Lower hybrid oscillations in multicomponent space plasmas subjected to ion cyclotron waves

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Abstract. It is found that in multicomponent plasmas subjected to Alfvén or fast magnetosonic waves, such as are observed in regions of the outer plasmasphere and ring current-plasmapause overlap, lower hybrid oscillations are generated. The addition of a minor heavy ion component to a proton electron plasma significantly lowers the low-frequency electric wave amplitude needed for lower hybrid wave excitation. It is found that the lower hybrid wave energy density level is determined by the nonlinear process of induced scattering by ions and electrons; hydrogen ions in the region of resonant velocities are accelerated; and nonresonant particles are weakly heated due to the induced scattering. For a given example, the light resonant ions have an energy gain factor of 20, leading to the development of a high-energy tail in the H+ distribution function due to low-frequency waves.

1. Introduction

1.1. Low-Frequency Wave-Plasma Interaction

Low-frequency Alfvén, fast magnetosonic, and ion cyclotron (IC) waves are commonly found in space plasmas. One example is IC waves generated by ion temperature anisotropies in the ring current [Cornwall, 1965, 1966; Kennel and Petschek, 1966; Liemohn, 1967]. IC waves have also been observed at geostationary orbit [Young et al., 1981; Mauk, 1982; and references therein], in the outer magnetosphere [Anderson et al., 1992], at midlatitude high latitudes [Erlandson et al., 1990], and at ionospheric altitudes [Iyemori and Hayashi, 1989].

Low-frequency waves (LFWs) in the IC range can effectively interact with different components of the magnetospheric plasma through heating and scattering (e.g., reviews by Gendrin [1981, 1983, 1985] and Chang and Andre [1993]). Such LFWs are believed to be responsible for precipitation of ring current protons [Kennel and Petschek, 1966; Cornwall et al., 1970], formation of SAR arcs [Cornwall et al., 1971] and acceleration of heavy ions [Omura et al., 1985; Tanaka, 1985]. Noncollisional damping of IC waves may lead to the heating of cold electrons and ions in the outer plasmasphere [Galeev, 1975; Gorbachev et al., 1988; Konikov et al., 1989], deduced from the observed increase of plasma temperatures in this region [Decreau et al., 1982; Gringauz, 1983, 1985; Comfort et al., 1985]. The transport of the dissipating wave energy from the plasmasphere along the geomagnetic field lines into the ionosphere below could account for the observed subauroral rise in the electron temperature [e.g., Kotyra et al., 1986, and references therein].

By interacting with charged particles, the waves influence the behavior of the plasma as a whole, and therefore wave effects must be included in corresponding models. Ganguli and Palmadesso [1987, 1988], Singh [1988], Singh and Tord [1990], Brown et al. [1991, 1995], and Lin et al. [1992, 1994] took into account effects from the low-frequency (LF) electrostatic turbulence, and they demonstrated that wave-particle interactions lead to significant effects on the evolution of the core plasma distribution functions.

In developing a mathematical model to describe plasma transport in the magnetosphere-ionosphere system that accounts for the active wave processes which occur there, we must develop a general scheme to include an analysis of the dispersion characteristics of the medium in order to choose suitable wave modes, a wave-particle interaction mechanism, and a system of hydrodynamical equations governing macroscopic plasma parameters which properly accounts for the presence of wave-particle interactions. One basis for this scheme’s development has been described elsewhere [Khazanov and Chernov, 1988; Konikov et al., 1989; Chernov et al., 1990; Gamayunov et al., 1991, 1992; Gorbachev et al., 1992].

When such LF waves are propagating in a plasma, a ponderomotive force is produced, which leads to significant effects in the plasma. Boehm et al. [1990] suggested that this ponderomotive force can be the cause of significant density perturbations in the auroral zone. Allan et al. [1991] and Allan [1992] have investigated mass transport caused by the ponderomotive force of hydrodynamic waves in the middle magnetosphere. It was also suggested that significant plasma energization could occur in these regions [Allan, 1993]. Li and Temerin [1993] and Lee and Parks [1996] have used similar ideas to evaluate ion energization by large-amplitude Alfvén waves (E>100 mV/m; \omega<\Omega_i) in the auroral zone.
It was recently demonstrated by Khazanov et al. [1996] that large-amplitude LFWs can generate lower hybrid waves (LHWs) in the auroral zone and ring current region. The ion energization due to the LHWs may be comparable with that produced by the ponderomotive force of the LFWs. The purpose of this paper is to extend the analysis of Khazanov et al. [1996] to multicomponent plasmas and to small LFW electric fields.

1.2. LHW Excitation Due to LFWs and Description of the Problem

It is well known that in the electric field of a LFW, electrons and ions move with different velocities. The arising ion-electron relative velocity can lead to a current instabilities and, in particular, LHW excitation. Depending on the frequency of the LFW, the magnitude of the electric field, plasma composition, temperatures of different plasma species, and the external magnetic field, strong or weak LHW turbulence may develop.

In the case of large-amplitude LFWs, strong lower hybrid (LH) turbulence could be produced [Garmanyuk et al., 1997; Khazanov et al., 1996]. Strong LH turbulence results in plasma heating and acceleration [Shapiro et al., 1993], the formation of ion conics [Chang and Coppi, 1981; Retiere et al., 1986], and possibly the saturation of Alfvén oscillations in the ring current region [Garmanyuk et al., 1992]. Weak LH turbulence can also lead to important changes in the plasma state; for example, Omelenko et al. [1994] found that the LH “fan” instability may accelerate ions.

