A model for lower hybrid wave excitation compared with observations by Viking

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Abstract. The mechanism of lower hybrid wave (LHW) excitation due to the O+ relative drift in a plasma subjected to low-frequency waves (LFWs) is used for analysis of Viking satellite data for events in the cusp/eleft region. In some cases, such a mechanism leads to LHW energy densities and ion distribution functions close to those observed, suggesting that the proposed mechanism is a plausible candidate to explain certain classes of LHW generation events in space plasmas.

Introduction

The excitation of LHWs is a widely discussed mechanism of interaction between plasma species in ionospheric and magnetospheric plasmas. Such waves are particularly interesting because they couple well to both electrons and ions. Various mechanisms for LHW excitation have been studied as well as the phenomena produced by such waves [Davidson et al., 1977; Sonnerup, 1980; Chang and Coppi, 1981; Bingham et al., 1984; Garnett et al., 1984; Pottelette et al., 1990; Omelchenko et al., 1994].

In some cases, the LHW activity has been observed simultaneously with LFWs [LaBelle et al., 1988; Pottelette et al., 1990]. The reason for this may be that there is a common source for both waves, but another possible explanation is that the LHW generation is due to the LFW activity [Khazanov et al., 1996, 1997]. This process results because the particle drift velocities in the LFW electric field is mass dependent. For heavy ions, the relative drift velocity is greater than for protons. LHW excitation can be due to any transverse electric field fluctuation with a frequency comparable to \( \omega_{ph} \), in particular due to ion cyclotron, Alfven, and fast magnetosonic waves. Such electric field fluctuations are frequently observed in the ionosphere and plasmasphere, and the electric fields are large enough to excite the LH instability [LaBelle et al., 1988; Pottelette et al., 1990; Erlandson et al., 1990; Fraser et al., 1992; Anderson et al., 1992].

A mechanism for the outer plasmasphere in the presence of a small O+ mixture was discussed by Khazanov et al. [1997]. It was found that LFWs may cause weak LH turbulence leading to the development of a high-energy tail in the H+ distribution function. For the auroral zone, the plasma energization by LFWs due to the excitation of strong LH turbulence and by the ponderomotive force was compared by Khazanov et al. [1996] for a proton-electron plasma. In the auroral region the O+ concentration may be relatively high [Moore et al., 1996], so the development of strong LH turbulence can be expected for lower LFW activity than in a purely proton-electron plasma.

Below, the excitation of strong LH turbulence due to LFW activity in a plasma containing hydrogen and oxygen ions will be analyzed as a plausible candidate to explain several observed events. We will focus on Viking satellite measurements in the cusp/eleft region [Pottelette et al., 1990, hereafter referred to as P90].

We will discuss only the events in P90 when the LH and LF activities were simultaneously observed. As it was stressed in P90 that different mechanisms for plasma heating were acting in concert, and all of these processes should be taken into account to explain the complicated structure of the observed particle distributions. Robinson et al. [1996b] found that for the events observed by Viking, the transverse ion acceleration due to LHW turbulence is possible. We will calculate the LHW energy density and analyze the differential ion fluxes produced by this turbulence, supposing that the LHWs are excited by the observed LFWs and also that the plasma contains hydrogen and oxygen ions.

A detailed description of the two events to be discussed is given in P90; here we present only the parameters that will be relevant to the following analysis (Table 1). Here, \( h \) is altitude in km, \( f_{p}\) and \( f_{LH} \) are the LF and LH frequencies in Hz, \( f_{p} \) and \( f_{c}\) are the plasma and electron cyclotron frequencies in kHz; and \( f_{p} \) and \( f_{LH} \) are the LF and LH electric field strengths in V/m. \( B_{z} \) is integrated when the antenna was at large angles to the magnetic field over the 4, 8, and 12 kHz channels for orbit 869 and over the 4 and 8 kHz channels for orbit 1199.

At the same time as these events, counterstreaming electron beams and ion comas were also observed. The differential ion fluxes observed in the equatorward edge of the cusp (orbit 869) are given in Table 2, segmented into three energy ranges for two times during the event.

In both cases, the ion and electron temperatures were assumed to have been equal and of the order of 1 eV. We will also suppose that the O+ concentration in all of these events was 15-25% [Moore et al., 1996].

