Lecture 7
SUSY breaking

Outline

- Spontaneous SUSY breaking in the WZ-model.
- The goldstino.
- Goldstino couplings.
- The goldstino theorem.

Reading: Terning 5.1, 5.3-5.4.

Spontaneous SUSY Breaking

Reminder: the SUSY algebra gives

$$\langle 0|H|0\rangle \geq 0$$

with "=" exactly when SUSY is preserved.

The SUSY potential is

$$V = \mathcal{F}^{i*}\mathcal{F}_i + \frac{g^2}{2} D^a D^a \ ,$$

so models with broken SUSY are those where all of $\mathcal{F}_i = 0$, $D^a = 0$ conditions cannot be simultaneously solved.

Later, we may use this SUSY breaking sector to generate the SUSY breaking needed in the MSSM.

O'Raifeartaigh Model

O'Raifeartaigh model: WZ model such that the extremization conditions of the superpotential $\mathcal{F}_i$ cannot all be solved simultaneously.

Example:

$$W_{O'R} = -k^2\Phi_1 + m\Phi_2\Phi_3 + \tfrac{y}{2}\Phi_1\Phi_3^2.$$

$\Rightarrow$ the scalar potential:

$$\begin{aligned} V &= |\mathcal{F}_1|^2 + |\mathcal{F}_2|^2 + |\mathcal{F}_3|^2 \\ &= |k^2 - \tfrac{y}{2}\phi_3^{*2}|^2 + |m\phi_3^*|^2 + |m\phi_2^* + y\phi_1^*\phi_3^*|^2. \end{aligned}$$

Key remark: there is no solution where both $\mathcal{F}_1 = 0$ and $\mathcal{F}_2 = 0$

For large m, minimum is at $\phi_2 = \phi_3 = 0$ with ϕ_1 undetermined. In this parameter range, the vacuum energy density is

$$V = |\mathcal{F}_1|^2 = k^4 \ .$$

O'Raifeartaigh Model: Spectrum

The spectrum depends on the value of $\langle \phi_1 \rangle$ (the position in moduli space).

Expanding around $\langle \phi_1 \rangle = 0$, the mass spectrum of scalars is

$$0,\ 0,\ m^2,\ m^2,\ m^2 - yk^2,\ m^2 + yk^2\ .$$

There are also three fermions with masses

$$0,\ m,\ m\ .$$

The masses satisfy a sum rule (that is in fact general for tree-level SUSY breaking):

$$\mathrm{Tr}[M^2_{\mathrm{scalars}}] = 2\mathrm{Tr}[M^2_{\mathrm{fermions}}]\ .$$

O'Raifeartaigh Model: One Loop

For coupling constant $k^2 \neq 0$, loop corrections give a mass to ϕ_1.

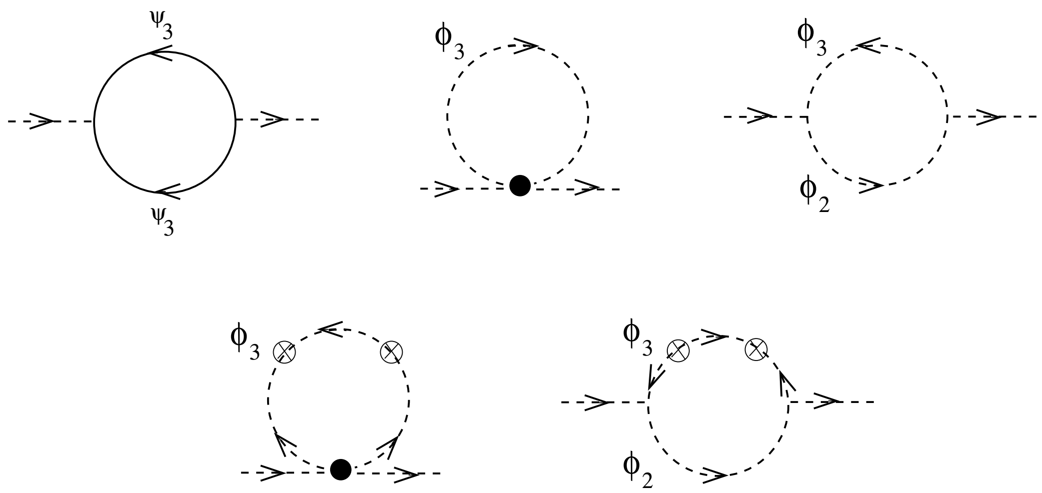

Figure 1: Crosses mark an insertion of yk^2.