The level of energy density of LFWs which leads to excitation of LHWs in a hydrogen plasma, was discussed by Khazanov et al. [1996]. It can be estimated by taking into account that the ion-electron relative velocity needed for LHW excitation is of the same order of magnitude as the thermal ion velocity. This leads to a requirement for the LFW energy density \(E_0^2\):

\[
\frac{E_0^2}{8\pi nT} > \frac{\omega_{pi}}{\omega_{ci}}\frac{\omega_{ci}^2}{\omega_{pi}^2} \tag{1}
\]

where \(\omega_{pi}\) and \(\omega_{ci}\) are the cyclotron and plasma frequencies for ions, respectively; \(nT\) is the thermal plasma energy density; and \(\omega_0\) is the LFW frequency.

Khazanov et al. [1996] assumed that the plasma consists of only electrons and protons and that the LFW electric field is strong enough not only to make the relative proton velocity greater than its thermal velocity, but also to create strong LHW turbulence. From (1), however, it can be seen that the LFW energy density needed for LHW excitation may be lower in the presence of a heavy ion component. Since the magnetosphere and ionosphere have multicomponent plasmas, it can be supposed that LHWs can be excited in such plasmas by LFWs with amplitudes less than that needed for a proton-electron plasma.

This leads us to the problem of LFW interaction with a multicomponent plasma due to LHW excitation. In accordance with this problem, we will study the following questions:

1. How does a small mixture of heavy ions influence the LFW electric field needed for LHW excitation? What are the growth rates of the LHWs and how do they depend on the heavy ion population, LFW electric field, and core plasma parameters? What are these dependencies in the kinetic and hydrodynamic regimes of LHW excitation? (These are discussed in section 2.)

2. What is the physical mechanism that determines the level of LHW energy density? Does the quasilinear interaction of LHWs with oxygen, helium, and hydrogen ions as well as with electrons lead to saturation of the LHW instability, or is it the nonlinear-induced scattering mechanism that determines the LHW energy density level? What is the level of LHW energy? (These are discussed in sections 3.1 and 3.2.)

3. What is the influence of weak LHW turbulence on different plasma species? (This is discussed in section 3.3.)

In section 4 the suppositions used and results obtained are summarized. Fulfillment of these suppositions and the possibility of LFW interaction with magnetospheric plasma due to LHW excitation are analyzed, and an estimation of particle energization for a magnetospheric plasma is conducted. Section 5 contains a short summary of the results.

2. Linear Analysis

2.1. Ion Drift in a LFW Electric Field

Let us consider a multicomponent plasma containing electrons and oxygen, helium, and hydrogen ions subjected to LFW. In a variable electric field \(\mathbf{E}\) (for instance, the electric field of an Alfvén or fast magnetosonic wave, which has a component normal to the external magnetic field \(\mathbf{B}_0\)) the electrons and ions will acquire different velocities, \(\mathbf{w}_e\) and \(\mathbf{w}_i\), due to this field. If the frequency \(\omega_0\) of the field is considerably lower than the electron cyclotron frequency, the electron and ion velocities will be given by [e.g., Akhiezer, 1975]

\[
w_e = \Re \left[ \frac{e \mathbf{E} \times \mathbf{B}_0}{B_0^2} - i \mathbf{E} \cdot \mathbf{b} \frac{e}{m_e \omega_0} \right], \quad w_i = \Re \left[ \frac{e}{m_i (\omega_{bi}^2 - \omega_0^2)} \frac{\mathbf{E} \times \mathbf{b} + i \mathbf{b} \omega_0 (\mathbf{E} \times \mathbf{b})}{\mathbf{b}} \right] \tag{2}
\]

where \(\mathbf{b}\) is a unit vector in the direction of the external magnetic field \(\mathbf{B}_0\) and \(\omega_{bi}\) is the ion cyclotron frequency of the ion species \(i\). In a LFW the parallel electric field component can be small and the perpendicular drift velocities of the electrons and ions may be comparable to or even larger than the parallel electron velocity. This study will be restricted to the case where the parallel electric field and the parallel electron velocity can be neglected.

In a coordinate system where the magnetic field \(\mathbf{B}_0\) is parallel to the \(z\) axis and the LFW electric field \(\mathbf{E}\) is parallel to the \(x\) axis, the relative velocity of the ion species \(i\) with respect to the electrons, \(\mathbf{u}_i = \mathbf{w}_i - \mathbf{w}_e\), is

\[
u_i = \frac{e E_0 \omega_0}{m_i (\omega_0^2 - \omega_{bi}^2)} \left[ \sin \omega_0 t + \frac{\omega_0}{\omega_{bi}} \cos \omega_0 t \right] \tag{3}
\]

For LFW with left-hand and right-hand circular polarization propagating along the \(z\) axis, the relative velocity is given by the expression

\[
u_i = \frac{e E_0 \omega_0}{\sqrt{2} m_i (\pm \omega_0 - \omega_{bi}) \omega_{bi}} \left[ \sin \omega_0 t + j \cos \omega_0 t \right] \tag{4}
\]

If \(\omega_0\) is not near the cyclotron frequency of the light or heavy ions, \(\omega_{bi}^2 < \omega_0 < \omega_{bi}^2\), then the ratio of the velocity of the heavy ions \(u^H\) to the velocity of light ions \(u^L\) is of the order
for waves with linear or circular polarization. For \( \omega_0 - \omega_H \), again for both cases, we get
\[
\frac{u_H^*}{u_e} = \frac{\omega_H^*}{\omega_0}\quad \text{and} \quad \frac{\omega_H^*}{\omega_0} - \frac{m_H^*}{m_e^*} \left( \frac{\omega_0}{\omega_H^*} - 1 \right)^{-1}
\]

It may be concluded from the estimations of (3) and (4) that the relative ion velocity in the electric field of LFW with fixed frequency increases with ion mass. The reason is that the relative velocity of the various ions is caused by the different inertia of ions and electrons in the oscillating electric field. For a LFW frequency near \( \omega_H^* \) the resonance effect also becomes important. The role of the resonance denominator in (3) and (4) can be significant in some cases. For example, IC waves with a frequency near the helium gyrofrequency, generated in the ring current region and propagating in the direction of stronger magnetic field, may move to a region where \( \omega_0 \) is closer to the oxygen gyrofrequency. For waves observed by the Active Magnetopsheric Particle Tracer Explorers (AMPTE) Ion Release Module (IRM) satellite [LaBelle and Treumann, 1992], the relation \( \omega_0 / \omega_{He} \), was 1.2 and the relative velocity of oxygen ions was 20 times greater than that of hydrogen. It follows from the above argument that in a multicomponent plasma, LFWs due to inertial drift of the heavy ions form a current across the magnetic field.

