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Alfvén waves as a source of lower-hybrid activity in the ring current region

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Abstract. A coupling mechanism between lower-hybrid waves and low-frequency electromagnetic activity in the Earth's ring current region is analyzed. In the ring current region the observed amplitudes of Alfvén and/or fast magnetosonic waves, as a rule, do not exceed the threshold of flux instability of lower-hybrid waves in an electromagnetic field of a low-frequency wave. Even for these conditions, however, a mechanism of parametric generation of lower-hybrid waves from low-frequency Alfvén or fast magnetosonic waves is possible, and this theory is presented. In the studied case, lower-hybrid waves are saturated because of induced scattering with plasma electrons. For typical plasma and low-frequency wave parameters in the Earth's ring current region the saturation level of lower-hybrid waves, excited by the proposed mechanism, is in good agreement with experimental data.

1. Introduction

Space plasmas support a wide variety of waves. Wave-particle interactions as well as wave-wave interactions are of crucial importance to magnetospheric and ionospheric plasma dynamics. The excitation of lower-hybrid waves (LHWs) is widely discussed as a mechanism of interaction between plasma species and waves in space. These waves are particularly interesting because they couple well to both electrons and ions, providing an energy channel to the thermal plasma. Different mechanisms of LHW excitation have been studied as well as the phenomena produced by these waves. One of the possible mechanisms of LHW excitation is instabilities associated with electron beams, accelerated by double layers in the auroral region [*Maggs,* 1976; *Chang and Coppi,* 1981]. Other possibilities of LHW generation are the anomalous Doppler resonance of the waves with precipitating electrons [*Omelchenko et al.,* 1994] and the lower-hybrid drift instability in an inhomogeneous plasma [*Davidson et al.,* 1977; *Huba et al.,* 1978]. The last mechanism was discussed for the magnetopause in connection with ion heating and cross-field transport of electrons [*Shapiro et al.,* 1994] and maintenance of the magnetopause boundary layer [*Treumann et al.,* 1991] (see also recent review by *Winske et al.* [1995] for the role and importance of wave-particle transport at the magnetopause) for the cleft/cusp region [*Pottelette et al.,* 1990] and, with incorporation of the hot ion population, for the inner edge of the magnetospheric ring current [*LaBelle and Treumann,* 1988; *Roth et al.,* 1990].

Another possible mechanism of LHW excitation is wave-wave interaction. Lower-hybrid and low-frequency waves have been simultaneously observed in the cleft/cusp region [*Pottelette et al.,* 1990] and in the region of magnetospheric ring current/plasmasphere overlap [*LaBelle et al.,* 1988]. Such simultaneous wave activities were also observed during active ionospheric sounding rocket experiments [*Arnoldy,* 1993; *Bale et al.,* 1998]. In these situations, LHWs can be generated because of low-frequency wave (LFW) activity [*Khazanov et al.,* 1996, 1997a, b]. It is well known from the plasma physics literature that LFWs with frequencies $\omega < \omega_{Bi}$, where ω_{Bi} is the ion cyclotron frequency, could drive a host of high-frequency waves through drifts of plasma particles produced by the former waves [see, e.g., *Akhiezer et al.,* 1975]. If the drift velocity exceeds the ion thermal velocity, LHW generation is possible because of the flux instability. Only recently has this idea been adopted to explain some observations in space plasmas [*Khazanov et al.,* 1996, 1997a, b; *Bale et al.,* 1998]. As far as we know, these studies were the first time attention has been paid to the special role of heavy plasma ions in this well-known mechanism. It was shown that the relative drift between electrons and ions across the ambient magnetic field, from the observed amplitudes of Alfvén waves, is sufficiently large to drive

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LHWs in a magnetospheric plasma. This process results because particle drift velocities in the LFW electric field are mass-dependent. LHW excitation is possible from any transverse electric field fluctuations with frequencies comparable to the ion cyclotron frequency. These fields are frequently observed in the Earth's ionosphere and plasmasphere; in particular, there are ion cyclotron waves, fast magnetosonic waves, and so on.

