**PHY 513: HW 1 (due Tue 9/15/09)**

# 1 Two Particle Decay

In atomic, nuclear, and particle physics, a common situation is when an unstable particle decays into two others. Consider a process $1 \to 2 + 3$ involving particles with four-momenta $p_1, p_2, p_3$ and masses $m_1, m_2, m_3$. Use the conservation of four-momentum and the relativistic invariance of the scalar product to determine the energy of particle 2 in the rest frame of particle 1 as function of the masses.

# 2 Two Particle Scattering

A collision process: a particle of mass $m_2$ is at rest in the laboratory. It is struck by a particle of mass $m_1$, momentum $p_{\text{lab}}$ and total energy $E_{\text{lab}}$. In the collision, the two initial particles are transformed into two others of mass $m_3$ and $m_4$. The direction of motion of these final particles are $\theta_3$ and $\theta_4$ with respect to particle 1. In the center of mass system (the frame where the total momentum vanishes) the initial momenta are $\vec{p'}$ (and $-\vec{p'}$), the final momenta are $\vec{q'}$ (and $-\vec{q'}$), and particle 3 moves at an angle $\theta'$ with respect to particle 1.

a) Draw the kinematic configuration in the laboratory frame and the cms.

b) Use invariant scalar products to show that the total energy $E_{\text{cms}}$ in the cms is given through

$$E_{\text{cms}}^2 = m_1^2 + m_2^2 + 2m_2 E_{\text{lab}} \tag{1}$$

c) Show that the 3-momentum of the beam particle in the cms is given by

$$p' = \frac{m_2 p_{\text{lab}}}{E_{\text{cms}}} \tag{2}$$

where $p' = |\vec{p'}|$.

A simple way to solve this problem is to consider the tensor $T_{\mu\nu} \equiv \epsilon_{\mu\nu\rho\sigma}(p_1 + p_2)^\sigma (p_1 - p_2)^\rho$ in the two frames. This way of solving the problem is also an instructive use of tensors.

d) Define the four-momentum transfer $q = p_1 - p_3$. Specializing to the case of elastic scattering ($m_1 = m_3, m_2 = m_4$) show that

$$t \equiv q^2 = -2m_4(E_4 - m_4) \tag{3}$$

where $E_4$ is the final energy in the lab frame, of the particle that was initially at rest.

e) Show that for elastic scattering

$$s \equiv (p_1 + p_2)^2 = E^2_{cms} \ , \quad t = -4p'^2 \sin^2(\frac{\theta'}{2}) \tag{4}$$

where $p'$ is the momentum and $\theta'$ the scattering angle in the cms.

f) Use these results to explain why, to explore high-energy processes, it is much more efficient to collide beams in their cms than exposing a stationary target to a high energy beam. What features of the beam or target might lead one to choose nevertheless to perform lab experiments?

# 3 Linear Terms in the Lagrangean

Consider the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - a\phi - \frac{b}{2}\phi^2 - \frac{\alpha}{3!}\phi^3 - \frac{\beta}{4!}\phi^4 \tag{5}$$

Show that, by means of a transformation

$$\phi \to \phi' = \phi + c \tag{6}$$

for some constant $c$, the linear term may be removed, *i.e.* $\mathcal{L}$ may be brought to the form

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi'\partial_\mu\phi' - \frac{m^2}{2}\phi'^2 - \frac{g}{3!}\phi'^3 - \frac{\lambda}{4!}\phi'^4 + \text{constant} \tag{7}$$

What is the relation between the parameters $\{m^2, g, \lambda\}$ and $\{a, b, \alpha, \beta\}$?