2.2. Lower Hybrid Wave Generation

Let us consider the LHW generation in the linear approximation. The dispersion equation in this case has the form [e.g., Akhiezer, 1975]
\[
1 + \frac{\omega^2_p}{k^2 v_T^2} \left[ 1 + I_0(x) e^{-x} Z(x) \right] + \sum_{\alpha} \frac{\omega^2_{pa}}{k^2 v_T^2} \left[ 1 + z_\alpha Z(z_\alpha) \right] = 0
\]
where
\[
x = \frac{k^2 v_T^2}{m_e^*} ; \quad z_e = \frac{\omega}{\sqrt{2} v_T e} ; \quad z_\alpha = \frac{\omega - k_\alpha u_{He}^*}{\sqrt{2} k v_T} ;
\]
\[
v_T^2 = \frac{T_e}{m_e^*} ; \quad \omega_p^2 = \frac{n_e e^2}{m_e^* e_0} ; \quad \omega_{pa}^2 = \frac{n_e e^2}{m_e^* e_0} ; \quad \omega_{He}^* = \frac{e B}{m_e^*}
\]

where \( I_0(x) \) is the modified Bessel function; \( Z(z_{\alpha}) \) is the plasma dispersion function; and \( \omega \) and \( k \) are the frequency and the wave vector of the LHW oscillations, respectively.

In obtaining (5), it was assumed that \( \omega_{Ba}^2 << \omega^2 << \omega_{Be}^2 \) and the motion of all ion species is unmagnetized, i.e., \( k_1^2 v_0^2 / \omega_{Ba}^2 \gg 1 \). This inequality is easily satisfied, taking into account that the ions with velocity \( u \) mainly generate LHWs with wave vector \( k - \omega / u \), where \( \omega - (\omega_{Be} \omega_{Ba})^{1/2} \) and \( u \geq v_T \). The dispersion equation (5) is valid if during a time of order \( \gamma^{-1} \) (where \( \gamma \) is the growth rate of LHW), the ion motion can be considered regular. This leads to the condition
\[
\gamma > \omega_0 \quad \text{(6)}
\]
We will restrict our analysis of (5) to the case \( u_{He}^* << u_{He}^* << c_{He}^* \), where \( c_{He}^* = n_{He}^* \).

2.2.1. LHW generation in the kinetic limit. Let us examine the kinetic limit of LHW generation. Considering \( c_{He}^* \), \( c_{He}^* \), \( c_{He}^* \), \( c_{He}^* \), neglecting the first term and \( u_{He}^* \), \( u_{He}^* \) in (5), and taking into account only the oxygen term in the imaginary part of (5), the dispersion law becomes
\[
\omega^2 = \omega_{LH}^2 \left( c_{He}^* + \frac{c_{He}^*}{4} + \frac{m_H^*}{m_e^*} \cos^2 \theta + \frac{k^2 v_T^2}{\omega_{LH}^2} \right)
\]
where \( k^2 v_T^2 / \omega_{LH}^2 < 1 \), \( \omega_{LH} = \sqrt{\omega_{He}^* \omega_{He}^*} \), \( \theta \) is the angle between the wave vector \( k \) and the magnetic field, and \( 2.5 \leq \Gamma(\theta) \leq 4 \), \( \Gamma(\theta) \leq 4 \). The function \( \Gamma(\theta) \) depends on the different ion species' concentrations \( c_{\alpha} \), and the minimum \( \Gamma(\theta) \) is at \( \theta \), satisfying the condition \( m_i / m_e \cos^2 \theta = c_{He}^* \).

The growth rate of LHW is
\[
\gamma = \frac{-\sqrt{\pi}}{2 R(\theta)} \left[ \frac{\omega^3}{k^2 v_T} \right]^{1/2} \frac{T_i}{T_e} \left( \frac{T_i}{T_e} \right)^{1/2} \left[ 1 + \frac{3}{\epsilon_e} \right] \frac{m_{He}^*}{m_e^*} \cos^2 \theta \geq 1
\]
where \( R(\theta) \) is a slowly changing function of \( k \) and \( \theta \),
\[
R(\theta) = \left( 1 + \frac{3}{\epsilon_e} \right) \frac{m_{He}^*}{m_e^*} \cos^2 \theta + \left( 1 + \frac{3}{\epsilon_{He}^*} \right) c_{He}^*
\]

It is clear from (8) that a positive \( \gamma \) can be obtained for weak LFW electric fields due to the oxygen ion drift (which is the largest drift) when in the oxygen term, \( \omega - k_1 u_{He}^* < 0 \) and \( \omega - k_1 u_{O}^* / k_{1 \perp} v_{Te} < 1 \). For a small oxygen concentration, \( \gamma \sim 0 \) in a narrow range of wave velocities, \( \omega / k_1 \sim u_{O}^* \), with \( u_{O}^* > v_{Te} \). In this case, the oxygen term is at a maximum and the number of hydrogen and helium ions in resonance with LHW is small. Also, electron Cherevskii damping limits the generation, so LHW excitation is possible only for
\[
\frac{T_i}{T_e} \frac{m_i}{m_e} \cos^2 \theta \leq 1
\]

Figure 1 illustrates these features of LHW generation, showing \( \gamma > 0 \) only for large \( \theta \) and a certain range of \( \omega / k \).

