magnetic field in a dense enough plasma can significantly reduce the total collected current and should be taken into account in the design of a space tether system.

4. Discussion and Summary

A crucial question in the problem of current collection is that of the potential distribution around the tether, i.e., the sheath potential profile. This problem requires a self-consistent analysis of current collection and is out of the scope of the presented study. Therefore we should make a few comments related to the assumptions about the potential distribution used above.

In the upper bound limit current calculation presented above, we supposed that the closed magnetic surfaces are electric equipotentials. Such a potential distribution will arise in particular if there is some trapped electron population on these surfaces. Owing to the magnetic field inhomogeniety and end effects, such a population should be formed by particles that are not collected. Besides this, the estimations for the range of possible upper limit currents using (18), which is useful for practical applications, will only overestimate the possible collected current if the potential changes along the closed magnetic surface. Therefore (18) still presents the upper bcund limit of current collection in this case.

Another assumption in this calculation is related to the region of current collection. In obtaining (8) and (11), i.e., the borders of this region at infinity, the perpendicular kinetic energy on the wire surface was set to zero. This choice can lead to an overestimated current if, owing to the fields' approximate rotational symmetry near the wire, the angular velocity cannot be treated as an independent variable.

When calculating the tether effectiveness, the expression for the plasma sheath radius based on the results of *Szuszczewicz and Takacs* [1979] and *Laframboise and Rubinstein* [1976] was used. We assumed that their sheath potential calculations are still a reasonable approximation for the potential distribution in our case. These calculations are based on two main assumptions: (1) there is a three-dimensional particle velocity distribution inside the plasma sheath, and (2) the potential has cylindrical symmetry. The first is valid for a two-dimensional potential well in the presence of a magnetic field, at least for noncollected particle orbits. We have the same situation in our case. In addition, there is also the magnetic field inhomogeniety that will work in favor of a three-dimensional particle velocity distribution. As for the second assumption, the potential distribution in the presence of the wire's magnetic field is actually closer to being symmetric for the closed field lines, particularly near the wire. The results obtained for the total current collected in the orbit-limited case are not sensitive to the temperature taken for the Larmor radius calculation. For a temperature three times larger the change in current is only a few percent. Therefore the fast part of a Maxwellian distribution can not change the results. The current collecting part of the wire and the total collected current strongly depend on the radius of the wire. The reason is clearly evident: the current, and as a result the region of closed magnetic surfaces, are proportional to the wire radius, as seen in (21). As can be expected, the current reduction is approximately the same for $r_w L$=const (see Figure 4) because the effect depends on the collected current, which is roughly proportional to this quantity.

It should be stressed that our estimations of tether system current reduction because of the current-induced magnetic field

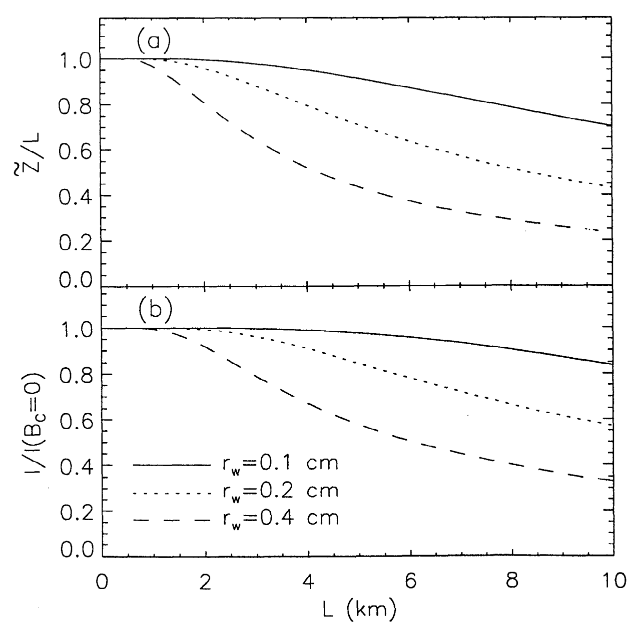

Figure 4. Dependence on the wire radius as a function of L for (a) $\tilde{z}/L$ and (b) $I/I(B_c=0)$. A thermal plasma density of 1.6×10^6 cm^{-3} was assumed.

are calculated under assumptions that minimize the possible reduction. Comparing the plasma sheath radius with the region of closed magnetic surfaces in (23), we substituted the region of such surfaces by the circular cylinder with a radius $0.3r^*$. This is the smallest distance from the wire to the separatrix (Figure 2, point B). That is, from the side of weaker magnetic field, the region of closed magnetic surfaces is more then three times larger (out to point A in Figure 2) than used in the calculation. In addition, we analyzed the case with the wire perpendicular to the external magnetic field. The size of the region of the closed magnetic surfaces around the wire depends on the ratio of the current-induced magnetic field and the component of the external magnetic field perpendicular to the wire. This region is minimal for the external magnetic field perpendicular to the wire. This also diminished the current reduction caused by the current-induced magnetic field. It is more reasonable to substitute the region of closed magnetic surfaces by a circular cylinder with a perimeter equal to the length of the separatrix. For such a cylinder (radius ~ $0.5r^*$), the current is reduced more than 30% compared the $0.3r^*$ cylinder results for a tether with a positive biased segment of 5 km (see Figure 5). In this calculation the Larmor radius is taken twice as large as in (23), but the growth of the magnetic field closer to the wire is still neglected. If this is taken into account, the current reduction will increase compared to the results presented in Figure 5.