In the auroral and cusp plasmas, where O^+ and H^+ ions are the main constituents, the O^+ drift produces an O^+ beam in a core e-H^+ plasma because of the electric field of the LFWs. This beam drives LHWs that in turn transfer wave energy to the particles by accelerating ions transversely to the ambient magnetic field and electrons parallel to it. *Khazanov et al.* [1996 1997a, b] demonstrated these by linear and nonlinear analytical calculations and applied their theoretical predictions to some wave events observed by the Viking satellite [*Pottelette et al.,* 1990].

The magnetospheric ring current is one of the objects that have a significant effect on interactions between the Earth's ionosphere and magnetosphere. A wide variety of waves are generated in the ring current region, in particular, Alfvén and fast magnetosonic waves [*Kennel and Petschek,* 1966; *Lyons and Williams,* 1984; *Young et al.,* 1981; *Roux et al.,* 1982], and LHWs [*Olsen et al.,* 1987; *LaBelle et al.,* 1988]. If the relative velocity of the electron and ion plasma components in the electromagnetic field of the Alfvén (or fast magnetosonic) wave is high enough, namely,

$$|\mathbf{u}| = |\mathbf{u}_i - \mathbf{u}_e| > 1.3 v_{Ti}, \qquad (1)$$

then there is a flux instability that excites LHWs [*Akhiezer et al.,* 1975]. Here v_{Ti} is the ion thermal speed. Figure 1 shows the mutual orientation of all of the vectors, presented in a Cartesian coordinate system with the external magnetic field along the z axis. In this context we can write explicit expressions for plasma particle velocities $\mathbf{u}_\alpha$ in the Alfvén wave field [*Akhiezer et al.,* 1975]:

$$u_{\alpha,x}(t) = \frac{e_\alpha v_A B \sin\omega_0 t}{m_\alpha c(\omega_0 - \omega_{B\alpha})}$$
$$u_{\alpha,y}(t) = \frac{e_\alpha v_A B \cos\omega_0 t}{m_\alpha c(\omega_0 - \omega_{B\alpha})}. \qquad (2)$$

Here e_α, m_α, and $\omega_{B\alpha}$ are the electric charge, mass, and gyrofrequency of type α particles, respectively; B is the magnetic field amplitude of the Alfvén wave with frequency ω_0; $v_A = B_0/\sqrt{4\pi\rho}$ is the Alfvén velocity (B_0 is the ambient geomagnetic field and ρ is the plasma density), and c is the speed of light. Now we can rewrite (1) in terms of the Alfvén wave magnetic field amplitude. For typical plasma parameters in the ring current region ($B_0 = 125\ \gamma$, $T_i = 1 - 30$ eV, and $n_e = 10 - 100\ \mathrm{cm}^{-3}$) this inequality takes the following form:

$$\frac{B}{B_0} > \frac{B_t}{B_0} \sim 10^{-2} - 10^{-1}, \qquad (3)$$

where B_t is a threshold Alfvén wave magnetic field amplitude to excite the LHW flux instability. These values of B/B_0 are only registered during disturbed geomagnetic conditions (e.g., *Anderson et al.* [1990] reported Pc 1 waves with amplitude $B/B_0 \sim 6 - 7 \times 10^{-2}$). During quiet geomagnetic conditions, Alfvén waves in the ring current region have a smaller amplitude than the lower border in inequality (3) (for instance, in the August 16, 1977, event observed by the GEOS 1 satellite [*Young et al.,* 1981], the normalized wave amplitude is $\sim 3 \times 10^{-3}$) or at best it is comparable with this border ($B/B_0 \sim 2.5 \times 10^{-2}$ was reported by *LaBelle et al.* [1988]). Thus amplitudes of Alfvén waves (and fast magnetosonic waves also) in the ring current region, as a rule, do not exceed the critical value in (3) to drive the LHW flux instability. However, in these conditions, parametric excitation of LHWs by the low-frequency Alfvén or fast magnetosonic waves may take place. In the present paper we study this as an additional mechanism of LHW instability in the conditions of the magnetospheric ring current. We obtain an expression of the growth rate for the proposed LHW instability and also estimate the energy density of LHWs excited by Alfvén or fast magnetosonic waves.