An even number of yk^2 insertions appear, to preserve the orientation of the arrows flowing into the vertices.

The correction to the ϕ_1 mass from the top three graphs vanishes by SUSY.

O'Raifeartaigh model: One Loop

The bottom two graphs give

$$-im_1^2 = \int \frac{d^4p}{2\pi^4}\left[(-iy^2)\frac{iy^2k^4}{(p^2-m^2)^3} + (iym)^2\frac{i}{p^2-m^2}\frac{iy^2k^4}{(p^2-m^2)^3}\right] ,$$

A finite and positive result:

$$m_1^2 = \frac{y^4k^4}{48\pi^2m^2} = \frac{y^4}{48\pi^2}\frac{|\mathcal{F}_1|^2}{m^2} .$$

Conclusion: the classical flat direction is lifted by quantum corrections, so the quantum potential is stable around $\phi_1 = 0$.

The massless fermion ψ_1 stays massless since it is the Nambu–Goldstone particle for the broken SUSY generator: it is a goldstino.

Remark: ψ_1 is the fermion in the multiplet with the nonzero $\mathcal{F}$ component.

Fayet–Iliopoulos Mechanism

Another simple mechanism for breaking SUSY: employ a nonzero D-term for a $U(1)$ gauge group.

Add a term linear in the auxiliary field to the theory:

$$\mathcal{L}_{\mathrm{FI}} = \kappa^2 D \ ,$$

where κ is a constant parameter with dimensions of mass.

The scalar potential:

$$V = \tfrac{1}{2} D^2 + gD \textstyle\sum_i q_i \phi^{i*} \phi_i - \kappa^2 D \ .$$

The D equation of motion:

$$D = \kappa^2 - g \textstyle\sum_i q_i \phi^{i*} \phi_i \ .$$

If the ϕ_i's have large positive mass squared terms (in the superpotential), then the minimum is at $\langle \phi \rangle = 0$ and $D = \kappa^2 \Rightarrow$ SUSY is broken.

Dynamical SUSY Breaking

Fayet–Iliopoulos and O'Raifeartaigh models set the scale of SUSY breaking by hand.

Dynamical SUSY breaking: the SUSY breaking scale is naturally small (compared to the e.g. M_{Pl}) if an asymptotically free gauge theory gets strong through RG evolution

$$\Lambda \sim e^{-\frac{8\pi^2}{b g_0^2}} M_{\mathrm{Pl}}$$

$\Rightarrow$ SUSY is broken nonperturbatively.

Scenario:

- Dynamical SUSY breaking in a "hidden sector".
- SUSY breaking is communicated to MSSM by non-renormalizable interactions or through loop effects.

Gauge-Mediated Scenario

Scenario:

- SUSY is broken in the hidden sector by a gauge group becoming strong.
- There are "messenger" chiral supermultiplets that are charged under the hidden gauge group. Their fermions and bosons masses are therefore split.
- The messenger fields also couple to the MSSM so the superpartners acquire masses through loops:

$$m_{\text{soft}} \sim \frac{\alpha_i}{4\pi} \frac{\langle \mathcal{F} \rangle}{M_{\text{mess}}}$$

- If $M_{\text{mess}} \sim \sqrt{\langle \mathcal{F} \rangle}$, then the SUSY breaking scale can be as low as $\sqrt{\langle \mathcal{F} \rangle} \sim 10^4$–$10^5$ GeV.

The Goldstino

Goldstone's theorem: any spontaneously broken continuous symmetry gives rise to a massless particle (due to the generator of the broken symmetry acting on vacuum).

For spontaneously broken SUSY: predict the goldstino, a massless fermion due to a SUSY current acting on vacuum.

Fermions in a general SUSY gauge theory: $\Psi = (\lambda^a, \psi_i) \Rightarrow$ the fermion mass matrix (from the $\phi^* \lambda \psi$ Yukawa, and from the superpotential):

$$\mathbf{M}_{\text{fermion}} = \begin{pmatrix} 0 & \sqrt{2} g_a (\langle \phi^* \rangle T^a)^i \\ \sqrt{2} g_a (\langle \phi^* \rangle T^a)^j & \langle W^{ij} \rangle \end{pmatrix}$$

Eigenvector with eigenvalue zero:

$$\begin{pmatrix} \langle D^a \rangle / \sqrt{2} \\ \langle \mathcal{F}_i \rangle \end{pmatrix}$$

