

PHY 513: HW 10 (due tue 11/24/09)

1 The Dirac Propagator

The Dirac propagator is defined as the time-ordered two point correlator

$$S_F(x-y)_{ab} = \langle 0|T\psi_a(x)\bar{\psi}_b(y)|0\rangle = \begin{cases} \langle 0|\psi_a(x)\bar{\psi}_b(y)|0\rangle & , x^0 > y^0 \\ -\langle 0|\bar{\psi}_b(y)\psi_a(x)|0\rangle & , y^0 > x^0 \end{cases} \quad (1)$$

We can write this definition as

$$S_F(x-y)_{ab} = \theta(x^0 - y^0)\langle 0|\psi_a(x)\bar{\psi}_b(y)|0\rangle - \theta(y^0 - x^0)\langle 0|\bar{\psi}_b(y)\psi_a(x)|0\rangle \quad (2)$$

a) Evaluate $S_F(x-y)$ for a free Dirac field ψ , by inserting the appropriate oscillator expansion. For each time ordering, express your answer as a linear differential operator acting on

$$D(x-y) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E} e^{-ip(x-y)} \quad (3)$$

b1) Starting afresh from the definition (2), show that the Dirac propagator satisfies the Dirac equation *with* a source

$$(i\cancel{\partial} - m)_{ca}S_F(x-y)_{ab} = i\delta(x-y)^{(4)}\delta_{cb} \quad (4)$$

i.e., that $S_F(x-y)$ is a Green's function. You will need the equal time commutation relations.

b2) Find the propagator by solving the equation for the Green's function (4). You can do this by introducing the Fourier transform

$$S_C(x-y)_{ab} = \int_C \frac{d^4p}{(2\pi)^4} \tilde{S}_C(p)_{ab} e^{-ip(x-y)} \quad (5)$$

Express your result for $S_C(x-y)_{ab}$ as a linear differential operator acting on the *scalar* propagator

$$D_C(x-y) = \int_C \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2} e^{-ip(x-y)} \quad (6)$$

where C is some contour in the complex p^0 -plane. Alternatively the integral could be specified by providing some suitable $i\epsilon$ -prescription to the bosonic propagator $\tilde{D}_C(p) = \left(\frac{i}{p^2 - m^2}\right)_C$.

c) Using the Feynman contour or, alternatively, the prescription $\tilde{D}_F(p) = \frac{i}{p^2 - m^2 + i\epsilon}$ for the propagator, it is shown in **PS** 2.60 that the bosonic propagator can be written

$$D_F(x - y) = \theta(x^0 - y^0)D(x - y) + \theta(y^0 - x^0)D(x - y) \quad (7)$$

Use this expression to show that your result in b) is consistent with the one found in a).

2 Mott's Formula Revisited

Solve the second part of **PS** prob 5.1. This involves the following steps:

a) Compute the spin-averaged amplitude for $e^- \mu^-$ scattering for general m_e and m_μ .

b) Take the limit where the mass of the μ is large, keeping the electron energy and scattering angle fixed.

Hints: in this limit the muon has negligible momentum and its energy $E_2 \simeq m_\mu$. The answer for $\frac{1}{4} \sum_{\text{spin}} |\mathcal{M}|^2$ will take the form of a power of m_μ , multiplied by a function that does not depend on m_μ .

c) Write the formula for the cross-section in the cms-frame. Simplify the kinematic factors multiplying the amplitude-squared by taking m_μ to be large. The power of m_μ from the kinematical factors should cancel the one from the amplitude, and the final result should be the Mott-formula.