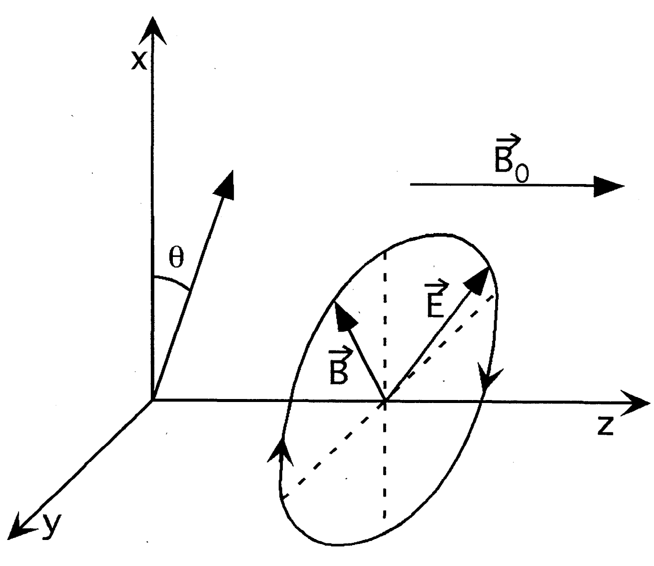

Figure 1. Mutual orientation of the lower-hybrid wave (LHW) normal vector ($\mathbf{k}_{LH}$), the Alfvén wave magnetic ($\mathbf{B}$) and electric ($\mathbf{E}$) fields, and the geomagnetic field ($\mathbf{B}_0$). Here $\cos\psi \simeq \sqrt{m_e/m_i}$.

2. Parametric Excitation of LHWs

Let us consider a homogeneous magnetoactive plasma. Assuming that the field of the pumping wave, $\mathbf{E}$, is uniform with frequency ω_0, we can write the expressions for the velocity and particle oscillation coordinates. Neglecting the magnetic field of the pumping wave and assuming that this wave has only x and y components, we get [*Gamayunov et al.,* 1992]:

$$\mathbf{u}_\alpha(t)=\begin{pmatrix} \frac{e_\alpha(\omega_0 e_x+i\omega_{B\alpha}e_y)E}{m_\alpha(\omega_0^2-\omega_{B\alpha}^2)}\sin\omega_0 t \equiv u_{\alpha,x}\sin\omega_0 t \\ \frac{e_\alpha(i\omega_0 e_y+\omega_{B\alpha}e_x)E}{m_\alpha(\omega_0^2-\omega_{B\alpha}^2)}\cos\omega_0 t \equiv u_{\alpha,y}\cos\omega_0 t \end{pmatrix} \quad (4a)$$

$$\mathbf{r}_\alpha(t)=\begin{pmatrix} -\frac{e_\alpha(\omega_0 e_x+i\omega_{B\alpha}e_y)E}{m_\alpha\omega_0(\omega_0^2-\omega_{B\alpha}^2)}\cos\omega_0 t \equiv x_\alpha\cos\omega_0 t \\ \frac{e_\alpha(i\omega_0 e_y+\omega_{B\alpha}e_x)E}{m_\alpha\omega_0(\omega_0^2-\omega_{B\alpha}^2)}\sin\omega_0 t \equiv y_\alpha\sin\omega_0 t \end{pmatrix} \quad (4b)$$

Here we introduced the polarization vector $\mathbf{e} = (e_x, e_y)$ of the pumping wave. For example, for Alfvén (fast magnetosonic) waves propagating along an external magnetic field this vector is $\mathbf{e} = (1, -i)$ ($\mathbf{e} = (1, i)$). Also, in (4b) we defined the amplitudes x_α and y_α of the displacements of the plasma particles in the pumping wave field.

