Lecture 9
Soft SUSY Terms and Spurions

Outline

- Soft SUSY breaking: a general discussion.
- Spurions.
- SUSY breaking scenario: gravity mediation.
- SUSY breaking scenario: gauge mediation.
- Computation of soft SUSY breaking terms: an example.

Reading: Terning 2.8, 5.2, 6.1, 6.2.

Chiral Symmetry Protects Masses

Quantum corrections to fermion masses are logarithmically divergent (at most). For cutoff Λ,

$$m_{\text{ferm}} = m_0 + c_f \, \frac{\alpha}{16\pi^2} m_0 \ln\left(\frac{\Lambda}{m_0}\right) \ .$$

The reason: chiral symmetry for $m_0 = 0$ is preserved by quantum effects so corrections vanish for $m_0 = 0 \Rightarrow$ linearly divergent corrections are absent even though they are allowed by dimensional analysis.

In SUSY QFT: the scalar mass is given by the same formula. In other words, quadratic corrections (to m^2_{bos}) are absent even though they are allowed by dimensional analysis.

The challenge: break SUSY such that quadratic corrections to scalar masses are absent also after SUSY breaking.

Soft Breaking of SUSY

Reminder: Higgs receives quadratically divergent corrections from the Yukawa coupling to the top.

The quadratic divergence cancels if the top has a scalar partner (a stop) with quartic coupling to the Higgs taking the SUSY value $\lambda = |y_t|^2$:

$$\delta m_h^2 \propto (\lambda - |y_t|^2)\Lambda^2$$

Analysis: we want to break SUSY such that SUSY relations between dimensionless coupling are protected.

Conclusion: SUSY breaking couplings with explicit mass dependence preserve SUSY cancellations of quadratic divergences. Those are relevant terms, i.e. terms with dimension< 4.

Terminology: relevant terms in the Lagrangian are called soft SUSY breaking terms.

Effective Theory of Soft Breaking

The effective theory of softly broken SUSY allows all relevant terms consistent with symmetries.

General soft terms in a SUSY gauge theory coupled to chiral matter fields:

$$\begin{aligned}\mathcal{L}_{\rm soft} &= -\tfrac{1}{2}(M_\lambda \lambda^a \lambda^a + h.c.) - (m^2)^i_j \phi^{*j}\phi_i \\ &\quad -(\tfrac{1}{2} b^{ij}\phi_i\phi_j + \tfrac{1}{6} a^{ijk}\phi_i\phi_j\phi_k + h.c.) \\ &\quad -\tfrac{1}{2} c_i^{jk}\phi^{i*}\phi_j\phi_k + e^i\phi_i + \text{h.c.}\end{aligned}$$

Comments:

- $e^i\phi_i$ is only allowed if ϕ_i is a gauge singlet.
- The c_i^{jk} term may introduce quadratic divergences if there is a gauge singlet multiplet in the model.
- The e^i and c_i^{jk} couplings are not allowed in the MSSM.