LHW Generation

Let us calculate the ion relative drift velocities in the LFW electric fields [e.g., Akhiezer et al., 1975]. In a coordinate system where the magnetic field \( \mathbf{B}_{0} \) is parallel to the z axis, and if the LF electric field \( \mathbf{E} = E_{0} e^{-i0_{l}t} \) is parallel to the x axis \( (\omega_{l} \) is the LF frequency), the relative velocity of ion species \( i \) with respect to the electrons, \( \mathbf{u}_{i} = \mathbf{w}_{i} - \mathbf{w}_{e} \), is

\[
\mathbf{u}_{i} = \frac{e}{B} \left( \frac{\omega_{l}}{\omega_{Bi}} \right) \frac{\omega_{0} / \omega_{Bi} - 1}{\sin \omega_{lt} + \frac{\omega_{0}}{\omega_{Bi} \cos \omega_{lt}}} \right]
\]

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where $\omega_B$ is the ion gyrofrequency and $c$ is the speed of light.

It is clear from (1) that, for a fixed LPW, the relative ion drift velocity is mass dependent. If the ion relative velocity (and, therefore, also the LHW phase velocity) is greater than the ion thermal velocity, then LHW are excited. The smallness of Cherenkov damping with electrons determines the possible wave normal angles (the angle between the wave vector and the geomagnetic field) where generation will occur.

The dispersion equation for LHWs in a multicomponent plasma subjected to LPWs is [Khazanov et al., 1997]

$$1 + \frac{\omega_B^2}{\omega_{Be}^2} - \frac{\omega_{pe}^2}{\omega^2} \cos^2 \theta - \frac{\omega_{pi}^2}{\omega^2} \frac{\omega_{pe}^2}{\omega_{pi}^2} - \frac{\omega_{Te}^2}{\omega^2} \frac{\omega_{Te}^2}{\omega_{Te}^2} = 0 \tag{2}$$

where $\omega_{pi}^2 = \omega_{pe}^2 m_e / m_i^*$ and $\omega, \omega_{pe}, \omega_{pi}$, and $\omega_{Be}$ are the LHW, plasma electron, plasma proton and electron cyclotron frequencies, respectively; $\vec{k}$ is the LHW wave vector; $\theta$ is the LHW wave normal angle; $\vec{v}$ is the relative $O^+$ velocity; and $c_H$ and $c_{Te}$ are concentrations, determined so that $n_cH = n_H$ and $n_{cTe} = n_{Te}$, where $n_{cH}, n_{cTe}$, and $n$ are the oxygen, hydrogen, and electron densities, respectively.

In obtaining (2), it was supposed that a hydrodynamic description is valid, i.e.,

$$\frac{\omega - \vec{k} \cdot \vec{u}}{\sqrt{2 \kappa V_{Te}}} \gg 1, \quad \frac{\omega}{\sqrt{2 \kappa V_{Te}}} \gg 1, \quad \frac{\omega_{Be}}{\sqrt{2 \kappa V_{Te}}} \gg 1 \tag{3}$$

$$\frac{k^2 \gamma_{Te}}{\omega_{Be}^2} \ll 1, \quad \frac{k^2 \gamma_{Te}}{\omega_{Be}^2} \ll 1, \quad \frac{\gamma_{Te}}{\omega_{Be}} = T_{ec} / m_i$$

Taking into account that the LHW frequency and growth rate (see below) are much greater than the LPW frequency $\omega_0$, we supposed $\vec{u}$ is constant and near it's maximum value. The $H^+$ drift is neglected in (2) due to smallness compared to that of $O^+$. The LLH ls are excited with phase velocities $\gamma_{ph} = \omega / k - u \gamma_{ph}$, the last three inequalities in (3) are satisfied for the events under consideration if $1 < \gamma_{Te} / \gamma_{ph} < 15$. The second inequality in (3) leads to this restriction on the wave normal angles of generation:

$$\tan \theta = \frac{m_i}{m_e} \cos \theta \ll \frac{u}{\gamma_{Te}} \tag{4}$$

Finally, the first inequality in (3) defines a lower boundary for the LLH growth rate. The maximum growth rate $\omega^*$ can be found from (2) for the plasma in the absence of LPWs,

$$\left(\omega^* \right)^2 = \frac{\omega_{pe}^2 (m_i^* / m_e) \cos^2 \theta + c_H^* + c_{Te}^* / 14}{1 + \omega_{pe}^2 / \omega_{Be}^2} \tag{5}$$