For a purely hydrogen plasma, \( \Gamma > 0 \) if \( \omega - k_1 u_{He}^* < 0 \) and \( \omega - k_1 u_{O}^* / k_{1 \perp} v_{Te} < 1 \), which means \( u_{He}^* < v_{Te} \). This condition leads to much greater required LFW electric fields for the \( u_{O}^* \) drift wave. From (8) the maximum growth rate (neglecting the damping) is
\[
\text{Figure 1. The normalized growth rate of lower hybrid waves (LHWs) as a function of the wave normal angle } \left( T_i m_e / (T_e m_e \cos^2 \theta) \right) \text{ and wave velocity } \left( \omega / (2 k v_T) \right). \]
\[
\gamma_{\text{max}} = 0.3 c_{\text{O}^+} \frac{u_{\text{O}^+}^2}{v_{\text{T}H^+}} \omega
\]  
(9)

The restriction given in (6) leads to an inequality that is easy satisfied when \( u_{\text{O}^+} \) equals several times \( v_{\text{T}H^+} \) and \( c_{\text{O}^+} \geq 10^{-2} \).

Note that LHW excitation is possible if the oxygen velocity is of the order of the thermal hydrogen velocity. Taking \( z_{\text{O}^+} \gg 1 \), the dispersion equation (5) can be presented in the form

\[
1 + \delta e^e(k, \omega) + \delta e^{\text{H}^+}(k, \omega) + \delta e^{\text{He}^+}(k, \omega) - \frac{\omega^2}{c_{\text{O}^+}^2 - k \cdot u_{\text{O}^+}} = 0
\]  
(10)

where \( \delta e^e, \delta e^{\text{H}^+}, \) and \( \delta e^{\text{He}^+} \) are the electron, hydrogen, and helium dielectric permittivities, respectively.

Assuming \( \omega \approx k \cdot u_{\text{O}^+} + \eta, \) \( \eta \ll \omega, \) we can find the growth rate from (10)

\[
\eta \approx \frac{\omega}{\sqrt{1 + \delta e^e(k, k \cdot u_{\text{O}^+}) + \delta e^{\text{H}^+}(k, k \cdot u_{\text{O}^+}) + \delta e^{\text{He}^+}(k, k \cdot u_{\text{O}^+})}}
\]  
(11)

If \( u_{\text{O}^+} \ll v_{\text{T}H^+} \), the term \( \delta e^{\text{H}^+} \) has a imaginary part caused by the presence of the resonant hydrogen ions, and so one root of (11) leads to an instability. For \( \frac{\omega^2}{k^2} \ll \frac{v_{\text{T}H^+}}{2} \) the estimation of the growth rate is

\[
\gamma \approx \frac{c_{\text{O}^+}^{1/2}}{4} k \cdot u_{\text{O}^+}
\]  
(12)

2.2.2. LHW generation in the hydrodynamic limit.

Let us examine the hydrodynamic region of LHW generation, \( z_{\text{O}^+} \ll 1, \) \( k \ll 1 \). For angles \( \cos \theta - m_e/m_i \), taking into account only the relative motion of oxygen ions, (5) can be transformed into

\[
1 + \frac{\omega_{\text{pe}}^2}{\omega^2} \sin^2 \theta + \frac{\omega_{\text{ph}^+}^2}{\omega^2} + \frac{\omega_{\text{He}^+}^2}{\omega^2} + \frac{\omega_{\text{O}^+}^2}{(w - k \cdot u_{\text{O}^+})^2} = 0
\]  
(13)

Assuming \( \omega = k \cdot u_{\text{O}^+} + \eta, \) where \( \omega = k \cdot u_{\text{O}^+} \) is the solution of (13) for \( c_{\text{O}^+} = 0 \), the maximum growth rate of LHW comes out of this expression as

\[
\eta \approx \frac{\sqrt{1 + k \cdot u_{\text{O}^+}^2}}{\sqrt{4}} c_{\text{O}^+}^{1/2}
\]  
(14)

The hydrodynamic consideration is valid if \( z_{\text{O}^+} \gg 1 \), i. e.,

\[
\frac{u_{\text{O}^+}}{v_{\text{T}O^+}} \gg \frac{\sqrt{4}}{c_{\text{O}^+}^{1/2}}
\]  
(15)

and \( \eta \ll \omega_{\text{O}^+} \) (see (6)). Both of these conditions are easily satisfied.

Using the estimations for \( u_{\text{O}^+} \) and \( u_{\text{H}^+} \) (see (3), (4), and below), (15) may be rewritten as

\[
\frac{u_{\text{H}^+}}{v_{\text{T}H^+}} \gg \frac{m_{\text{H}^+}}{m_{\text{O}^+}} \sqrt{\frac{\omega_{\text{O}^+}}{\omega_{\text{BO}^+}}} - 1
\]

For a purely hydrogen plasma the LHW hydrodynamic instability is possible if \( u_{\text{H}^+}/v_{\text{T}H^+} \gg 1 \). It follows from the last two inequalities that for a multicomponent plasma the LHW hydrodynamic instability occurs for a smaller drift velocity and LFW electric field than for a purely hydrogen plasma.

Note that the obtained results are valid for plane polarized as well as circularly polarized LFW. In the case of circular polarization the oxygen ions act as a beam of oscillators, with a rotation frequency \( \omega_{\text{O}^+} \) (see (4)). The radius of rotation, \( \frac{u_{\text{O}^+}}{\omega_{\text{O}^+}} \), is much greater than the LH wavelength, because

\[
\frac{k_l^2 u_{\text{O}^+}^2}{\omega_{\text{O}^+}^2} > \frac{k_l^2 v_{\text{T}O^+}^2}{\omega_{\text{BO}^+}^2} \gg 1
\]

and for this linear analysis of LHW generation the rotation can be neglected.