- Proof, part 1: gauge invariance of the superpotential implies

$$(\phi^* T^a)^i W_i^* = -(\phi^* T^a)^i \mathcal{F}_i = 0$$

- Proof, part 2: the derivative of the scalar potential vanishes in the vacuum

$$\begin{aligned}\langle \frac{\partial V}{\partial \phi_i} \rangle &= \langle W_j^* \rangle \langle \frac{\partial W^j}{\partial \phi_i} \rangle - g_a (\langle \phi^* \rangle T^a)^i \langle D^a \rangle \\ &= -\langle \mathcal{F}_j \rangle \langle W^{ji} \rangle - g_a (\langle \phi^* \rangle T^a)^i \langle D^a \rangle = 0\end{aligned}$$

- Remark: the eigenvector is only nontrivial if SUSY is broken: at least one of the $\langle D^a \rangle, \langle \mathcal{F}_i \rangle \neq 0$.

- The canonically normalized massless goldstino:

$$\Pi = \frac{1}{F_\Pi} \left(\frac{\langle D^a \rangle}{\sqrt{2}} \lambda^a + \langle \mathcal{F}_i \rangle \psi_i \right)$$

where

$$F_\Pi^2 = \frac{1}{2} \sum_a \langle D^a \rangle^2 + \sum_i \langle \mathcal{F}_i \rangle^2$$

The Supercurrent

Rewrite the supercurrent for SYM coupled to chiral matter:

$$\begin{aligned} J^\mu_\alpha &= \tfrac{i}{\sqrt{2}} D^a (\sigma^\mu \lambda^{\dagger a})_\alpha + \mathcal{F}_i i (\sigma^\mu \psi^{\dagger i})_\alpha + (\sigma^\nu \overline{\sigma}^\mu \psi_i)_\alpha \, D_\nu \phi^{*i} \\ &\qquad\qquad - \tfrac{1}{2\sqrt{2}} (\sigma^\nu \overline{\sigma}^\rho \sigma^\mu \lambda^{\dagger a})_\alpha \, F^a_{\nu\rho} \ , \\ &= i F_\Pi (\sigma^\mu \bar{\Pi})_\alpha \, + (\sigma^\nu \overline{\sigma}^\mu \psi_i)_\alpha \, D_\nu \phi^{*i} - \tfrac{1}{2\sqrt{2}} (\sigma^\nu \overline{\sigma}^\rho \sigma^\mu \lambda^{\dagger a})_\alpha \, F^a_{\nu\rho} \ , \\ &\equiv i F_\Pi (\sigma^\mu \bar{\Pi})_\alpha \, + j^\mu_\alpha \ . \end{aligned}$$

Terms included in j^μ_α contain two or more fields.

The effective Lagrangian for the goldstino

$$\mathcal{L}_{\text{goldstino}} = i \bar{\Pi} \overline{\sigma}^\mu \partial_\mu \Pi + \tfrac{1}{F_\Pi} (\Pi \, \partial_\mu j^\mu + h.c.) \ ,$$

is such that the e.o.m. is supercurrent conservation:

$$\partial_\mu J^\mu_\alpha = i F_\Pi (\sigma^\mu \partial_\mu \bar{\Pi})_\alpha + \partial_\mu j^\mu_\alpha = 0 \ .$$

Goldstino Couplings

The couplings of the goldstino:

$$\mathcal{L}_{\text{goldstino int}} = \frac{1}{F_\Pi}(\Pi\,\partial_\mu j^\mu + h.c.).$$

where

$$j^\mu_\alpha = (\sigma^\nu\overline{\sigma}^\mu\psi_i)_\alpha\,D_\nu\phi^{*i} - \frac{1}{2\sqrt{2}}(\sigma^\nu\overline{\sigma}^\rho\sigma^\mu\lambda^{\dagger a})_\alpha\,F^a_{\nu\rho}\ .$$

Comments:

- The goldstino–scalar–fermion and goldstino–gaugino–gauge boson interactions allow the heavier superpartner to decay.
- The interaction terms have two derivatives.
- The coupling constant has dimension of inverse mass squared, with scale set by the VEV responsible for spontaneous SUSY breaking.

The Goldstino Theorem

The goldstino theorem: there is a goldstino in any SUSY QFT with spontaneously broken SUSY.

The point: it is not important that SUSY is broken at tree level. There is a goldstino no matter how SUSY is spontaneously broken, even if it is dynamical.

This is invaluable for the analysis of QFTs with broken SUSY.

The proof uses techniques that are common in QFT so it is instructive to present it.