To illustrate the possible role of the current-induced magnetic field on current collection, it is interesting to discuss the next situation. Suppose the tether system moves from an ionospheric region where the electron density is n~10^6 cm^{-3} into a region where the density happens to be n~4×10^6 cm^{-3}. The parameters of the plasma-tether system are presented in Figure 6. If the current-induced magnetic field is neglected it is expected that the collected current will be four times larger according to the plasma density change. If the current-induced magnetic field is taken into account, a 25% current decrease should be expected (Figure 6) for a tether with a 5-km-long positively biased segment for this change in plasma density.

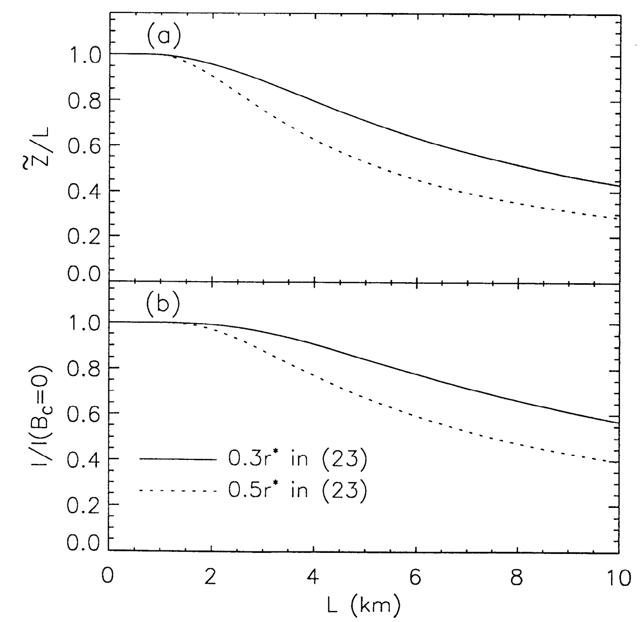

Figure 5. Dependence on the radius of the cylinder with that replaced the real region of closed magnetic surfaces (from (23)) as a function of L for (a) $\tilde{z}/L$ and (b) $I/I(B_c=0)$. A thermal plasma density of 1.6×10^6 cm^{-3} and a wire radius of 0.2 cm was assumed.

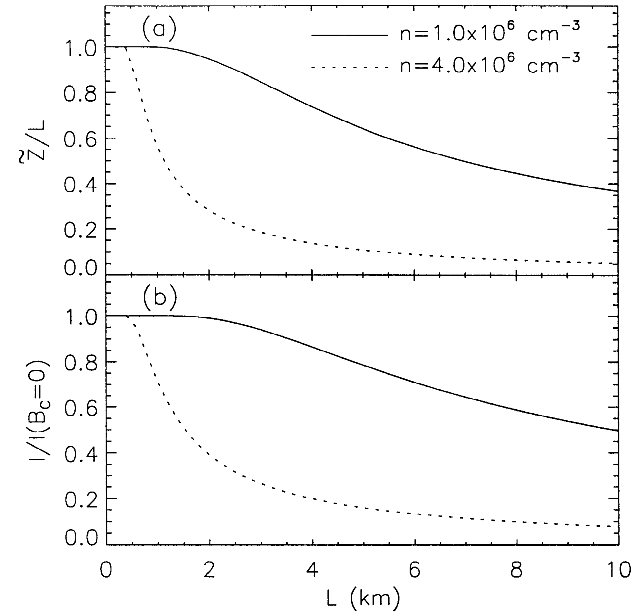

Figure 6. Dependence on the ambient thermal plasma density assuming $0.5r^*$ in (23) as a function of L for (a) $\tilde{z}/L$ and (b) $I/I(B_c=0)$. A wire radius of 0.3 cm was assumed.

Therefore, the current-induced magnetic field can reduce the current collected by an infinite cylinder. The current creates a region of closed magnetic surfaces around the cylinder disconnected from infinity. Because of this, the collected current depends on the ratio between the size of this region and that of the plasma sheath region. If the potential becomes zero inside the region of closed magnetic lines, the collected current will be limited by the end effects and particle flow across the magnetic field. In the case when the collected current can be characterized by the upper bound limit of current collection, the magnitude of this limit is reduced because of the current-induced magnetic field.

In the case when the orbit-limited model can describe the current collection, the region of current collection as well as the total collected current also can be reduced by the current-induced magnetic field. The part of the wire effectively collecting the current, and the total collected current, strongly depend on the plasma density. Both of them can be significantly reduced for conditions typical of the ionospheric altitude of ~300 km with a dayside plasma density of $n\approx1.6\times10^6$ cm^{-3} for a tether with a positive biased segment longer than 5 km. As it can be seen from these results, the current-induced magnetic field for a dense enough plasma may reduce the total collected current and should be taken into account in tether system design.

It should be remembered that in these calculations the wire resistance was neglected. Therefore the results do not present a systematical study of current reduction for a real space-borne current system. The main point of this study is to demonstrate that an effect exists, which depends on the system parameters, and should be carefully analyzed in the design of a real tether system. In addition, it is clear that the wire resistance will not change the physical phenomenon, and it is unlikely that the quantitative changes will be dramatic, especially for long tethers. It should also be noted that the shear of the total magnetic field can suppress the plasma instabilities around the tether, reducing the level of turbulence and the possible role of this turbulence in particle transport.

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