

The Leontief Input-Output Model

Text Reference: Section 2.6, p. 155

The purpose of this set of exercises is to provide three more examples of the Leontief Input-Output Model in action. The basic assumptions of the model and the calculations involved are reviewed first. Refer to Section 2.6 of your text for more complete information.

Recall that the input-output model requires that the economy in question be divided into sectors. Each sector produces goods or services except for the **open sector**, which only consumes goods and services. A production vector x lists the output of each sector. A final demand vector (or bill of final demands) d lists the values of the goods and services demanded from the productive sectors by the open sector. As the sectors strive to produce enough goods to meet the final demand vector, they make intermediate demands for the products of each sector. These intermediate demands are described by the **consumption matrix**. This matrix is constructed as follows.

The description of the economy begins with a collection of data called an **input-output table** (or an **exchange table**) for an economy. This table lists the value of the goods produced by each sector and how much of that output is used by each sector. For example, the following table is derived from the table Leontief created for the American economy in 1947. (See References 1 or 2 for the complete table.) For purposes of this example the data from the 42 sectors has been collected into just 3: agriculture, manufacturing, and services. Of course, the open sector is also present.

| | Agriculture | Manufacturing | Services | Open Sector |
|--------------------|-------------|---------------|----------|-------------|
| Agriculture | 34.69 | 4.92 | 5.62 | 39.24 |
| Manufacturing | 5.28 | 61.82 | 22.99 | 60.02 |
| Services | 10.45 | 25.95 | 42.03 | 130.65 |
| Total Gross Output | 84.56 | 163.43 | 219.03 | |

Table 1: Exchange of Goods and Services in the U.S. for 1947 (in billions of 1947 dollars)

Reading the table is straightforward; for example, in 1947 the agriculture sector spent 84.56 billion dollars for the inputs it needed. These inputs were divided among the sectors as follows: 34.69 billion dollars of agricultural output was consumed by the agriculture sector itself, 5.28 billion dollars of manufacturing output was consumed by the agriculture sector, etc.

To create the consumption matrix from the table, divide each column of the 3×3 table by the Total Gross Output for that sector. The result is Table 2, which appears on the following page.

The matrix with entries taken from this table is the consumption matrix C for the economy.

$$C = \begin{bmatrix} .4102 & .0301 & .0257 \\ .0624 & .3783 & .1050 \\ .1236 & .1588 & .1919 \end{bmatrix}$$

| | Agriculture | Manufacturing | Services |
|---------------|-------------|---------------|----------|
| Agriculture | .4102 | .0301 | .0257 |
| Manufacturing | .0624 | .3783 | .1050 |
| Services | .1236 | .1588 | .1919 |

Table 2: Inputs Consumed Per Unit of Sector Output

For the 1947 economy, the final demand vector \mathbf{d} is the column of the table associated with the open sector:

$$\mathbf{d} = \begin{bmatrix} 39.24 \\ 60.02 \\ 130.65 \end{bmatrix}$$

The equilibrium levels of production for each sector may now be calculated. These equilibrium levels are the production levels which will just meet the intermediate demands of the sectors of the economy plus the final demands of each sector. If \mathbf{x} is the desired production vector, \mathbf{x} must satisfy

$$\mathbf{x} = C\mathbf{x} + \mathbf{d}$$

This equation may be solved for \mathbf{x} to find that

$$\mathbf{x} = (I - C)^{-1}\mathbf{d}$$

where I is the identity matrix.

In the example,

$$(I - C)^{-1} = \begin{bmatrix} 1.7203 & .1006 & .0678 \\ .2245 & 1.6768 & .2250 \\ .3073 & .3449 & 1.2921 \end{bmatrix}$$

and thus

$$\mathbf{x} = (I - C)^{-1}\mathbf{d} = \begin{bmatrix} 82.40 \\ 138.85 \\ 201.57 \end{bmatrix}$$

Question:

1. Suppose the bill of final demands is changed to

$$\mathbf{d} = \begin{bmatrix} 40.24 \\ 60.02 \\ 130.65 \end{bmatrix}$$

What is the new equilibrium production vector? Find the difference between this new vector and the old equilibrium vector? How much extra production must each sector provide?