The real part of the LHW frequency, $\omega$, is close to $\omega^*$, and, in the estimations below, it will be taken that $\Re e \omega = \omega^*$. The growth rate is given by

$$\gamma = \frac{1}{\alpha} \omega \left[ \frac{c_{Te}^*}{4 (m_i^* / m_e) \cos^2 \theta + c_{H}^*} \right]^{1/3} \tag{6}$$

The condition $\gamma / \omega^* \ll 1$ and the first inequality from (3) lead to the restrictions

$$\gamma / \omega^* < 1 \ll \frac{\gamma}{\sqrt{2 \omega^*}} < \frac{u}{\gamma_{Te}} \tag{7}$$

### Table 1. Event Characteristics

<table>
<thead>
<tr>
<th>Orbit/UT</th>
<th>$h_0$</th>
<th>$f_0$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$E_0$</th>
<th>$E_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>869/20:03</td>
<td>13,500</td>
<td>530</td>
<td>25</td>
<td>53</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>1199/20:11</td>
<td>12,000</td>
<td>500</td>
<td>20</td>
<td>64</td>
<td>0.1</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Since the peak of the LFW activity is near 1 Hz and close to $\omega_{Be}^*$, $u$ is much greater than the $H^+$ drift velocity. It then follows from (7) that the O$^+$ can be treated as an ion beam spreading in a proton-electron plasma. For a beam instability when the beam density is small compared to the core density, these results are valid even for $(\omega - k u) / \sqrt{2 \kappa V_{Te}} \ll 1$.

Let us estimate $u$ from (1). In both cases (the equatorward edge of the cusp and the cleft region), the LFW electric fields are the same, $E_{eq}=100$ mV/m with a frequency $f_{eq}=1$ Hz. The O$^+$ cyclotron frequencies are 1.8 and 2.2 Hz, respectively. It can be concluded from the data presented in F90 that the LFWs are nonmonochromatic. We will suppose that the energy density of the LFW electric field fluctuations is primarily within the frequency range 0.2-4 Hz, with a Gaussian distribution around 1 Hz and halfwidth $\sigma_{Te} / (\sqrt{2} \cdot 0.7)$. This supposition is in agreement with data published by Lundin et al. [1990], but in their case the altitude of the event is about 9.3 km. The maximum $O^+$ relative velocity is then $u=9.3 \times 10^6 \text{cm/s}$ for the equatorward edge of the cusp and $u=6.5 \times 10^6 \text{cm/s}$ for the cleft region. For such velocities, the restrictions in (3) and (7) are easily satisfied.

The core oxygen ion density is not measured in these events. From (6), however, the growth rate weakly depends on this parameter ($\gamma = \omega / (2 \pi)$). Returning to (5) we can now find a LHW frequency for the discussed events, $f_{LLH} = \omega / (2 \pi)$, that is of order 0.5 kHz and significantly less than the observed frequencies, i.e. 4-8 kHz (see below). It follows from this linear analysis that the instability in a plasma with $O^+$ that is subjected to a LFW electric field can be treated as a beam instability, and this will be used in the next section.

### The LHW Turbulence and Ion Acceleration

The increase of LH oscillation energy density is limited by nonlinear effects. For weak turbulence, the main nonlinear process is induced scattering by the particles [Musher et al., 1978]. In the present study, induced scattering by electrons and ions are the same order of magnitude,

$$\gamma_{SC} = \omega_{pe}^2 \omega_{Be}^2 \frac{W}{\omega_{LLH} n T} \tag{8}$$

with $\omega_{LLH} = \omega_{pe}^2 (1 + \omega_{pe}^2 / \omega_{Be}^2)^{1/2}$. If the saturation of the instability is due to this process, the LHW energy density $W/nT$ for a quasi-stationary state can be found by equating the linear growth rate (6) with (8).

### Table 2. Differential Ion Fluxes

<table>
<thead>
<tr>
<th>n</th>
<th>$E$(keV)</th>
<th>$J_n$</th>
<th>$f_n$</th>
<th>$f_n^a$</th>
<th>C_n</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.09</td>
<td>15.4</td>
<td>20.0</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>2</td>
<td>0.33</td>
<td>1.5</td>
<td>2.3</td>
<td>1.2</td>
<td>1.3</td>
</tr>
<tr>
<td>3</td>
<td>0.52</td>
<td>0.5</td>
<td>0.4</td>
<td>0.9</td>
<td>0.8</td>
</tr>
</tbody>
</table>

