PHY 513: HW 3 (due Tue 9/29/09)

1 Quantization of the Complex Scalar Field

Problem 2.2 in **PS**, questions a-c (*i.e.* skip question d). In b) the conventional way of expanding the field is

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2\omega_p}}\left(a_{\vec{p}}e^{i\vec{p}\vec{x}} + b^\dagger_{\vec{p}}e^{-i\vec{p}\vec{x}}\right) \tag{1}$$

2 Green's Functions: Explicit Expressions

PS introduces the two-point correlation function

$$D_W(x) \equiv \langle 0|\phi(x)\phi(0)|0\rangle = \int \frac{d^3p}{(2\pi)^3}\,\frac{1}{2E_p}\,e^{-ipx} \tag{2}$$

for a real scalar field $\phi(x)$. It is also useful to consider

$$\begin{aligned} iD(x) &\equiv \langle 0|[\phi(x),\phi(0)]|0\rangle \\ D_1(x) &\equiv \langle 0|\{\phi(x),\phi(0)\}|0\rangle \end{aligned}$$

In the literature it is conventional to refer to D_W as the Wightman function, D as the commutator function, and D_1 as the Hadamard function. Beware that **PS** uses an unconventional notation where the Wightman function D_W is denoted D and no notation is introduced for the other two-point correlators.

The purpose of this problem is to compute these functions explicitly. You will need properties of Bessel functions from a table of integrals or a computer program such as Mathematica. One good source is the *Handbook of Mathematical Functions* by Abramowitz and Stegun (AS); another is *Table of Integrals, Series, and Products* by Gradshteyn and Ryzhik (GS).
a) Consider the free scalar field of mass m. Show that for spacelike x

$$D_W(x) = \frac{m}{4\pi^2\sqrt{-x^2}}K_1(m\sqrt{-x^2}) \tag{3}$$

where $K_1(z)$ is a Bessel function.
b) Compute the commutator and Hadamard functions from the Wightman function at spacelike x. Then determine their behaviors at large distance (including the overall prefactor that is a power of r).

3 Green's Functions; the Massless Case

This problem is a continuation of the previous problem with the difference that we now consider the case of a *massless* field.
a) Compute the integral (2) for $D_W(x)$ using elementary integrals. To avoid ambiguities, introduce a regulator so that the integrals converge properly. One way to do this is to take $(t, \vec{x}) \to (t - i\epsilon, \vec{x})$. The result should be valid to spacelike, timelike, and null x.
b) Compute the commutator function $D(x)$ and the Hadamard function $D_1(x)$. For example you can start from the result in a) and then use

$$\frac{1}{s - i\epsilon} = \frac{P}{s} + i\pi\delta(s) \tag{4}$$

where P denotes the principal value prescription (you will not need to know precisely what this means, although you are encouraged to look it up if you are curious). In this formula the first term is real and odd while the second term is purely imaginary and even.