PHY 621: HW 1 (due mon. sep 20, 2010)
SUSY in Component Formalism

1 Vacuum Conditions

Consider the WZ-model with the superpotential

$$W = m\phi_2\phi_3 + \tfrac{1}{2}y\phi_1\phi_3^2$$

1) Calculate the scalar potential.

2) Show that the SUSY vacua are given by

$$\langle\phi_1\rangle = v \ , \quad \langle\phi_2\rangle = 0 \ , \quad \langle\phi_3\rangle = 0 \ ,$$

for arbitrary v.

3) Expanding around the vacua $\phi_1 = v + \tilde{\phi}_1$ and writing the scalar potential out to quadratic prder, find the scalar mass squared matrix M_ϕ^2.

4) Find the fermion mass matrix M_ψ and verify that $M_\psi M_\psi^\dagger = M_\phi^2$.

2 WZ-Model with Nonrenormalizable Terms

Consider a Wess-Zumino model with the superpotential

$$W = \tfrac{1}{3}y\phi^3 + \tfrac{\lambda}{4M}\phi^4$$

1) What are the off-shell SUSY transformations of the scalar ϕ and its superpartner fermion ψ, expressed only in terms of ϕ and ψ?

2) For the superpotential given above, what is the corresponding Lagrangian in terms of ϕ and ψ?

3) Schematically (include the parametric dependence on the couplings but not numerical coefficients), what are the Feynman rules for the cubic and quartic interactions?

3 SUSY invariance of SYM

Check that

$$S = \int d^4x \left(-\tfrac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\lambda^{\dagger a} \bar{\sigma}^\mu D_\mu \lambda^a + \tfrac{1}{2} D^a D^a \right)$$

is invariant under the SUSY transformations

Hints:

- After writing the SUSY variation it is extremely helpful to go to gauge where at the point of interest x_0^μ the gauge field vanishes $A^a_\nu(x_0) = 0$.
- You will need to use the Bianchi identity

$$\epsilon^{\mu\nu\alpha\beta}(D_\nu F_{\alpha\beta})^a = 0$$

4 SUSY Commutators

Check that the commutator of two SUSY transformations close in super Yang-Mills theory:

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2}) X^a = -i(\epsilon_1 \sigma^\mu \epsilon_2^\dagger - \epsilon_2 \sigma^\mu \epsilon_1^\dagger) D_\mu X^a$$

for $X^a = F^a_{\mu\nu}, \lambda^a, \lambda^{\dagger a}, D^a$.

You need the identities:

$$\begin{aligned}
\xi \sigma^\mu \bar{\sigma}^\nu \chi &= \chi \sigma^\nu \bar{\sigma}^\mu \xi = (\chi^\dagger \bar{\sigma}^\nu \sigma^\mu \xi^\dagger)^* = (\xi^\dagger \bar{\sigma}^\mu \sigma^\nu \chi^\dagger)^* \\
\bar{\sigma}^\mu \sigma^\nu \bar{\sigma}^\rho &= -\eta^{\mu\rho}\bar{\sigma}^\nu + \eta^{\nu\rho}\bar{\sigma}^\mu + \eta^{\mu\nu}\bar{\sigma}^\rho + i\epsilon^{\mu\nu\rho\kappa}\bar{\sigma}_\kappa \\
\sigma^\mu_{\alpha\dot{\alpha}} \bar{\sigma}^{\dot{\beta}\beta}_\mu &= 2\delta^\beta_\alpha \delta^{\dot{\beta}}_{\dot{\alpha}}
\end{aligned}$$

5 SUSY invariance of Gauge Theory

Verify that the SUSY transformation of the gauge field A^a_μ in

$$\mathcal{L}_{\psi \text{ gauge int.}} = -g\psi^\dagger \bar{\sigma}^\mu A^a_\mu T^a \psi$$

cancels in

$$\mathcal{L}_{\text{Yukawa}} = -\sqrt{2} g[(\phi^* T^a \psi)\lambda^a + \lambda^{\dagger a}(\psi^\dagger T^a \phi)]$$

You will need to use the generalized Pauli identity (A.27).