Comparing the growth rates of (9), (12), and (14), it can be found that the strongest dependence on the oxygen ion density as well as the LFW electric field takes place in the kinetic regime of LHW generation in (9). Of course, in this regime the LFW electric field needed for instability excitation is smaller than in the hydrodynamic case (15). From the above derivation it follows that in a plasma subjected to LFW the heavy plasma species acts as a beam of particles with a relative drift velocity. This heavy ion beam determines the character of the LHW excitation, growth rate, and restrictions on the excitation. It also follows that in a plasma with some oxygen ions, the LHW instability can be excited by LFW with an electric field amplitude less than in a purely hydrogen plasma.

3. Quasi-Linear Approximation

The LHW instability excited by the oxygen ion drift in the LFW electric field leads to the development of LH turbulence. The growth of the LHW can be restricted owing to their quasi-linear interaction with different plasma species. If such interaction does not lead to the instability saturation, the level of LH turbulence is determined by nonlinear processes, in particular, by induced scattering of LHW by ions and electrons. Below, these processes are studied and the quasi-steady-state level of the LHW energy density is found.

Let us summarize the characteristics of LHWs excited in a multicomponent plasma subjected to plane polarized LFW, with \( \omega_{\text{O}^+} > \omega_{\text{BO}^+} \) and some other results of the previous analysis. From (3) it follows that the main component of the relative velocity, \( \delta u_{\text{O}^+} \), is directed along the \( y \) axis in the same phase as the LFW electric field. We will restrict our consideration to weak LFW electric fields (small \( E_0 \)). It follows from (8) that in this case the LHW may be generated in a narrow region of angles near \( u_{\text{O}^+} \) (the LFW electric field needed for excitation of LHW is at a minimum when \( k \cdot (\delta u_{\text{O}^+}) \)). It was found above that the hydrogen and helium ion motions can be treated as unmagnetized, so the influence of the magnetic fields on their motion will only lead to phase mixing. Taking into account that the thermal velocity of oxygen ions is much smaller than their relative velocity, we can consider their motion as one-dimensional. The region of wave velocities in which the LHW generation is possible is determined by the condition \( \gamma > 0 \), where the lower limit is given by (8) and the upper limit is \( u_{\text{O}^+} \) and is narrow for small LFW electric fields (Figure 1).

3.1. Quasi-Linear Equations

Let us now obtain the quasilinear equations describing the behavior of the LHW spectral energy density and distribution functions of different species of plasma particles.

The kinetic equation for oxygen ions in the presence of a LFW electric field is
The changes in the LHWP energy density spectrum are described by an equation which takes into account the energy generation due to oxygen ion drift and energy damping due to the absorption by all other species [Shapiro and Shevchenko, 1988]:

\[
\frac{\partial |E|^2}{\partial t} = \frac{\pi e^2}{\omega_0} \int \frac{d\omega}{\omega} \int dv_\perp J_\perp^2(x) \frac{\partial |E|^2}{\partial v_\perp} + \frac{m_0^2}{\omega_0} \int \frac{d\omega}{\omega} \int dv_\perp \frac{|E|^2}{\sqrt{k^2 v_\perp^2 - \omega^2}} + NLT
\]

(22)

The last term, NLT, takes into account nonlinear interaction processes of LHWP with particles. In this case, it is induced scattering by ions and electrons [Musser et al., 1978]. The system of equations (19)-(22) describes the evolution of distribution functions and LHWP electric field energy density.

3.2 LHW Energy Density

Our purpose here is to determine the LHWP energy density and the influence of LHWP on oxygen, hydrogen, and helium ions and electrons. We will avoid the beginning stage of LHWP instability development and restrict the analysis to a time long enough for significant changes to occur in the initial distribution functions of the particles.

Let us examine (20) describing the distribution functions of hydrogen and helium ions. Owing to the interaction with the resonant LHWP, these ions are accelerated, and, for large times, it can be supposed that the resonant ions have velocities such that \(v_{\perp} > \omega/k_{\perp}\). Then (20), for \(v_{\perp} = 0\), have an asymptotic solution for \(\tau \rightarrow \infty\) that does not depend on the initial distribution function and is self-similar in form [Shapiro and Shevchenko, 1988].

\[
f_{H^+}(v_{\perp}) = \frac{5\pi^2 (m_e)}{2} \Gamma(2/5) \exp[-v_{\perp}^2/\bar{\tau}^2]
\]

(23)

where \(\Gamma(2/5)\) is the Gamma function and \(n_{H^+}(v_{\perp})\) is the number of resonant particles.

From (23) it follows that the distribution functions of hydrogen and helium ions, and therefore the energy absorption, only weakly depend on fast energy density oscillations and thus are determined by the slow changes in the average energy density. For \(T_{\text{H}^+} - T_{\text{He}^+}\), the thermal velocity of helium ions is half that of the hydrogen ions, so the number of helium ions in the region of resonant velocities (for Maxwellian initial distribution functions) is \(n_{\text{He}^+}/n_{\text{H}^+}\) exp\(-3v_{\perp}^2/m_e^2\) times less than the number of hydrogen ions. This means that the absorption by hydrogen ions is much greater than by helium ions.

From (21) it follows that the electron distribution function is one-dimensional. Therefore, for large times, a plateau in the region of resonant velocities along the \(z\) axis will eventually form and the energy absorption will cease. Thus, for large times, only the term describing the energy absorption by hydrogen ions needs to be kept in (22).
Note that the maximum level of LIHW energy density is limited by the nonlinear term in (22). This energy level can be estimated from the balance of the quasilinear oxygen term and the nonlinear term.