Notice that in the above exercise the only difference in the old and new demand vectors is the addition of one unit of demand to the agricultural sector. Also notice that the difference in the old and new production vectors is just the first column of the matrix $(I - C)^{-1}$. This is a valuable interpretation of the entries of $(I - C)^{-1}$:

Observation: The (i, j) entry in the matrix $(I - C)^{-1}$ is the amount by which sector i must change its production level to satisfy an increase of 1 unit in the final demand from sector j .

Question:

2. How much would the service production level need to increase if agricultural demand increased by 1 unit? How much would the manufacturing production level need to increase in this situation?

Now consider a less abrupt consolidation of the 1947 economic data: the economy is now divided into 25 sectors. These sectors are:

- | | |
|--|--------------------------------------|
| 1. Agriculture and Fisheries | 13. Motor Vehicles |
| 2. Food and Kindred Products | 14. Other Transportation Equipment |
| 3. Textiles and Apparel | 15. Miscellaneous Manufacturing |
| 4. Lumber, Wood, and Furniture | 16. Coal, Gas, and Electric Power |
| 5. Paper, Printing, and Publishing | 17. Transportation Services |
| 6. Chemicals, Petroleum Products, Rubber | 18. Trade |
| 7. Leather and Leather Products | 19. Communications |
| 8. Stone, Clay, and Glass Products | 20. Finance, Insurance, Real Estate |
| 9. Primary Metals | 21. Business Services |
| 10. Fabricated Metal Products | 22. Personal and Repair Services |
| 11. Machinery (non-electric) | 23. Miscellaneous Services |
| 12. Electrical Machinery | 24. New Construction and Maintenance |
| | 25. Undistributed |

The consumption matrix C and final demand vector d for this model accompany this exercise set.

Questions:

3. Find the equilibrium production vector for the consumption matrix $C1$ and final demand vector $d1$.
4. If the final demand for motor vehicles increases by one billion dollars, how much will the production of fabricated metal products have to increase to compensate?

The input-output model is still used to model economies throughout the world, as well as the global economy itself. The most up-to-date available input-output table for the American economy is the 1998 table. A copy of the entire table (which is officially called an I-O Use table) is an appendix to the document "Annual Input-Output Accounts of the U.S. Economy, 1998," which can be viewed or downloaded from the Bureau of Economic Analysis website at <http://www.bea.gov/bea/an2.htm>. This document provides a good overview of different types of tables, as well as some applications of the 1998 table. The table for 1998 divides the economy into nearly 500 sectors, which are then consolidated into over 90 sectors. These categories have been carefully compressed by the author into the 33 listed below. The resulting table is given in Table 3; the consumption matrix $C2$ and final demand vector $d2$ which accompany this set are derived from that data.

- | | |
|--|--|
| 1. Agriculture, Forestry, Fisheries | 18. Non-Electrical Machinery |
| 2. Mining, Petroleum and Natural Gas | 19. Electrical Machinery |
| 3. Construction | 20. Motor Vehicles |
| 4. Food and Kindred Products | 21. Other Transportation Equipment |
| 5. Tobacco Products | 22. Miscellaneous Manufacturing |
| 6. Textile Products | 23. Transportation Services |
| 7. Lumber and Wood Products | 24. Communication Services |
| 8. Furniture and Fixtures | 25. Utilities |
| 9. Paper Products | 26. Trade |
| 10. Printing and Publishing | 27. Finance, Insurance, Real Estate |
| 11. Chemicals, Plastics, Drugs, Paints | 28. Personal and Professional Services – non-medical |
| 12. Petroleum Refining | 29. Miscellaneous Services |
| 13. Rubber and Miscellaneous Plastics | 30. Health Services |
| 14. Footwear and Leather Products | 31. Educational and Social Services |
| 15. Glass, Stone, and Clay Products | 32. Government |
| 16. Metals Manufacturing | 33. Miscellaneous – Imports, Scrap, etc. |
| 17. Fabricated Metal Products | |

Questions:

5. Find the equilibrium production vector for the consumption matrix $C2$ and final demand vector $d2$.
6. Which three sectors are most affected by an increase in one million dollars of final demand for motor vehicles?

