PHY 513: HW 11 (due tue 12/1/09)

1 Electron-Electron scattering

Consider the elastic scattering of two electrons in QED $e^-\,e^- \to e^-\,e^-$. Denote the scattering amplitude in helicity basis as

$$\langle p'\lambda'\; k'\mu', \text{out}|p\lambda\; k\mu, \text{in}\rangle \equiv i\mathcal{M}_{\lambda'\mu';\lambda\mu} \tag{1}$$

a) Draw the two Feynman diagrams for the scattering amplitude in tree approximation. Write the corresponding transition amplitudes. Justify your choice of relative sign between the two terms.
b) The vertices in this amplitude are of the form $\bar{u}_{\lambda'}(p')\gamma^\mu u_\lambda(p)$. Show that, in the forward direction where $p'_\mu = p_\mu$, the vector coupling is helicity-conserving,

$$\bar{u}_{\lambda'}(p)\gamma^\mu u_\lambda(p) = 2p^\mu\delta_{\lambda\lambda'}. \tag{2}$$

Hint: Refer to the Gordon identity.
c) Referring to the result in part (b), which of the helicity amplitudes $\mathcal{M}_{\lambda'\mu';\lambda\mu}$ are largest in the near-forward direction?
d) Which helicity amplitudes are largest near the backward direction? Hint: this is an easy question.
e) In the center of mass frame, evaluate the dominant diagram approximately for $M_{LR;LR}(E_{\text{cm}},\theta)$ for angles near the forward direction.
f) Assuming that the result of part (e) were the only contribution to that helicity amplitude, determine the differential cross section $d\sigma/d\cos\theta$ in the center of mass at near forward angles for that process. Express your result in terms of the cms energy E_{cm}, the scattering angle θ, the electron mass m, and the fine-structure constant α.

2 Detailed Balance

Consider the reactions

$$a + b \longrightarrow a' + b'$$

and

$$a' + b' \longrightarrow a + b$$

Here a, b, a', b' are four possibly different particles of spins $s_a, s_b, s_{a'} s_{b'}$. Let $\vec{k}'$ and $\vec{k}$ represent the center-of-mass momenta of particles a', b' and a, b, respectively. Assuming that the interactions are time-reversal invariant, show that the *unpolarized* differential cross sections in the center-of-mass frame satisfy

$$\frac{\frac{d\sigma}{d\Omega'}(a+b \to a'+b')}{\frac{d\sigma}{d\Omega}(a'+b' \to a+b)} = \frac{\vec{k}'^2(2s_{a'}+1)(2s_{b'}+1)}{\vec{k}^2(2s_a+1)(2s_b+1)}. \tag{3}$$

Relations such as these are often used in thermodynamics for determining equilibrium conditions.

3 Fermion Annihilation in Yukawa Theory

In the Yukawa model consider the process of fermion-anti-fermion annihilation to two scalars $f\bar{f} \to \phi\phi$

a) Draw the two Feynman diagrams for the S-matrix in the tree approximation.

b) Write the corresponding expressions for the scattering amplitude $\mathcal{M}$.

c) Give a physical argument that justifies the relative sign between the two terms in the amplitude found in b).