The damping rates due to induced scattering by electrons and ions in the case under consideration are of the same order of magnitude [Musher et al., 1978],

\[ \gamma_{NL} = \frac{\omega_{pe}^2 \omega_{HI}^2}{\omega_{HI}^2 - \omega_{HI}^2} e \frac{\gamma}{\gamma} \]

where

\[ \gamma = \frac{1}{8\pi} \int \frac{dE}{dk} \Delta k = \frac{E_k^2}{8\pi} \frac{\omega_{HI}^2}{nT} \]

From the equality of the linear growth rate \( \gamma \) (the maximum value of the quasilinear term) and the nonlinear damping rate \( \gamma_{NL} \), the maximum value of the LIHW energy density can be estimated,

\[ \frac{E_k^2}{8\pi} = \frac{\omega_{HI}^2 nT}{\omega_{pe}^2 \omega_{HI}^2} \]

(25)

If we suppose that the system of LHWs and particles progresses to a steady-state condition due to quasilinear interactions, then we can consider the slow evolution of the oxygen distribution function and the LHW spectral energy density.

Let us investigate the solution of (20) and (22) with the form

\[ f_0 = F(v,i) + a(v) \cos \omega_{HI} + b(v) \sin \omega_{HI} \]

\[ E_k^2 = E_k(v,i) + c(v) \cos \omega_{HI} + d(v) \sin \omega_{HI} \]

where \( F \) and \( E_k \) are slowly varying functions of time. We will assume that the growth rate determined by the slowly changing part of the oxygen ion distribution function \( F \) and the damping rate caused by hydrogen ions are less than \( \omega_0 \), and this damping rate can be estimated from the hydrogen term in (22) using the distribution function (23) and the LHW energy density (25) and is in accordance with this assumption.

In this case, the equation for \( F \) may be written in the form

\[ \frac{\partial F}{\partial t} + \frac{\omega_e^2}{m_0^2} \frac{\partial^2 F}{\partial v^2} - \frac{\omega_0^2}{m_0^2} \frac{\partial F}{\partial v} + \frac{\omega_e^2}{m_0^2} \frac{\partial^2 F}{\partial v^2} \]

(27)

The second term on the left-hand side shows that the asymptotic influence of the LFW electric field on the oxygen ion motion is determined by the "friction" of the ions with the LHW electric field. Equation (22) for the slowly varying part of LHW energy density \( E_k \) has the form

\[ \frac{\partial E_k}{\partial t} = \frac{\varepsilon}{m_0^2} \frac{\omega_e^2}{m_0^2} \frac{\partial F}{\partial v} + \frac{\omega_0^2}{m_0^2} \frac{\partial F}{\partial v} - \frac{\partial E_k}{\partial v} \]

(28)

As was discussed above, only the absorption by the hydrogen ions is taken into account in (28).

We will suppose below, as is usual in a quasilinear approximation, that the LHW spectrum has a sharp maximum at the wave vector \( k = \alpha v \). Equation (27) can then be reduced to the following form:

\[ \frac{\partial F}{\partial t} + \frac{\omega_e^2}{2 m_0^2} \frac{\partial^2 F}{\partial v^2} + \frac{\omega_0^2}{2 m_0^2} \frac{\partial F}{\partial v} + \frac{1}{\alpha v} \frac{\partial E_k}{\partial v} \]

(29)

where

\[ \tau = \frac{m_e^2 v}{m_0^2} \frac{\partial E_k}{\partial v} \]

Let us estimate the role of the different terms on the time behavior of the oxygen ion distribution function. If we neglect the quasilinear term, then the asymptotic self-similar solution can be found. It does not depend on the initial conditions and can be calculated by the perturbation method as a series of powers of \( \eta \). The term which determines the LHW evolution for \( t \rightarrow \infty \) is \( F \). Conversely, if we neglect the term with \( \omega_0^2 \), the analogous quasilinear solution is \( F \). The first expression shows that the LFW electric field causes a positive value for the term \( \partial F/\partial v \) and leads to LHW generation. From the second expression for \( F \) it follows that the quasilinear interaction gives a negative \( \partial F/\partial v \) term. Such a result means that by neglecting the term with the LFW electric field, we reduce the problem to a completely different one with the oxygen particles being accelerated by the LHW electric field.

We will suppose for further estimations that, for large times, the behavior of the derivative, \( \partial F/\partial v \), is determined by the second term in expression (29). This means that the energy flow to the plasma is directly related to the LFW electric field and leads to

\[ \frac{\partial F}{\partial v} = \frac{\alpha v}{\omega_0^2} \frac{\partial E_k}{\partial v} \]

(30)

Taking into account (23) and (30), the equation for the LHW spectral energy density (28) may be reduced to the form

\[ \frac{D}{Dx} = \frac{\pi \omega_0}{m_0^2} \frac{\partial F}{\partial v} \frac{m_0^2}{n D} \frac{\partial E_k}{\partial v} \]

(31)

where \( \omega_0 \) and

\[ D = \frac{\pi \omega_0^3}{m_0^2} \frac{\partial E_k}{\partial v} \]

and we have assumed \( \omega_0^2 \kappa \cdot u \).

In (31), the asymptotic dependence of the LHW energy density is different for the oxygen and hydrogen terms (the first and second terms on the right-hand side, respectively). The oxygen term yields \( E \sim \omega_0^2 \), a growth of energy, while the hydrogen term has the form \( E \sim (t-t_0)^2 \), a damping of energy. Examining the powers shows that the oxygen term will dominate for large times and the LHW energy will grow.

