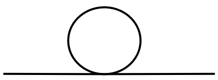
PHY 513: HW 7 (due tue. nov 3, 2009)

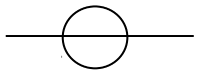
1 Symmetry Factors

a) Determine the symmetry factor for each of the following diagrams in scalar ϕ^4 theory.

a



b



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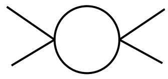

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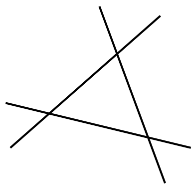
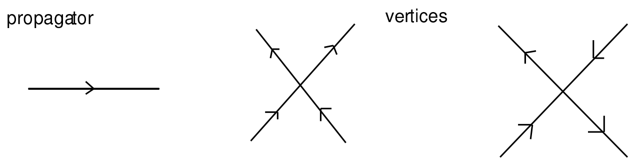


b) Consider the field theory associated with a *complex* scalar field ϕ given by Lagrangian

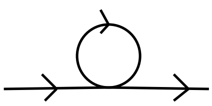
$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi - \frac{\lambda}{4}(\phi^\dagger \phi)^2 \tag{1}$$

The Feynman rules for this theory has the propagator $i/(p^2 - m^2 + i\epsilon)$ and vertices $-i\lambda$, just as the real theory. However, for the complex field the lines are *oriented* in the direction of the charge flow. This is implemented in the Feynman rules by drawing diagrams with arrows, as in:



What is the symmetry factor for each of the following Feynman diagrams?

a



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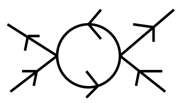

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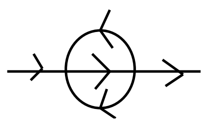
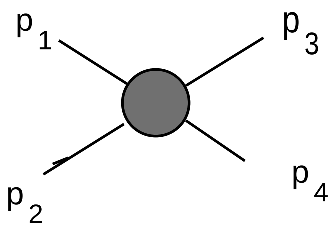


2 Feynman Diagrams in ϕ^4-theory

The ϕ^4-theory has the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{24}\lambda\phi^4$$

In this theory consider an amplitude with 4 external particles (2 in, 2 out):



The solid bubble denotes the collection of diagrams with four legs attached.

a) Draw all the distinct Feynman diagrams for the four-point function through order λ^2. Do not draw any self-energy corrections to external legs, nor disconnected diagrams.

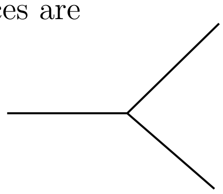
b) Write out the corresponding expressions in momentum space for each Feynman diagram in part (a). You do not have to consider the symmetry factor.

3 Symmetry Factors in Cubic-Quartic Theory

Consider the theory defined by the Lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \frac{m^2}{2}\phi^2 - \frac{g}{3!}\phi^3 - \frac{\lambda}{4!}\phi^4.$$

The vertices are



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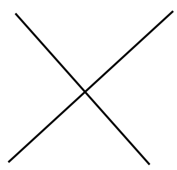

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(the l is a λ). In the following, you should consider only the so-called "amputated, connected" Feynman diagrams.

a) For two-particle scattering, $p_A + p_B \to p_1 + p_2$, draw all the Feynman diagrams to $\mathcal{O}(g^2)$ or $\mathcal{O}(\lambda)$

It is natural to treat these all diagrams as if they are of the same order, *i.e.* to think of λ as being of order g^2/m^2 (where the factor of m^2 is inserted simply to have a dimensionless quantity to compare with λ).

b) Consider the production of a third particle, $p_A + p_B \to p_1 + p_2 + p_3$. Draw all the Feynman diagrams of order $\mathcal{O}(g\lambda)$ for this process. The diagrams of order $\mathcal{O}(g^3)$ are of the same order but they are (even) more tedious and you do not need to consider those.

c) Show that all the diagrams in part (b) have symmetry factor 1.

4 $\phi_1\phi_2\phi_3$-theory

Consider a theory with three distinct scalars, interacting through a cubic interaction

$$\mathcal{L} = \frac{1}{2}\sum_{i=1}^{3}\left(\partial_\mu\phi_i\partial^\mu\phi_i - m_i^2\phi_i^2\right) - g\phi_1\phi_2\phi_3$$

a) Write the Feynman diagrams for scattering of one ϕ_1-particle and one ϕ_2-particle, with the same particle content in the final state.

b) Compute the amplitude for these Feynman diagrams. Denote the initial momenta $p_{A,B}$ (for particle of type $1, 2$) and the final momenta $p_{1,2}$.

c) Compute the differential scattering cross-section $\frac{d\sigma}{d\Omega}$ for scattering in the center of mass frame. You can express your answer in terms of the total energy E_{cms}, the spatial momenta $\vec{p}_A, \vec{p}_1$ and the matrix element from b).