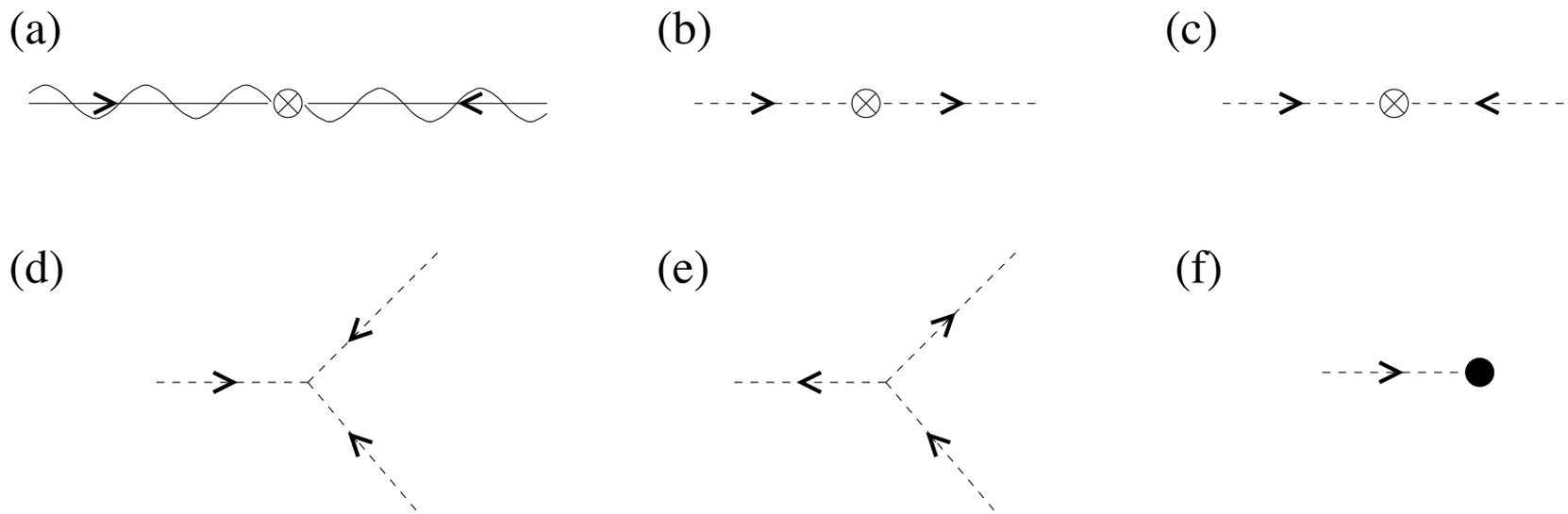


Figure 1:

Soft SUSY breaking interactions:
(a) gaugino mass M_λ.
(b) non-holomorphic mass m^2.
(c) holomorphic mass b^{ij}.
(d) holomorphic trilinear coupling a^{ijk}.
(e) non-holomorphic trilinear coupling c_i^{jk}.
(f) tadpole e^i.

Spurions

Spurions are convenient book-keeping devices for broken symmetries.

Spurions are fictitious background fields that transform under a symmetry group $\Rightarrow$ spurions break the symmetry "spontaneously".

Example: a chiral superfield Φ with a real superfield Z as wavefunction renormalization factor that breaks SUSY (but not Lorentz):

$$Z = 1 + b\,\theta^2 + b^*\bar{\theta}^2 + c\,\theta^2\bar{\theta}^2$$

Thus (recalling $\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2\mathcal{F}(y)$):

$$\int d^4\theta\, Z\Phi^\dagger\Phi = \mathcal{L}_{\text{free}} + b\mathcal{F}^*\phi + b^*\phi^*\mathcal{F} + c\phi^*\phi$$

Integrate out the auxiliary field $\mathcal{F}$:

$$\int d^4\theta\, Z\Phi^\dagger\Phi = \partial^\mu\phi^*\partial_\mu\phi + i\psi^\dagger\overline{\sigma}^\mu\partial_\mu\psi + (c - |b|^2)\phi^*\phi$$

Conclusion: the spurion field Z encodes the non-holomorphic soft SUSY breaking mass: $m^2 = |b|^2 - c$.

Superpotential Spurions

Holomorphic spurions are θ^2 components in:
a) Yukawa couplings.
b) Masses.
c) The coefficient of $W_\alpha W^\alpha$.
These generate a, b, and M_λ soft SUSY breaking terms.

The c term requires a term like

$$\int d^4\theta \, C_i^{jk} \Phi^{i\dagger} \Phi_j \Phi_k + h.c.$$

where C_i^{jk} has a nonzero $\theta^2 \bar{\theta}^2$ component.

Thus the spurion interpretation applies to all soft SUSY breaking terms:

$$\begin{aligned} \mathcal{L}_{\text{soft}} \quad = \quad & -\tfrac{1}{2}(M_\lambda \lambda^a \lambda^a + h.c.) - (m^2)^i_j \phi^{*j} \phi_i \\ & -(\tfrac{1}{2} b^{ij} \phi_i \phi_j + \tfrac{1}{6} a^{ijk} \phi_i \phi_j \phi_k + h.c.) \\ & -\tfrac{1}{2} c_i^{jk} \phi^{i*} \phi_j \phi_k + e^i \phi_i + h.c. \end{aligned}$$

Gravity-Mediated Scenario

Scenario:

