PHY 621: HW 4 (due mon. oct 11, 2010)
Aspects of SUSY Phenomenology

1 Decay of a Slepton

a) Estimate the decay width of a slepton decaying to an electron and a gravitino. You can neglect the electon and gravitino masses and take the slepton mass of order $M_{\text{soft}} \sim M_{\text{weak}}$.

b) Estimate the SUSY breaking scale $\sqrt{\langle F_X \rangle}$ in gravity mediation and in gauge mediation with comparable messenger scale and breaking scale.

c) Give numerical estimate for the decay width in a), in the two scenarios in b). Give you answer for the decay length of the slepton in meters.

2 Quadratic Divergences of the Higgs Mass

Starting with the superpotential

$$W = y_t H_u Q_3 \overline{u}_3$$

verify in the SUSY limit the conditions given in Terning sec 1.1 (and in Lecture 1) to cancel the logarithmic divergent contributions to the HIggs mass are indeed satisfied.

3 Spontaneous Breaking of $U(1)_{\text{em}}$

Check whether the Higgs potential in the MSSM has any stable minima which break $U(1)_{\text{em}}$.

A possible approach is, using the gauge choice $\langle H_u^+ \rangle = 0$, the constraint $\partial V(H_u, H_d)/\partial H_u^+ = 0$, and assuming $\langle H_d^- \rangle \neq 0$, find the stationary points of $V(|H_u^0|^2, |H_d^-|^2)$. Then check if they correspond to stable minima of the full potential.

4 Soft SUSY Breaking from Spurions

For a general SUSY gauge theory with renormalizable interactions, find the soft SUSY breaking terms that are produced by giving θ^2 spurion components to the background chiral superfields corresponding to holomorphic couplings in the superpotential, the coefficient of $W_\alpha W^\alpha$, as well as the wave function renormalization

$$Z = 1 + b\theta^2 + b^*\overline{\theta}^2 + c\theta^2\overline{\theta}^2 \ .$$

5 Soft Slepton Mass

Using

$$m_Q^2 = -\left.\frac{\partial^2 \ln Z}{\partial \ln X \partial \ln X^\dagger}\right|_X = M\frac{FF^\dagger}{MM^\dagger}$$

show that

$$m_Q^2 = 2C_2(r)\frac{\alpha(\mu)^2}{16\pi^2}N\left(\xi^2 + \frac{N}{b}(1-\xi^2)\right)\left(\frac{F}{M}\right)^2$$

where

$$\xi = \frac{\alpha(M)}{\alpha(\mu)} = \frac{1}{1+\frac{b}{2\pi}\alpha(\mu)\ln\frac{M}{\mu}}$$

and

$$\mu\frac{d\ln z}{d\mu} = \frac{C_2(r)}{\pi}\alpha(\mu)$$