$^{a}J_n$ in $10^6 \text{(keV-cm}^{-2}\text{s}^{-1})$
\[
\frac{W}{nT} = \frac{1}{2} \left( \frac{C_{0T}}{4} \right)^{1/3} \frac{m_e}{m_{He}} \frac{m_{He}^2}{m_{He}^*} \left( 1 + \frac{\omega_{pe}^2}{\omega_{Be}^2} \right)
\]  
(9)

Let us compare this energy density with the energy density required to excite the modulation instability [Musher et al., 1978]. The latter occurs if

\[
\frac{W}{nT} > \frac{m_e}{m_{He}^*} \frac{k^2 v_{Te}^2}{\omega_{Be}^2} \left( 1 + \frac{\omega_{pe}^2}{\omega_{Be}^2} \right)
\]  
(10)

From (9) and (10), the modulation instability is excited for

\[
u^2 \left( \frac{C_{0T}}{4} \right)^{1/3} > 2 \frac{m_e}{m_{He}^*} \text{cos}^2 \theta + c_{H^+}^2 v_{Te}^2
\]  
(11)

Taking into account (4), it can be found from (11) that, for the Viking satellite data events under consideration, strong LHW turbulence excitation is possible.

In the description of strong turbulence and the resulting particle distributions below, we will follow Musher et al. [1986] who discussed strong LH turbulence excited by an ion beam propagating across the magnetic field. As in that paper, the growth rate in our case has a maximum near \( k \sim \omega_{ci} / \nu_e \), but the angle dependence of the growth rate is different from the previous study and this is taken into account below.

Due to induced scattering, the LHWs may transfer from the angles of generation along an inertial interval to angles where cavities are formed and Landau damping is strong. This is valid if the growth rate of the modulation instability, \( \gamma_m \), satisfies the conditions

\[
\gamma_m < k_z v_{Te} < k_y v_{Te}^* \quad \text{where} \quad \gamma_m = \omega_{LH} \frac{W}{nT} \frac{m_{He}^*}{m_e} \frac{\omega_{pe}^2 + \omega_{Be}^2}{4nT}
\]  
(12)

Then, comparing the energy flux into the plasma due to induced scattering in the weak turbulence region with the absorption due to the collapse, the energy density of a LH oscillation in the strong turbulence region can be found. The details of such calculations are given in the paper by Musher et al. [1986], and the equation for the energy density level is

\[
y^4 = \left[ \frac{2}{\pi} \left( \frac{C_{0T}}{32} \right)^{1/3} \left( 1 + \frac{\omega_{pe}^2}{\omega_{Be}^2} \right) \frac{u}{v_{Te}^*} \right]^2
\]  
(13)

\[
\times y_0 \left( y'^2 + c_{H^+}^2 \right)^{1/6} \frac{dy'}{y'} \frac{dy''}{y''} (y' y'' + c_{H^+}^2)^{1/3}
\]

where

\[
y_0 = \sqrt{\frac{m_{He}^*}{m_e} \text{cos} \theta}; \quad y_0 = \frac{u}{3 v_{Te}^*}; \quad y^2 = \frac{m_{He}^*}{m_e} \frac{W}{nT}
\]

The upper boundary in the integral, \( y_0 \), is found by the angles where Landau damping on electrons is strong (4). The lower boundary, \( y_* \), is the limit where strong turbulence arises.

Numerical calculation of (13) yields

\[
\frac{W}{nT} = 3.2 \frac{m_e}{m_{He}^*} \quad \text{and} \quad \frac{W}{nT} = 5.5 \frac{m_e}{m_{He}^*} \quad \frac{E^2}{8 \pi m_e nT} \left( 1 + \frac{\omega_{pe}^2}{\omega_{Be}^2} \right)
\]  
(14)

for the cleft region and the equatorward edge of the cusp, respectively. The LHW electric field is then \( E \sim 17 \text{ mV/m} \) in the cleft region and \( E \sim 30 \text{ mV/m} \) in the equatorward edge of the cusp. Comparing these results with the observed data, \( E \sim 100 \text{ mV/m} \) and \( E \sim 50 \text{ mV/m} \), respectively, it can be concluded that the proposed mechanism leads to LHW electric fields close to those observed for the equatorward edge of the cusp, but not for the cleft region.