The SUSY Algebra Revisited

The SUSY algebra:

$$\{Q_\alpha, Q^\dagger_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu$$

where, by Noether's theorem, the charge is related to the supercurrent

$$Q^\dagger_{\dot\alpha} = \sqrt{2}\int d^3x J^{\dagger 0}_{\dot\alpha}$$

Combine and write a local version:

- Remove $\int d^3x$ on LHS.
- Interpret P_μ as the four momentum density $T_\mu{}^0$ on the RHS.
- Boost the result to an arbitrary frame.
- Local version of the SUSY algebra:

$$\{Q_\alpha, J^{\dagger\nu}_{\dot\alpha}\} = \sqrt{2}\sigma^\mu_{\alpha\dot\alpha} T_\mu{}^\nu$$

The VEV of the SUSY algebra:

$$\langle 0|\{Q_\alpha, J^{\mu\dagger}_{\dot\alpha}(y)\}|0\rangle = \sqrt{2}\sigma^\nu_{\alpha\dot\alpha}\langle 0|T^\mu_\nu(y)|0\rangle = \sqrt{2}\sigma^\nu_{\alpha\dot\alpha}\,E\,\eta^\mu_\nu\ ,$$

where E is the vacuum energy density.

When $E \neq 0$, SUSY is spontaneously broken.

Introduce supercurrent for the first SUSY charge as well:

$$\sqrt{2}\sigma^\mu_{\alpha\dot\alpha}\,E \quad = \quad \int d^3x \langle 0|\{\textstyle\int J^0_\alpha(x), J^{\mu\dagger}_{\dot\alpha}(0)\}|0\rangle$$

Strategy: rewrite RHS in two different ways.

Rewriting I

Insert a sum over a complete set of states

$$\sqrt{2}\sigma^{\mu}_{\alpha\dot{\alpha}} E = \sum_n \int d^3x \left(\langle 0|J^0_\alpha(x)|n\rangle\langle n|J^{\mu\dagger}_{\dot{\alpha}}(0)|0\rangle + \langle 0|J^{\mu\dagger}_{\dot{\alpha}}(0)|n\rangle\langle n|J^0_\alpha(x)|0\rangle\right)$$

Translational invariance (with generator P^μ), applied at time $x^0 = 0$:

$$\begin{aligned}\langle 0|J^0_\alpha(x)|n\rangle &= \langle 0|e^{iP\cdot x}J^0_\alpha(0)e^{-iP\cdot x}|n\rangle \\ &= \langle 0|J^0_\alpha(0)e^{-i\vec{p}_n\cdot\vec{x}}|n\rangle\end{aligned}$$

So:

$$\begin{aligned}\sqrt{2}\sigma^{\mu}_{\alpha\dot{\alpha}} E &= \sum_n (2\pi)^3\delta(\vec{p}_n)\left(\langle 0|J^0_\alpha(0)|n\rangle\langle n|J^{\mu\dagger}_{\dot{\alpha}}(0)|0\rangle + \langle 0|J^{\mu\dagger}_{\dot{\alpha}}(0)|n\rangle\langle n|J^0_\alpha(0)|0\rangle\right) \\ &\equiv \sum_n (2\pi)^3\delta(\vec{p}_n) f_n(E_n, \vec{p}_n)\end{aligned}$$

Rewriting II

Rewrite the anticommutator using current conservation, and only then insert a sum over a complete set of states:

