PHY 513: HW 9 (due tue 11/17/09)

1 Two-body Decay

There are many two-body decays in AMO (photoemission from an excited level, positronium decay to two photons, ...), condensed matter (exciton decay to phonon and electron,...), nuclear (fission,...), and elementary particle physics. As a specific example consider the decay of a neutral vector particle to two charged scalars and assume that the effective interaction responsible for this decay is

$$if\rho^\mu\,\phi^\dagger\overleftrightarrow{\partial}_\mu\phi$$

where f is some coupling constant. Let the mass of the vector particle be M and the mass of the scalar be m. Calculate the decay rate for particles in a sample of unpolarized vector particles. Express you result in terms of f, M, and m.

You will need the Feynman rule associated with the vertex. It is

$$if(\bar{p}-p)^\mu$$

where p and $\bar{p}$ are the four momenta of the outgoing particle and anti-particle, respectively.

The polarization vector ϵ^μ for a massive spin-one particle is very similar to that for a photon, except that there are 3 helicity states instead of two. The completeness relation needed to perform the sum over spin states of the vector-boson is

$$\sum_{\text{spins}}\epsilon_\mu\,\epsilon^*_\nu = -\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{M^2}\right)$$

The form of the completeness relation is clear in the rest frame of the vector particle $k^\mu=(M,0,0,0)$. There, the three polarization vectors can be chosen as

$$\epsilon^\mu_{R,L}=\frac{1}{\sqrt{2}}(0,1,\pm i,0)$$

for the right-handed and left-handed helicity states, and

$$\epsilon^\mu_l=(0,0,0,1)$$

for the longitudinal or zero-helicity state.

2 Mott's Formula

Solve the first part of **PS** problem 5.1

The problem asks you to compute Mott's formula using two different strategies. It is enough to compute it using the first (and easier) method, which is simply a generalization of **PS** prob.4.4 (HW8 prob.2) to the relativistic case.

3 Helicity Amplitudes in Yukawa Theory

The Yukawa model of a Dirac fermion f (of mass M) interacting with a real scalar (of mass m) is described by the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 + \overline{\psi}(i\partial\!\!\!/ - M)\psi - g\phi\overline{\psi}\psi. \tag{1}$$

The ϕ^4 term will play no role in this problem.

Consider the elastic scattering of two fermions ff in the center of mass frame and assume that the energies of the particles are very large compared to their masses. The amplitude $i\mathcal{M}$ is given in terms of Feynman diagrams and their associated formulas in **PS** eq. 4.119. The scattering cross sections for polarized particles (*i.e.* particles with definite helicities) can teach us much about the nature of their interactions. We will denote denote the various helicity amplitudes as

$$\langle p'\lambda'\ k'\mu', out|p\lambda\ k\mu, in\rangle \equiv i\mathcal{M}_{\lambda'\mu';\lambda\mu}(E_{\text{cm}}, \theta) \tag{2}$$

where θ is the scattering angle in the center of mass frame. The greek indices denote the helicities of each of the particles (L or R, in each case).

a) Derive the selection rule satisfied by helicity in the present context. To do so consider the helicity spinors, *i.e.* the eigenspinors of the projection operators $P_{R,L} = \frac{1}{2}(1 \pm \gamma^5)$ taking the form

$$u_\lambda(p) = P_\lambda u(p)$$

The notation is that λ takes either the value R, associated with the numerical value $+1$; or the value L, associated with the numerical value -1. Use this equation to find the selection rule (Do not consider the explicit form of $u_\lambda(p)$).

b) Varying all helicities over their possible values L/R there is a total of 16 amplitudes $\mathcal{M}_{\lambda'\mu';\lambda\mu}$. Use the selection rule derived in a) to determine which amplitudes may be nonvanishing. How many are there?
c) HW5 prob 5 determined the explicit form of the two-spinors $\xi_\lambda(p)$ with definite helicity (the result was also derived on **PS** p. 68). Use this result to find the explicit forms of the eigenspinors $u_\lambda(p)$ in the limit of very high energy $E/m \to \infty$ (with the scattering angle kept fixed).
d) Use the spinors found in c) to rederive the selection rule from a).
e) Using the explicit eigenspinors, compute the inner products $\bar{u}_R(p')u_L(p)$, $\bar{u}_R(k')u_L(k)$, $\bar{u}_R(p')u_L(k)$, and $\bar{u}_R(k')u_L(p)$.
f) Compute the amplitudes $\mathcal{M}_{RR;LL}$ and $\mathcal{M}_{LR;LR}$ in the limit of very high energies at fixed scattering angle θ (with $\theta \neq 0, \pi$). You may neglect the mass m_ϕ^2 of the boson as well in this limit.
g) Determine the spin-averaged form of the amplitude squared, by explicit averaging of the amplitudes found in question f).