Now we can analyze the particle distribution for the events near the equatorward edge of the cusp. From P90, the electron population was a composition of particles of magnetospheric and ionospheric origin. Therefore, we will restrict our analysis to the ion distribution function. In Musher et al. [1986], the formation of the ion distribution function was dominated by the strong LH turbulence and is found to be

\[
f = \frac{n}{v^2} \frac{m_e}{m_{He}^*} \frac{nT}{W} \frac{v_{Te}^*}{v_1} \left( 1 + \frac{\omega_{pe}^2}{\omega_{Be}^2} \right)^4
\]  
(15)

Expression (18) corresponds to the fact that LH turbulence leads to ion acceleration perpendicular to the magnetic field [Chang and Coppi, 1981].

Let us discuss the data presented in Table 2. The differential fluxes were obtained with an energy resolution of 0.1 FWHM (full width at half max), and a field of view of 4x4° [Sundahl et al., 1990]. The data was obtained for pitch angles ~ 90° [P90]. This means that the ion distribution function \( f \) is integrated over \( \theta_i \), \( \nu_i - v_T \), \( \nu_T \) during the measurements, with some weight determined by the transmission function [Ogelman and Wargulis, 1970]. Then the relation between the differential fluxes in Table 2 and the distribution function (15) is

\[
j = c v_T^2 f
\]  
(16)

where \( v_T^2 \) is equal to \( v^2 \) with the accuracy of order \( v_T^2 / v^2 \).

The law \( f \sim 1 / v^6 \) in (15) can be examined calculating the product \( j v^4 \), which in this case does not depend on the velocity. In the last two columns of Table 2, the observed differential fluxes were used to find the coefficients \( C_n \)

\[
C_n = 3 j_n \sum_{\alpha=1}^{3} j_{\alpha} v_{\alpha}^4
\]  
(17)

If the velocity distribution is \( f \sim 1 / v^6 \), then the quantities \( C_n \) in all of the cases ((15) and (16)) should be near unity. It follows from Table 2 that the ion distribution function depends on velocity with a power law close to \( 1 / v^4 \).

**Discussion and Summary**

The LH activity observed by the Viking satellite in the cleft/cusp region can be generated by different mechanisms. The first option is the LH drift instability in an inhomogeneous plasma [P90]. For the events when LH activity was observed simultaneously with LFWs in the range ~ 1 Hz, it can also be the LH instability due to the relative drift of O^+ in the LFW electric field. For both of these mechanisms, the frequency of the LHWs is less than that observed. It can be supposed, then, that the LH frequency is Doppler shifted due to the large plasma flow velocity in the observed region [P90]. The shift is the same for both mechanisms because of equal LH wave vectors. In the first case, it is the wave vector of the maximum growth rate for the drift instability. In the mechanism proposed in this study, it is the wave vector of the strong LH turbulence. Both are of order \( k \sim \omega_{Be} / \nu_{Te} \). Therefore the estimations for the Doppler shift of the LH frequency given in P90 are exactly the same for our case.
For the equatorward region of the cusp, when the proposed mechanism gives a LHW energy density close to that observed, the calculated shape of the ion distribution function tail is in agreement with the observed differential fluxes. Agreement between theory and observations only occurs for a limited time period when transverse ion acceleration can be supposed. When “elevated” conics appear, the acceleration due to the proposed mechanism is less than that due to others and can not be separated using the given observations [P00].

We restricted our analysis to two points: determining the energy density of LHWs and the shape of the ion distribution "tail." We used the theory of Mushot et al. [1986] because the source of the LHW generation in our case (accelerated O⁺) can be treated as an ion beam. Our estimations assume that the LHW energy density is greater than the threshold of the modulation instability. This threshold in the various theories is calculated taking into account the effects of polarization, temperature ratio, etc., but the values are not very different, so the choice of the theory for calculation of the threshold is not critical for our conclusions. The level of the LHW turbulence was determined as a result of the transfer of the LHW along the spectrum due to induced scattering. This effect will also work in the case of the “nucleation” mechanism of LHW collapse [Robinson et al., 1996a].

In addition to the choice of theory, several other factors were not included in this study because they will not change the conclusions. O⁺ phase mixing was neglected for several reasons, most notably because the time of LHW generation is comparable to \( \omega_{\text{L,H}} \) (see (12)). Another is the effect of plasma inhomogeneities due to the LFW ponderomotive force. An estimation of this yields an inhomogeneity of less than 25%, which can support localized LHW waves and will not significantly change the modulation instability threshold. A numerical model could address these processes.

It can be supposed from our previous work [Khazanov et al., 1996, 1997] and from these results that LFWs may excite LHWs due to the heavy ion relative drift in the LF electric field. Therefore, the proposed mechanism is a plausible candidate to explain certain classes of LHW generation events.

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