The dispersion equation of longitudinal oscillations in an external electric field $\mathbf{E} = (E_x, E_y)$ has the following form [*Kaw,* 1976; *Mima and Nishikawa,* 1984]:

$$\frac{\tilde{n}_e(\omega,\mathbf{k})}{\chi_e(\omega,\mathbf{k})} = -\sum_{p,q=-\infty}^{\infty}\exp\left[iq(\theta_i+\pi-\varphi)\right]J_p(\mu) \times J_{p-q}(\mu)\frac{\tilde{n}_e(\omega+q\omega_0,\mathbf{k})}{1+\chi_i^p}. \quad (5)$$

Here we introduce the following notation: $\chi_\alpha(\omega, \mathbf{k})$ is the linear dielectric susceptibility of the type α plasma component, obtained in the oscillating coordinate system, connected with a given type of particles ($\alpha = e/i$ for plasma electrons/ions, respectively); $\tilde{n}_e(\omega, \mathbf{k})$ is the perturbation of the electron density in the proper, oscillating coordinate system; the following definitions were used:

$$\chi_i^p = \chi_i(\omega + p\omega_0, \mathbf{k});$$
$$\mu^2 = a_i^2 + a_e^2 - 2a_i a_e\cos(\theta_i - \theta_e);$$
$$a_\alpha^2 = k_x^2 x_\alpha^2 + k_y^2 y_\alpha^2;$$
$$\theta_\alpha = \arctan(k_x x_\alpha / k_y y_\alpha);$$
$$\varphi = \arctan\left(\frac{a_e\sin(\theta_e-\theta_i)}{a_i - a_e\cos(\theta_i-\theta_e)}\right);$$

ω and $\mathbf{k}$ are the frequency of the longitudinal wave and its wave normal vector, respectively; ω_0 is the frequency of the pumping wave, and x_α and y_α are the amplitudes of the oscillation coordinates of type α particles in the field $\mathbf{E}$ of this wave; and $J_p(\mu)$ is the Bessel function. The components of vector $\mathbf{a}_\alpha = (k_x x_\alpha, k_y y_\alpha)$ are proportional to the ratios of x_α and y_α to the wave lengths in the x and y directions, respectively, and θ_α is just the angle between this vector and the y axis. So, the square of this vector, a_α^2, characterizes a ratio between two scales in the studied problem. With this interpretation the physical meaning of μ is obvious because $\mu^2 = (\mathbf{a}_i - \mathbf{a}_e)^2$ and μ^2 may be readily related to a current magnitude in the pumping field $\mathbf{E}$. It follows from the above definition of μ^2 that $\mu = [k_x(x_i - x_e), k_y(y_i - y_e)]$. So, if the plasma electrons and ions have the same displacements in the field of the pumping wave, the μ will be equal to zero. (This situation takes the place in the limit $\omega_0 \to 0$ and, of course, for $E \to 0$.) In this particular case, only terms $p = q = 0$ are nonzero in the right-hand side of (5), and (5) is the ordinary dispersion equation of the longitudinal oscillations in the absence of the pumping wave. In the present paper we shall analyze the inverse case, when $\mu >> 1$ and the influence of the pumping wave on the plasma is essential.

In the case of a low-frequency pumping wave, (5) was analyzed by *Gamayunov et al.* [1992]. (Parametric excitation of electromagnetic waves by a low-frequency pumping wave was discussed by *Gamayunov et al.* [1993].) The developed method permitted *Gamayunov et al.* [1992, 1993] to analyze the problem of high- frequency wave generation in a plasma subjected to LFWs in a general form. They found that the obtained dispersion equation reduces to the Mathieu equation when the flux instability is absent. In the framework of their approach, *Gamayunov et al.* [1992, 1993] examined excitation of magnetohydrodynamic waves, whistlers, LHWs, and Langmuir oscillations. The obtained general form for the growth rate is dependent on both the square of the low-frequency electric field amplitude and the linear dielectric permittivities of the plasma species.

The most favorable conditions for parametric excitation of LHWs take place if the LHW wave normal vector is oriented almost perpendicular to the ambient magnetic field $[k_\parallel^2/[k_\parallel^2 + k_\perp^2] \sim m_e/m_i]$. In this case the linear dielectric permittivities of plasma electrons and ions, in the electron proper coordinate system, have the following forms [*Akhiezer et al.,* 1975]:

$$\chi_\alpha = \frac{\omega_{p\alpha}^2}{k^2 v_{T\alpha}^2}\left[1 + i\sqrt{\pi}z_\alpha W(z_\alpha)\right],$$
$$z_e = \frac{\omega}{\sqrt{2}k_\parallel v_{Te}}, \qquad z_i = \frac{\omega - \mathbf{k}_\perp\mathbf{u}}{\sqrt{2}k v_{Ti}}, \quad (6)$$
$$W(z) = \exp(-z^2)\left[1 + \frac{2i}{\sqrt{\pi}}\int_0^z \exp(y^2)\mathrm{d}y\right].$$