- A hidden sector field X acquires a nonzero $\langle \mathcal{F}_X \rangle$ and SUSY breaking is transmitted by universal gravitational interaction.
- The MSSM soft terms are estimated as

$$m_{\rm soft} \sim \frac{\langle \mathcal{F}_X \rangle}{M_{\rm Pl}} \ .$$

- Scale: $m_{\rm soft}$ around the weak scale requires $\sqrt{\langle \mathcal{F}_X \rangle} \sim 10^{10}$–$10^{11}$ GeV.
- Example: gaugino condensation in the hidden sector $\langle 0 | \lambda^a \lambda^b | 0 \rangle = \delta^{ab} \Lambda^3 \neq 0$ implies

$$m_{\rm soft} \sim \frac{\Lambda^3}{M_{\rm Pl}^2},$$

which requires $\Lambda \sim 10^{13}$ GeV. (Equivalently, $\langle \mathcal{F}_X \rangle = \Lambda^3 / M_{\rm Pl}$.)

Effective Lagrangian

MSSM is decoupled from the hidden sector for $M_{\rm Pl} \to \infty$ so no soft terms in this limit. For finite $M_{\rm Pl}$ there may be couplings:

$$\begin{aligned}
\mathcal{L}_{\rm eff} \quad = \quad & -\int d^4\theta \, \frac{X^*}{M_{\rm Pl}} \hat{b}'^{ij} \psi_i \psi_j + \frac{X X^*}{M_{\rm Pl}^2} \left(\hat{m}^i_j \psi_i \psi^{j*} + \hat{b}^{ij} \psi_i \psi_j \right) + \text{h.c.} \\
& -\int d^2\theta \, \frac{X}{2M_{\rm Pl}} \left(\hat{M}_3 G^\alpha G_\alpha + \hat{M}_2 W^\alpha W_\alpha + \hat{M}_1 B^\alpha B_\alpha \right) + \text{h.c.} \\
& -\int d^2\theta \, \frac{X}{M_{\rm Pl}} \hat{a}^{ijk} \psi_i \psi_j \psi_k + \text{h.c.}
\end{aligned}$$

Notation: G_α, W_α, B_α, ψ_i are the chiral superfields of the MSSM; the hatted symbols are dimensionless.

SUSY breaking in the hidden sector gives $\langle X \rangle = \langle \mathcal{F}_X \rangle$ so

$$\begin{aligned}
\mathcal{L}_{\rm eff} \quad = \quad & -\frac{\langle \mathcal{F}_X \rangle}{2M_{\rm Pl}} \left(\hat{M}_3 \widetilde{G}\widetilde{G} + \hat{M}_2 \widetilde{W}\widetilde{W} + \hat{M}_1 \widetilde{B}\widetilde{B} \right) + \text{h.c.} \\
& -\frac{\langle \mathcal{F}_X \rangle \langle \mathcal{F}_X^* \rangle}{M_{\rm Pl}^2} \left(\hat{m}^i_j \widetilde{\psi}_i \widetilde{\psi}^{j*} + \hat{b}^{ij} \widetilde{\psi}_i \widetilde{\psi}_j \right) + \text{h.c.} \\
& -\frac{\langle \mathcal{F}_X \rangle}{M_{\rm Pl}} \hat{a}^{ijk} \widetilde{\psi}_i \widetilde{\psi}_j \widetilde{\psi}_k - \frac{\langle \mathcal{F}_X^* \rangle}{M_{\rm Pl}} \int d^2\theta \, \hat{b}'^{ij} \psi_i \psi_j + \text{h.c.}
\end{aligned}$$

Assumptions in Gravity Mediation

The messenger field X does not have MSSM quantum numbers so universality of gravity motivates assumptions of its couplings:

- Universal gaugino masses: $\hat{M}_i = \hat{M}$.
- Universal masses for scalars $\hat{m}^i_j = \hat{m}\delta^i_j$.
- Diagonal Yukawa couplings: $\hat{a}^{ijk} = \hat{a}Y^{ijk}$.
- The coefficients $\hat{b}'$ in the μ-term $\mu^{ij} = \hat{b}'\delta^i_{H_u}\delta^j_{H_d}\langle\mathcal{F}^*_X\rangle/M_{\rm Pl}$ and $\hat{b}$ in the $b = B\mu$ term $\hat{b}^{ij} = \hat{b}\delta^i_{H_u}\delta^j_{H_d}$ are of the same order of magnitude.