The numerical analysis of (31) gives the same results. In the calculations below a maximum time \( \alpha v t \sim 10^3 \) is used, supposing that ion velocities are \( 10^5-10^6 \) cm/s, the region of interaction with LFW is \( 10^{8}-10^{9} \) cm, and \( \omega_0 \) is a few \( 10^2 \). Figure 2 shows that the solution of (31) weakly depends on the initial conditions. The quantity \( \eta/n \) can be changed in magnitude 10-20 times without significant energy changes, allowing a wide range of concentrations of resonant hydrogen ions (Figure 3). The dependence on oxygen ion concentrations is more significant, however. If \( \eta/n \) is changed from \( 10^{-2} \) to \( 3 \times 10^2 \), the LHW energy growth for \( \omega_0^2 \sim 10^3 \) changes 20 times. Owing to the difference of the numerical coefficients in the terms on the right-hand side of (34), the exact dependence of the LHW energy on \( \tau \) in the derivative in (31) is not very significant.
Figure 2. The normalized LHW energy density $\varepsilon / \varepsilon_0$ from (31) for initial conditions of $\varepsilon / \varepsilon_0 = 0.5$ (solid curve), $\varepsilon / \varepsilon_0 = 0.05$ (dashed curve), and $\varepsilon / \varepsilon_0 = 0.005$ (dotted curve), as a function of normalized time $\omega_0 t$, with $\tilde{n}/n = 0.005$ and $D^{-2/5}$.

Figure 3. The normalized LHW energy density $\varepsilon / \varepsilon_0$ from (31) for the initial condition $\varepsilon / \varepsilon_0 = 0.005$ as a function of $\omega_0 t$ with $D^{-2/5}$. The solid curve is for $\tilde{n}/n = 0.1$, the dashed curve is $\tilde{n}/n = 0.05$, and the dotted curve is $\tilde{n}/n = 0.005$.

Figure 4. The normalized LHW energy density $\varepsilon / \varepsilon_0$ from (31) for the initial condition $\varepsilon / \varepsilon_0 = 0.005$ as a function of $\omega_0 t$ with $\tilde{n}/n = 0.005$. The solid curve is for $D^{-2/5}$, the dashed curve is $D^{-4/5}$, and the dotted curve is $D^{-2/5}$.

and the dependence of this energy on the time of interaction with LHW,

$$K(t) = \frac{\tilde{n}}{n_0} \int \frac{mv^2}{2} f_{\perp} dv \left[ \frac{m_{H^+} v_{H^+}^2}{2} \frac{u^2}{v_{H^+}^2} - \frac{3.5 \times 10^{-3} \sqrt{v_{eH}^2}}{u} \gamma t \right]^{2/5}$$ (32)

where $\tilde{n}$ is the number of resonant hydrogen ions.

Let us estimate the energy flow $Q$ in the plasma particles due to induced scattering by ions and electrons. For $\omega \sim 3 - 4 \omega_{LH}$ the result is of the same order of magnitude for both species [Musher et al., 1978]. For the stationary state,

$$\int \gamma W_k dk = Q$$ (33)

where $W_k$ is the spectral energy density of LH oscillations. Using (25), it is found that

$$\frac{Q}{nT} \sim \frac{\gamma^2 \omega}{\omega_{Be}^2}$$ (34)

4. Discussion and Results for Magnetospheric Plasma

Let us discuss the main results and the assumptions that are used in the analyses of LFW interaction with a multicomponent plasma due to LHIW excitation and the possibility of their applications to the magnetospheric plasma.

In obtaining the LHW frequency (8), it was supposed that the LFW frequency is small compared with the LHW frequency and growth rate. If the LFW is an ion cyclotron wave (ICW) with a frequency near the helium cyclotron frequency [Young et al., 1981] or oxygen cyclotron frequency [Fraser et al., 1992], the frequency of LHW is $\omega = 2 \omega_{LH}$ (7), $\gamma$ is determined by (9), and the condition $\gamma > \omega_0$ for $\omega_0 \sim \omega_{Be}$ leads to the inequality $c_0 u_{0H}^2 / v_{eH}^2 \gg 10^{-2}$. For the oxygen drift velocity $u_{0O} \approx 3 v_{eH}$, this condition is satisfied even for an oxygen ion concentration $c_{0O} \approx 1\%$. For $\omega_0 = \omega_{BO}$, the inequality is 4 times weaker.
Another assumption used in obtaining the LHW growth rate is about the uniformity of the plasma, external magnetic field, and the electric field of LFW. The wavelength of the LHW can be estimated as $\lambda \approx 2 \mu m_{e}/o_{LH}$, and even for $L_{T} - 7$, it is of order $\lambda \approx 10^{5}$ cm. Therefore, this assumption is usually fulfilled, because the LFW wavelength is greater than $10^{5}$ cm.

Let us discuss the LFW electric fields needed for LHW excitation. LHW excitation in a plasma with an oxygen mixture of 1-3% is possible if the oxygen ion drift velocity $u_{o}$ is 3.5-4 times greater than the thermal velocity of hydrogen ions. In this case, the LHW Cherenkov damping on hydrogen ions is small and the development of the instability is possible. Taking into account that the instability develops on a timescale much smaller than the LFW period, the LHW generation is possible even if the oxygen ions have such a drift velocity only during a short part of the LFW period.

As was found in section 2.1 in (3), the drift velocity of a particle in a LFW electric field depends on the mass of the particle and the ratio of the LFW electric field frequency to the particle's cyclotron frequency. Expressions (3) and (4) are correct if $\lambda_{o} - o_{L} \gg k_{B} v_{T}$, where $k_{B}$ is the wave vector of LFW. Owing to the large LFW wavelength, the right-hand side of this inequality is small. For ICW with a frequency of the order of the helium ion cyclotron frequency and an oxygen ion temperature 1-2 eV, $k_{B} v_{T} \approx 5 \times 10^{-4}$. Therefore (3) and (4) are correct for ICW frequencies close to the particle's gyrofrequency.