Here $\omega_{p\alpha}$ and $v_{T\alpha} = \sqrt{T_\alpha/m_\alpha}$ are the plasma frequency and thermal velocity of the type α plasma particles, respectively, and $\mathbf{u}$ is the relative ion velocity in the field of the pumping wave $\mathbf{E}$ and in the electron proper coordinate system $\mathbf{u} = (u_{i,x} - u_{e,x}, u_{i,y} - u_{e,y})$ obtained from (4a). The variables z_e and z_i are the ratios of the wave phase velocities to the thermal velocities in the electron and ion proper coordinate systems, respectively. Both the wave dispersion properties and the efficiency of the resonating wave-particle interaction will depend on the numerical values of these variables. To obtain a parametric growth rate of LHWs, one also should know the solution of the "unperturbed" dispersion equation, i.e., $1 + \sum_\alpha \chi_\alpha = 0$. At the boundary of the flux instability, where Landau damping of LHWs with the plasma electrons is balanced by the wave growth due to the ion flux, the solution of the equation $1 + \sum_\alpha \chi_\alpha = 0$ is [*Akhiezer et al.,* 1975]

$$z_e \simeq -z_i \quad z_e \simeq 0.9; \quad (7)$$

That is, the eigenmode frequency is $\omega_* \simeq 1.3kv_{Ti}$. Now we can get the parametric growth rate of LHWs for the case under consideration. Using the general results from *Gamayunov et al.* [1992], combining it with (6), and performing some simple calculations, we get the following expression for the LHW growth rate:

$$\gamma \simeq \frac{\omega_*}{4\sqrt{2}}\left(1+8z_e^2\right)^2\left(\frac{\omega_0}{\omega_*}\right)^3\left[\frac{\mu\omega_0}{\omega_*}z_e^2\left(\frac{T_i}{T_e}-1\right)\right]^2. \quad (8)$$

For the wave normal number for which the eigenfrequency $\omega_* \simeq \omega_{LH} = \sqrt{\omega_{Bi}|\omega_{Be}|(1+\omega_{Bi}^2/\omega_{pi}^2)/(1+\omega_{Be}^2/\omega_{pe}^2)} \approx \sqrt{\omega_{Bi}|\omega_{Be}|}$ (we used a dense plasma approximation, $\omega_{pe}^2/\omega_{Be}^2 >> 1$, that is valid for ring current conditions) and using (7), we can rewrite (8) in the form

$$\gamma \approx 10\omega_{LH}\left(\frac{\omega_0}{\omega_{LH}}\right)^3\left[\frac{\mu\omega_0}{\omega_{LH}}\left(\frac{T_i}{T_e}-1\right)\right]^2. \quad (9)$$

The physical mechanism of the examined instability is as follows. The interaction of the proper mode of the unperturbed plasma with the frequency ω_* and pumping wave **E** generates oscillations with the frequencies $\omega_* \pm \omega_0$. The interaction of these oscillations with the pumping wave leads to the increase of the amplitude of the proper plasma mode. In other words, the beating of high-frequency waves ω_* and $\omega_* \pm \omega_0$ are in resonance with the low-frequency pumping wave. As can be seen from (4b), the obtained growth rate is proportional to the square of the LFW amplitude through the parameter μ (which is related to the current induced by the pumping wave, as discussed above).