Simplifications in Minimal SUGRA

Assumptions give soft parameters a universal form (when renormalized at $M_{\rm Pl}$):

- Gaugino masses are equal:

$$M_i = m_{1/2} = \hat{M}\,\frac{\langle \mathcal{F}_X \rangle}{M_{\rm Pl}} \ .$$

- The scalar masses are universal:

$$\mathbf{m_f^2} = m_{H_u}^2 = m_{H_d}^2 = m_0^2 = \hat{m}\,\frac{|\langle \mathcal{F}_X \rangle|^2}{M_{\rm Pl}^2} \ .$$

- The A terms are given by:

$$\mathbf{A_f} \quad = \quad A\,\mathbf{Y_f} = \hat{a}\,\frac{\langle \mathcal{F}_X \rangle}{M_{\rm Pl}}\,\mathbf{Y_f} \ .$$

- The SUSY conserving μ^2 and soft breaking $b = B\mu = \frac{\hat{b}}{\hat{b}'}\,\frac{\langle \mathcal{F}_X \rangle}{M_{\rm Pl}}\mu$ are naturally of the same order of magnitude.

Discussion

Desirable consequence: the assumptions avoid problems with FCNCs.

Motivation: gravity is flavor-blind, as is hidden sector.

Problem: the motivation does not hold up. Specifically, there are non-universal terms like a Kähler function of the form

$$K_{\rm bad} = f(X^\dagger, X)^i_j \, \psi^{\dagger j} \psi_i \ ,$$

which leads directly to off-diagonal terms in the matrix $\hat{m}^i_j$.

Terminology: the minimal supergravity scenario assumes just μ and the four universal SUSY breaking parameters at unification scale (rather than the Planck scale).

Gauge-Mediated Scenario

Scenario:

- SUSY is broken in the hidden sector by a gauge group becoming strong.
- There are "messenger" chiral supermultiplets that are charged under the hidden gauge group. Their fermions and bosons masses are therefore split.
- The messenger fields also couple to the MSSM so the superpartners acquire masses through loops:

$$m_{\text{soft}} \sim \frac{\alpha_i}{4\pi} \frac{\langle \mathcal{F} \rangle}{M_{\text{mess}}}$$

- If $M_{\text{mess}} \sim \sqrt{\langle \mathcal{F} \rangle}$, then the SUSY breaking scale can be as low as $\sqrt{\langle \mathcal{F} \rangle} \sim 10^4$–$10^5$ GeV.

Messengers of SUSY breaking

Notation: goldstino multiplet X with an expectation value:

$$\langle X \rangle = M + \theta^2 \mathcal{F}$$

Assume N_f messengers ϕ_i, $\bar{\phi}_i$ coupling to hidden sector as

$$W = X \bar{\phi}_i \phi_i$$

Constraint: for gauge unification, ϕ_i and $\bar{\phi}_i$ should form complete GUT multiplets. Messengers shift the coupling at the GUT scale

$$\delta \alpha_{\rm GUT}^{-1} = -\frac{N_m}{2\pi} \ln\left(\frac{\mu_{\rm GUT}}{M}\right)$$

where

$$N_m = \sum_{i=1}^{N_f} 2T(r_i)$$

For the unification to remain perturbative we need

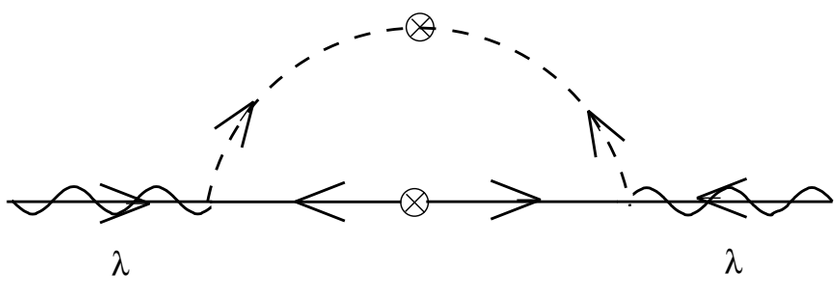
$$N_m < \frac{150}{\ln(\mu_{\rm GUT}/M)}$$

Soft Masses

VEV of goldstino multiplet:

$$\langle X \rangle = M + \theta^2 \mathcal{F}$$

$\Rightarrow$ messenger fermion mass $= M$
$\Rightarrow$ messenger scalars mass^2 $= M^2 \pm \mathcal{F}$



One-loop gaugino mass:

$$M_{\lambda i} \sim \frac{\alpha_i}{4\pi} N_m \frac{\mathcal{F}}{M}$$

Scalar Soft Masses

Messenger couple to MSSM gauge fields (by assumption) but not to MSSM matter (there are no superpotential couplings).