$$
\begin{aligned}
\sqrt{2}\sigma^{\mu}_{\alpha\dot{\alpha}}\,E &= \int d^4x \left(\langle 0|J^0_\alpha(x)J^{\mu\dagger}_{\dot{\alpha}}(0)|0\rangle + \langle 0|J^{\mu\dagger}_{\dot{\alpha}}(0)J^0_\alpha(x)|0\rangle\right)\delta(x^0) \\
&= \int d^4x\,\partial_\rho\left(\langle 0|J^\rho_\alpha(x)J^{\mu\dagger}_{\dot{\alpha}}(0)|0\rangle\Theta(x^0) - \langle 0|J^{\mu\dagger}_{\dot{\alpha}}(0)J^\rho_\alpha(x)|0\rangle\Theta(-x^0)\right) \\
&= \sum_n \int d^4x\ \partial_\rho\left(\begin{array}{l} \langle 0|J^\rho_\alpha(0)e^{ip_n\cdot x}|n\rangle\langle n|J^{\mu\dagger}_{\dot{\alpha}}(0)|0\rangle\Theta(x^0) \\ -\langle 0|J^{\mu\dagger}_{\dot{\alpha}}(0)|n\rangle\langle n|e^{-ip_n\cdot x}J^\rho_\alpha(0)|0\rangle\Theta(-x^0)\end{array}\right) \\
&= \sum_n \int d^4x \left[\begin{array}{l} \delta(x^0)\left(\begin{array}{l} e^{-i\vec{p}_n.\vec{x}}\langle 0|J^\rho_\alpha(0)|n\rangle\langle n|J^{\mu\dagger}_{\dot{\alpha}}(0)|0\rangle \\ +e^{i\vec{p}_n.\vec{x}}\langle 0|J^{\mu\dagger}_{\dot{\alpha}}(0)|n\rangle\langle n|J^\rho_\alpha(0)|0\rangle\end{array}\right) \\ -ip_{n\rho}\left(\begin{array}{l} e^{ip_n\cdot x}\langle 0|J^\rho_\alpha(0)|n\rangle\langle n|J^{\mu\dagger}_{\dot{\alpha}}(0)|0\rangle\Theta(x^0) \\ +e^{-ip_n\cdot x}\langle 0|J^{\mu\dagger}_{\dot{\alpha}}(0)|n\rangle\langle n|J^\rho_\alpha(0)|0\rangle\Theta(-x^0)\end{array}\right)\end{array}\right. \\
&= \sum_n (2\pi)^3\delta(\vec{p}_n)\ \left(f_n(E_n,\vec{p}_n,) - i\int_0^\infty dx^0\, e^{iE_nx^0}E_nf_n(E_n,\vec{p}_n)\right)
\end{aligned}
$$

Notation: $\Theta(x^0)$ is the step function.

The Goldstino Theorem

By comparison of the two rewritings:

$$\int_0^\infty dx^0\, e^{iE_n x^0} E_n f_n(E_n, \vec{0}) = 0$$

If SUSY is spontaneously broken the vacuum energy density $E \neq 0$ so

$$f_n(E_n, \vec{0}) \neq 0$$

The only possibility is that

$$f_n(E_n, \vec{0}) \propto \delta(E_n)$$

Conclusion:

- A state contributes to the current two-point function.
- The state has the quantum numbers of $J^0_\alpha \Rightarrow$ it is a fermion.
- The state has with $\vec{p} = 0$ and $E = 0 \Rightarrow$ it is massless.

- In other words, there must be a goldstino!

The physics content of the proof:

- The superalgebra relates the energy density E to the supercurrents.
- Current conservation further relates it to integral of a total divergence.
- The total divergence becomes an surface integral at infinity that vanishes for massive particles since those are exponentially damped.
- The energy density cannot vanish if there is spontaneous SUSY breaking.
- So: there must be a massless particle. This is the goldstino.

Super Higgs Mechanism

Spontaneously broken global symmetry: there is a Goldstone boson that is exactly massless.

Spontaneously broken gauge symmetry: the would-be massless Goldstone boson combines the would-be massless gauge bosons, and form massive gauge bosons (this is the Higgs mechanism).

Spontaneously broken global SUSY: there is a goldstino that is exactly massless.

Spontaneously broken supergravity (=local SUSY): the would be goldstino combines with the massless gravitino (the SUSY partner of the graviton) and form a massive gravitino.

This mechanism is (also) called the super Higgs Mechanism.

Reminder from lecture 6: the other super Higgs mechanism is spontaneously broken gauge symmetry in SUSY theory; a SUSY vector multiplet combines with a SUSY chiral multiplet, without breaking SUSY.

Super Higgs Mechanism

Details: for gravity Poincaré symmetry is local, so SUSY must be local as well.

The SUSY spinor $\epsilon^\alpha \to \epsilon^\alpha(x)$: supergravity.

In supergravity the spin-2 graviton has spin-3/2 fermionic superpartner, the gravitino. Notation: $\widetilde{\Psi}^\alpha_\mu$,

The gravitino transforms inhomogeneously under local SUSY transformations, as expected for the "gauge" particle of local SUSY transformations:

$$\delta\widetilde{\Psi}^\alpha_\mu = -\partial_\mu \epsilon^\alpha + \dots \ .$$

When SUSY is spontaneously broken, the gravitino acquires a mass by "eating" the goldstino: the other super Higgs mechanism.

Gravitino mass:

$$m_{3/2} \sim \frac{\langle \mathcal{F}_X \rangle}{M_{Pl}}$$