In the following estimations we shall assume that the Alfvén wave amplitude does not differ too much from the threshold of the flux instability obtained in (3). In this case we have $\mu\omega_0 \lesssim \omega_{LH}$ because in the case of the flux instability, $\mu\omega_0 \approx ku \sim \omega_{LH}$. In addition, we shall assume that the ion temperature is about twice the electron temperature by supposing that $|T_i/T_e - 1| \sim 1$ is valid, and we shall employ the pumping wave frequency $\omega_0 \sim \omega_{Bi}$. Then, from (9), we have the following estimate ($\omega_0/\omega_{LH} \approx (m_e/m_i)^{1/2}$):

$$\gamma \approx 10^{-4}\omega_{LH}. \quad (10)$$

One of the principal approximations under which the dispersion equation (5) was derived is an approximation of the regular phases for the pumping wave. In the case under consideration this assumption means that on the timescale of order $2\pi/\gamma$ the pumping wave may be treated as if it was coherent. For external magnetic field $B_0 = 125\ \gamma$ we have an estimate of $2\pi/\gamma \approx 125$ s. So in order for the above theory to be applied to LHW excitation the LFWs need to be regular for at least 2 min. The digitally filtered waveforms for a Pc 1 event, presented by *LaBelle and Treumann* [1992], show strong wave coherence during a time interval about of $\sim$2 min (these authors selected a time interval of only 110 s for their wave analysis). So, the required coherent time of order 2 min does not seem unusual for the low-frequency waves in the ring current region. Another assumption, the one that the LFW must be in a particular mode, is not too crucial because the growth rate (9) depends on μ^2 and is therefore not so sensitive to the LFW polarization.

Expressions (8) and (9) for the growth rate of LHWs are valid if the Alfvén wave amplitudes comply with the following inequality:

$$10^{-1} < \frac{B}{B_t} \leq 1. \quad (11)$$

For these Alfvén wave amplitudes we may consider that the wave normal number of LHWs is approximately constant ($k \simeq \omega_{LH}/v_{Ti}$). If the Alfvén wave amplitude exceeds B_t, the flux instability dominates the LHW growth. If the Alfvén wave amplitude runs below $10^{-1}B_t$, there is an additional requirement on the wave normal number of LHWs to avoid Landau damping of LHWs by the plasma particles. This requirement is that k should be at least 3 times less than the above value. In this case, LHWs may be considered to be in the cool plasma approximation. Parametric generation of LHWs in this approximation was discussed by *Gamayunov et al.* [1992]. For these Alfvén wave amplitudes and wave numbers of LHWs the parametric growth rate decreases dramatically to the value $\gamma \approx 10^{-7}\omega_{LH}$. So, this range of the low-frequency wave amplitudes does not have much interest in the context of the objectives of the present study. A sketch of the LHW growth rate versus amplitude of the low-frequency wave is presented in Figure 2. Two regions are highlighted here, the region of significant parametric instability (region I) and the region of flux instability (region II). In this study, only the excitation of LHWs in region I has been discussed. While LHWs can potentially be excited if the low-frequency wave amplitude is in region II, such amplitudes are rarely achieved in the magnetospheric ring current, and therefore LHW growth in this regime has not been considered further here.

3. Energy Density of LHWs

Let us estimate the energy density of the lower-hybrid waves generated from the discussed parametric instability. At $k_\parallel^2/(k_\parallel^2 + k_\perp^2) \leq m_e/m_i$ the main nonlinear process, within the framework of weak turbulence theory, is induced scattering of LHWs by plasma electrons, with the rate [*Musher et al.*, 1978]

$$\gamma_s = \frac{\omega_{Be}^2 W}{\omega_{LH} n T_e}.$$

Here W is the energy density of the LHWs and n is the plasma number density. By equating the parametric growth rate (9) to the induced scattering rate γ_s the saturation level of the lower hybrid waves can be written as