Now let us estimate the possibility of LHW generation using observed LFW magnetic and electric fields. The plasma composition and the hydrogen ion temperature are not given in the papers cited below, so it was assumed that oxygen concentration is of order 1% and the hydrogen ion temperature is 1 eV. First, let us present the cases to be discussed.

Young et al. [1981], for an event observed on board GEOS 1 (August 16, 1977; L = 7; 1200 LT), found that the electric fields were 2.9 mV/m (1120 UT) and 4.3 mV/m (1215 UT); the wave frequency was $f = 0.6$ Hz, the frequency range was $\Delta f = 0.1$ Hz, and the helium gyrofrequency $f_{He}^{*} = 4.5$ Hz.

The event described by ISEE 1 and 2 (August 22, 1978; L = 7; 1700 L1) is described by Fraser et al. [1992]. For this event the electric field was found to be 1.4 mV/m, the wave frequency $f = 0.12$ Hz, and the oxygen gyrofrequency $f_{O}^{*} = 0.06$ Hz.

In the case described by LaBelle et al. [1988] and LaBelle and Treumann [1992] (AMPTE/IRM observation, June 6, 1985; L = 4.5; 1547 UT) the electric field in full is not given. Our estimation based on this data leads to an electric field amplitude of $-1.8$ mV/m. The wave frequency in this case is close to hydrogen ion gyrofrequency, $f = 0.15$ Hz, $f_{O}^{*} = 0.125$ Hz; and the frequency region is $\Delta f = 0.15f$.

In all of these cases, the oxygen ion drift velocity is 3.5 or more times greater than the hydrogen ion thermal velocity, and so LHW can be excited. In the last two examples the resonance denominator in the expression for the oxygen ion drift velocity (3) is significant. Taking into account that the listed observations were obtained in the region near the equator for $L = 5-7$, it can be found that the difference $\omega_{B} - \omega_{O}$ does not change significantly in a region of order $R_{e}$ along the magnetic field lines.

The next supposition used is that the LFW is monochromatic. In two of the cases cited above (Fraser et al. [1992] and LaBelle et al. [1988] and LaBelle and Treumann [1992]) it is quasi-monochromatic with $\Delta f / f \sim 0.1$. Young et al. [1981] reported that the ICW emissions observed on board GEOS 1 and 2 are often quasi-monochromatic ($\Delta f / f \sim 0.1-0.2$).

From the previous analysis it can be concluded that LHW excitation due to LFW activity is a possible mechanism of LFW interaction with multicompont magnetospheric plasma, at least at $L < 5$ near the equatorial region. As discussed by Khazanov et al. [1996], the same mechanism of interaction is possible in the auroral zone.

Now we will estimate the plasma energization due to the discussed mechanism for magnetospheric plasma. For this calculation, let us take the magnetospheric plasma in the region of the plasmapause ($L = 4.5$) with the densities of the hydrogen, helium, and oxygen ions in the ratio $100:10:1$, respectively. The plasma is assumed to be isothermal with a temperature of 1 eV. For the LFW with the frequency $0.15$ Hz and oxygen ion gyrofrequency $f_{O}^{*} = 0.125$ Hz, the needed electric field amplitude of LFW, $E_{0}$ (see (8)), is 1.8 mV/m.

The 1.0H frequency in that region, given by (7), is $\omega \sim (0.5-1) \times 10^{-3}$ s$^{-1}$, and the growth rate is proportional to the oxygen ion concentration $c_{O}^{+} (9)$, $\gamma = 5c_{O}^{+} \omega$. The wavelength of LH oscillation is $\lambda \sim 2\pi u_{o} / \omega \sim 10^{5}$ cm, and the envelope of generation is $\Delta \lambda / \lambda$.

We will suppose that the LFW electric field as well as the external magnetic field and plasma parameters are the same in a region of order $L \sim 10^{5} - 10^{5}$ cm. Then the LHW turbulence in this region can be treated as uniform. The tracing time for hydrogen ions for such a region is of order $10^{2} - 10^{3}$ s. It can be taken that during this time the LFW and LHW activities are quasi-steady. Then, from (32), the energy obtained by the resonant hydrogen ions during the tracing time can be found. It is of order 20-40 eV. The number of resonant hydrogen ions can be estimated supposing that their initial distribution is Maxwellian. It is equal to the number of particles with velocities above the minimum LHW phase velocity. In the case under consideration, the LHW phase velocity is 3.5 $v_{th}$ and the number of resonant hydrogen ions is $-0.5\%$. The energy of the core plasma, found from (34), is doubled for the period of interaction owing to induced scattering of LHW.

5. Summary

The preceding analysis demonstrates that LFH in a multicompont plasma can generate LH oscillations. The energy density of the LFW electric field needed is significantly less than in the case of a purely hydrogen plasma. The oxygen ions play the role of a beam, and in the linear approximation the concentration dependence of the growth rate and the kinetic and hydrodynamic limits of LHW generation are the same as those of a beam instability. The quasilinear effects do not lead to saturation of the instability, and the energy density level is determined by the nonlinear processes of induced scattering.

In the region of resonant velocities the hydrogen ions are accelerated and the concentration of accelerated ions weakly depends on the oxygen ion concentration. The nonresonant particles are weakly heated owing to induced scattering of LH oscillations.

For a given region of the magnetosphere ($L = 4-5$) with LFW electric fields of 2 mV/m, plasma temperatures of 1-2 eV, and an oxygen ion concentration of $\sim 1\%$, the resonant hydrogen ions ($\sim 0.5\%$) obtain energies of 20-40 eV. Therefore the discussed mechanism may lead to the growth of a tail in the
hydrogen distribution function in the presence of low-frequency activity.

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