Soft mass for matter is therefore a two-loop effect: messenger loop corrections in the one-loop sfermion mass diagrams spoils the cancellation.

Two-loop mass^2 for squarks and sleptons

$$M_s^2 \sim \sum_i \left(\frac{\alpha_i}{4\pi}\frac{\mathcal{F}}{M}\right)^2 \sim M_\lambda^2$$

Direct computation at two loop is laborious but there is a short cut.

RG Calculation of Soft Masses

Effective Lagrangian below the messenger mass:

$$\mathcal{L}_G = -\tfrac{i}{16\pi} \int d^2\theta\, \tau(X,\mu) W^\alpha W_\alpha + \text{h.c.}$$

where

$$\tau = \tfrac{\theta}{2\pi} + \tfrac{4\pi i}{g^2} \ .$$

Recall

$$W^\alpha W_\alpha = \lambda^a_\alpha \lambda^{a\alpha} + 2i\lambda^{a\dagger}\overline{\sigma}^\mu D_\mu \lambda\ \theta^2 + \ldots \ .$$

so

$$M_\lambda \quad = \quad -\tfrac{1}{2\tau}\tfrac{\partial\tau}{\partial X}\Big|_{X=M} \mathcal{F} = -\tfrac{1}{2}\tfrac{\partial \ln\tau}{\partial \ln X}\Big|_{X=M} \tfrac{\mathcal{F}}{M}$$

The standard running of the gauge coupling can be recast as:

$$\mu \frac{d}{d\mu} \tau = \frac{ib}{2\pi} \ .$$

The β-function coefficients: b' at high scale (includes the messengers) but b in the MSSM at lower scale (where messengers are inert). Matching at the messenger scale μ_0:

$$\tau(X, \mu) = \tau(\mu_0) + i \frac{b'}{2\pi} \ln \left(\frac{X}{\mu_0} \right) + i \frac{b}{2\pi} \ln \left(\frac{\mu}{X} \right) \ .$$

The shift as messengers freeze out:

$$b' = b - N_m \ .$$

The gaugino mass is simply given by:

$$M_\lambda = -\frac{1}{2} \frac{\partial \ln \tau}{\partial \ln X} \bigg|_{X=M} \frac{\mathcal{F}}{M} = \frac{\alpha(\mu)}{4\pi} N_m \frac{\mathcal{F}}{M} \ .$$

Gaugino Masses

The gaugino masses in the gauge mediated model:

$$M_\lambda = \frac{\alpha(\mu)}{4\pi} N_m \frac{\mathcal{F}}{M} \ .$$

Remark: the ratio of the gaugino mass to the gauge coupling is universal

$$\frac{M_{\lambda_1}}{\alpha_1} = \frac{M_{\lambda_2}}{\alpha_2} = \frac{M_{\lambda_3}}{\alpha_3} = N_m \frac{\mathcal{F}}{M} \ .$$

Historical note: this was once thought to be a signature of gravity mediation models, but here it appears rather simply in gauge mediation.

Sfermion Masses

Consider wavefunction renormalization for the matter fields of the MSSM:

$$\mathcal{L} = \int d^4\theta \, Z(X, X^\dagger) Q'^\dagger Q' \ .$$

Notation: wave function renormalization factor Z is real and the prime indicates that the fields are not yet canonically normalized.