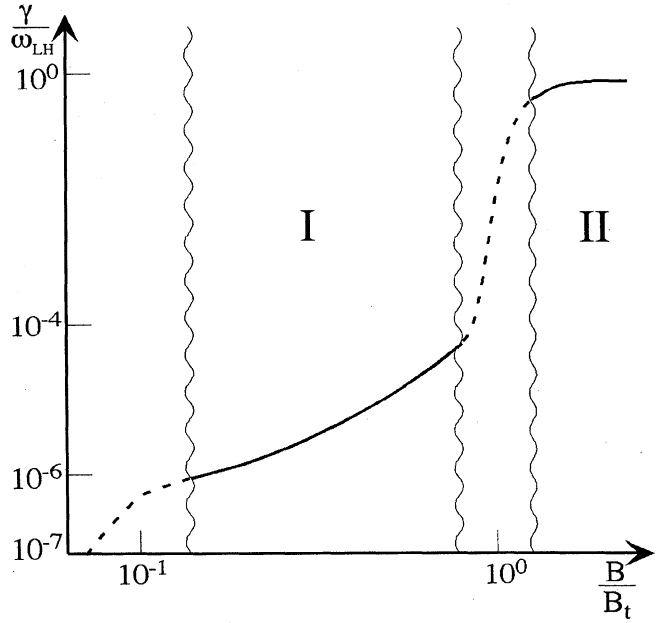

Figure 2. A sketch of LHW growth rate versus amplitude of the low-frequency pumping wave. Region I is the region of significant parametric instability, region II is the region of flux instability.

$$\frac{W}{nT_e} \approx 10 \left(\frac{\omega_{LH}}{\omega_{Be}}\right)^2 \left(\frac{\omega_0}{\omega_{LH}}\right)^3 \left[\frac{\mu\omega_0}{\omega_{LH}}\left(\frac{T_i}{T_e}-1\right)\right]^2. \tag{12}$$

Let us discuss the dependence of $\mu\omega_0/\omega_{LH}$ on the plasma and the low-frequency wave parameters. We suppose that the pumping wave is an Alfvén or fast magnetosonic wave propagating along an external magnetic field. In this case, $\theta_i = \theta_e$ and $\mu^2 = (\mathbf{a}_i - \mathbf{a}_e)^2 = (a_i - a_e)^2$. In the most favorable conditions, when the pumping wave amplitude does not differ too much from the threshold of the flux instability, the wave normal number of LHWs is $k \simeq \omega_{LH}/v_{Ti}$. Then the following dimensionless function

$$F = \frac{\frac{\mu\omega_0}{\omega_{LH}}}{\frac{B}{B_0}\frac{B_0}{\sqrt{8\pi n_e T_i}}}$$

depends only on $Y = \omega_0/\omega_{Bi}$. In Figure 3 the dependences of F on Y for Alfvén and fast magnetosonic waves are depicted. As can be seen, the parametric excitation of LHWs is more effective for an Alfvén pumping wave and for higher values of Y.

Data from the May 6, 1982 event obtained by the Dynamics Explorer 1 satellite gave the experimental value $W/(nT_e) \approx 1.7 \times 10^{-10}$ [*Olsen et al.*, 1987]. The plasma parameters observed during this event were $B_0 \approx 320\ \gamma$, $T_i \approx 1$ eV, $n_e \approx 20$ cm^{-3}. The typical frequencies and magnetic field amplitudes of Pc1-2 pulsations may be obtained from the Active Magnetospheric Particle Tracer Explorers (AMPTE) Charge Composition Experiment (CCE) satellite data. For the 3-5 L-shell range $f_0 = 0.5 - 3$ Hz, $B \approx 3.5\ \gamma$ [*Anderson et al.*, 1992a, b] and we can estimate the energy density of LHWs using (13). For selected parameters of $B/B_0 \approx 10^{-2}$, $Y = 0.1 - 0.6$, and $T_i/T_e \approx 2$ we have

$$\frac{W}{nT_e} = 6.9 \times 10^{-13} - 1.5 \times 10^{-8}. \tag{13}$$

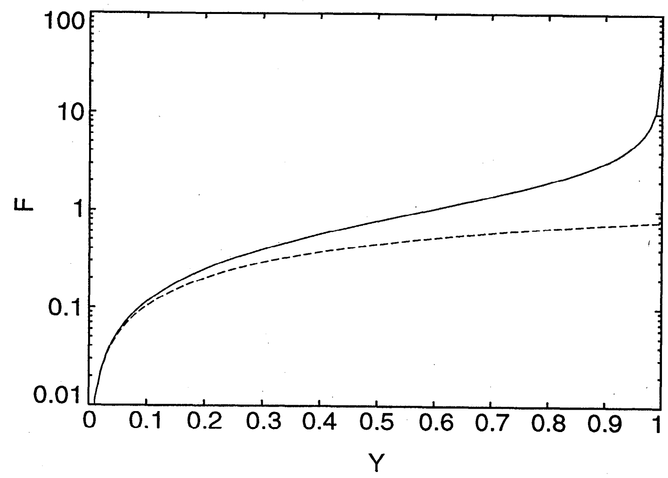

Figure 3. Dependence of the function $F = (\mu\omega_0\sqrt{8\pi n_e T_i})/(\omega_{LH}B)$ on $Y = \omega_0/\omega_{Bi}$. The solid line is for Alfvén pumping wave, and the dashed line is for fast magnetosonic pumping wave.

Thus for the cases of simultaneous observations of lower-hybrid and low-frequency waves, the proposed mechanism of lower-hybrid wave generation is able to take place.

4. Discussion and Conclusion

Excitation of lower-hybrid waves is widely discussed as a mechanism of interaction between plasma species and waves in ionospheric and magnetospheric plasmas. Such waves are particularly interesting because they couple well to both electrons and ions. Various mechanisms of lower hybrid wave excitation have been studied as well as the phenomena produced by these waves [*Davidson et al.*, 1977; *Chang and Coppi*, 1981; *Bingham et al.*, 1984; *Gurnett et al.*, 1984; *Pottelette et al.*, 1990; *Omelchenko et al.*, 1994]. In some cases, lower-hybrid wave activity has been observed simultaneously with the low-frequency waves [*LaBelle et al.*, 1988; *Pottelette et al.*, 1990; *McFadden et al.*, 1998]. Such simultaneous wave activities also have been observed in active ionospheric sounding rocket experiments [*Arnoldy*, 1993; *Bale et al.*, 1998]. A reason for this may be some wave–coupling mechanism between the two wave modes.

In the present paper we studied a possibility of lower-hybrid wave generation in the electromagnetic fields of Alfvén and/or fast magnetosonic waves for magnetospheric ring current conditions. The flux instability of lower-hybrid waves in the low-frequency wave field [*Khazanov et al.*, 1996, 1997a, b] does not take place in the ring current region if the amplitudes of the Alfvén or fast magnetosonic waves do not exceed the threshold of this instability, i.e., if inequality (3) is not satisfied. However, in these geophysical conditions a parametric excitation [*Mima and Nishikawa,* 1984] of lower-hybrid waves by the low-frequency Alfvén or fast magnetosonic waves (or by the mixture of these waves) is possible [*Gamayunov et al.,* 1992, 1993]. It was the intent of this study to present some theoretical aspects of the parametric excitation of lower-hybrid waves in the presence of low-frequency waves. A comparison of the LHW energy density, that follows from the studied instability, with the observed energy in the ring current region was also presented.

The parametric excitation of LHWs has been analyzed with an approximation of the regular phases for the low-frequency pumping waves. This assumption means that on the timescale of the order of the LHW growth time the pumping wave is treated as a coherent wave. In order to apply the discussed mechanism of the LHW excitation to conditions in the ring current, LFWs need to be regular for at least 2 min. *LaBelle and Treumann* [1992] selected a time interval of 110 s for their wave analysis of one Pc1 event in the evening ring current region. The digitally filtered waveforms show strong wave coherence during the selected time interval. So, the required coherent timescale of 2 min does not seem unusual for the low-frequency waves in the ring current region.

In our derivation we also assumed that LFWs are in a particular mode. This assumption is not crucial for the obtained results because the growth rate (9) depends on μ^2 and is not very sensitive to LFW polarization. However, some difference exists, especially when ω_0 approaches ω_{Bi}. In this case, Alfvén waves are more efficient than fast magnetosonic waves for producing LHWs (see Figure 3).

Specific quantities were derived for typical plasma and wave parameters in the ring current region. The obtained growth rate of lower-hybrid waves from the parametric instability was estimated to be $\gamma \approx 10^{-4}\omega_{LH}$. Here ω_{LH} is the frequency of lower-hybrid oscillations. The resulting energy density of the LHWs was determined by setting this value equal to the dominant damping mechanism (induced scattering) to obtain a normalized energy density of $W/nT_e \lesssim 10^{-8}$. This LHW energy density is in agreement with wave observations in the ring current region. Therefore we believe that these results, particularly the general formulations of (9) and (13), should be useful in the interpretation of plasma observations in the Earth's magnetosphere.

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