Taylor expand in the superspace coordinates

$$\mathcal{L} \quad = \quad \int d^4\theta \left(Z + \tfrac{\partial Z}{\partial X} \mathcal{F}\theta^2 + \tfrac{\partial Z}{\partial X^\dagger} \mathcal{F}^\dagger \overline{\theta}^2 + \tfrac{\partial^2 Z}{\partial X \partial X^\dagger} \mathcal{F}\theta^2 \mathcal{F}^\dagger \overline{\theta}^2 \right) \Big|_{X=M} Q'^\dagger Q'$$

Introduce canonical normalizations of the matter fields:

$$Q = Z^{1/2} \left(1 + \tfrac{\partial \ln Z}{\partial X} \mathcal{F}\theta^2 \right) \Big|_{X=M} Q'$$

so

$$\mathcal{L} = \int d^4\theta \left[1 - \left(\tfrac{\partial \ln Z}{\partial X} \tfrac{\partial \ln Z}{\partial X^\dagger} - \tfrac{1}{Z} \tfrac{\partial^2 Z}{\partial X \partial X^\dagger} \right) \mathcal{F}\theta^2 \mathcal{F}^\dagger \overline{\theta}^2 \right] \Big|_{X=M} Q^\dagger Q$$

Sfermion Masses

Read off the soft sfermion masses

$$m_Q^2 = -\frac{\partial^2 \ln Z}{\partial \ln X \partial \ln X^\dagger}\bigg|_{X=M} \frac{\mathcal{F}\mathcal{F}^\dagger}{MM^\dagger} \ .$$

Wave function renormalization also introduces an A term in the effective potential. The rescaled superpotential:

$$W(Q') = W\left(QZ^{-1/2}\left(1 - \frac{\partial \ln Z}{\partial X}\mathcal{F}\theta^2\right)\bigg|_{X=M}\right) \ ,$$

gives the A term ($\sim a\phi^3$) in the Lagrangian

$$Z^{-1/2}\frac{\partial \ln Z}{\partial X}\bigg|_{X=M} \mathcal{F}Q\frac{\partial W}{\partial(Z^{-1/2}Q)} \ .$$

Remark: it is proportional to the Yukawa coupling in the superpotential.

RG Calculation

So far: expressions for soft breaking terms from spurion.
Next: evaluate these expressions using the RG.

First, compute the wave function renormalization factor Z in the conventional manner. At one-loop

$$\mu \frac{d \ln Z}{d\mu} = \frac{C_2(r)}{\pi} \alpha(\mu)$$

Replace the RG-scale M by $\sqrt{XX^\dagger}$, so that $Z(X, X^\dagger)$ is invariant under $X \to e^{i\beta} X$.

RG Calculation

Integrate, using the β-function equation for the running of the coupling:

$$Z(\mu) = Z_0 \left(\frac{\alpha(\mu_0)}{\alpha(X)}\right)^{2C_2(r)/b'} \left(\frac{\alpha(X)}{\alpha(\mu)}\right)^{2C_2(r)/b} ,$$

where

$$\alpha^{-1}(X) = \alpha^{-1}(\mu_0) + \frac{b'}{4\pi} \ln\left(\frac{XX^\dagger}{\mu_0^2}\right) ,$$
$$\alpha^{-1}(\mu) = \alpha^{-1}(X) + \frac{b}{4\pi} \ln\left(\frac{\mu^2}{XX^\dagger}\right) .$$

Insert in expression for the soft mass and obtain

$$m_Q^2 = 2C_2(r)\frac{\alpha(\mu)^2}{16\pi^2} N_m \left(\xi^2 + \frac{N_m}{b}(1-\xi^2)\right)\left(\frac{\mathcal{F}}{M}\right)^2 ,$$

where

$$\xi = \frac{1}{1+\frac{b}{2\pi}\alpha(\mu)\ln(M/\mu)} .$$

Remark: two-loop scalar masses are determined by a one-loop RG equation!

Spurions: the Big Picture

Reminder: in SUSY gauge theory, there is an extended gauge symmetry, where the gauge parameter Λ is a chiral superfield.

Characterization of SUSY breaking: some chiral field has non-vanishing auxiliary component $\mathcal{F}$. (We do not consider $\mathcal{D}$-breaking in this discussion). This chiral field could be composite, the point is just its transformation property.

Computation of soft SUSY breaking terms: the renormalization scale is a chiral superfield. All counterterms (couplings and wave function renormalization factors) depend on that renormalization scale.

The big picture: in SUSY QFT the SUSY transformation properties are encoded in superfields so everything is a superfield.

Examples: masses, coupling constants, renormalization scale, gauge parameter...

These quantities typically have definite values, but treating them as fields give extended symmetries.