7. How many dollars worth of new production is produced **in the entire economy** by an increase in one million dollars of final demand for motor vehicles? Note: this amount is referred to in economic literature as a “backward linkage”
8. If demands are increased by 10% in all sectors, by what percentage must total production increase to maintain equilibrium?
9. What would it mean for the (i, j) entry in $(I - C)^{-1}$ to be zero?

References:

1. Leontief, Wassily W. “Input-Output Economics.” *Scientific American*, October 1951, pp.15-21.
This article explains the author’s input-output model, and includes the complete 42-sector exchange table for 1947.
2. Leontief, Wassily W. *Input-Output Economics*. New York: Oxford University Press, 1966.
This book contains the full 42-sector exchange table for 1947, as well as an 81-sector table for 1958.
3. Leontief, Wassily W. “The Structure of the U.S. Economy.” *Scientific American*, April 1965, pp. 25-35.
This article contains the 81-sector table mentioned above.

| Sect# | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. | 11. |
|-------|--------|--------|---------|--------|-------|--------|--------|-------|--------|--------|--------|
| 1. | 68681 | 78 | 5860 | 124826 | 3182 | 4088 | 8698 | 56 | 92 | 99 | 1207 |
| 2. | 367 | 31477 | 7368 | 188 | 24 | 53 | 3 | 13 | 668 | 0 | 9388 |
| 3. | 3368 | 4693 | 895 | 2498 | 126 | 941 | 389 | 306 | 1786 | 1177 | 3604 |
| 4. | 21245 | 0 | 0 | 78643 | 0 | 18 | 0 | 95 | 698 | 0 | 1971 |
| 5. | 0 | 0 | 0 | 0 | 3508 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6. | 1529 | 94 | 2693 | 98 | 8 | 54013 | 207 | 4872 | 964 | 138 | 50 |
| 7. | 529 | 121 | 54850 | 123 | 2 | 41 | 35867 | 4845 | 6741 | 30 | 46 |
| 8. | 0 | 0 | 2219 | 0 | 0 | 0 | 271 | 290 | 0 | 0 | 0 |
| 9. | 1836 | 35 | 3949 | 16615 | 677 | 333 | 340 | 1027 | 34698 | 23297 | 5315 |
| 10. | 57 | 3 | 200 | 957 | 233 | 41 | 19 | 14 | 192 | 16133 | 681 |
| 11. | 14107 | 2360 | 10344 | 4631 | 387 | 15791 | 1269 | 651 | 9864 | 3142 | 85977 |
| 12. | 4075 | 1810 | 13316 | 893 | 44 | 248 | 376 | 174 | 689 | 206 | 2839 |
| 13. | 1484 | 640 | 18237 | 10123 | 290 | 1161 | 942 | 2539 | 4278 | 1620 | 10249 |
| 14. | 53 | 1 | 0 | 0 | 0 | 953 | 9 | 57 | 0 | 3 | 0 |
| 15. | 164 | 498 | 51373 | 3565 | 4 | 334 | 1052 | 331 | 79 | 2 | 1055 |
| 16. | 63 | 2481 | 15247 | 0 | 0 | 2 | 112 | 2990 | 545 | 78 | 265 |
| 17. | 1048 | 1557 | 68795 | 11387 | 50 | 6 | 2572 | 4039 | 656 | 58 | 3138 |
| 18. | 1300 | 4254 | 10402 | 560 | 21 | 641 | 441 | 311 | 1093 | 845 | 1188 |
| 19. | 1284 | 452 | 40765 | 297 | 47 | 10 | 641 | 27 | 71 | 130 | 290 |
| 20. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21. | 389 | 175 | 1603 | 244 | 2 | 23 | 465 | 25 | 32 | 86 | 34 |
| 22. | 237 | 35 | 5438 | 50 | 10 | 638 | 85 | 31 | 40 | 832 | 283 |
| 23. | 7981 | 3127 | 18509 | 14828 | 438 | 3268 | 4479 | 1369 | 7966 | 4407 | 12129 |
| 24. | 829 | 345 | 4082 | 771 | 79 | 344 | 206 | 221 | 409 | 956 | 1196 |
| 25. | 3816 | 9181 | 2256 | 5807 | 103 | 2721 | 1318 | 606 | 4506 | 1434 | 10610 |
| 26. | 13948 | 3498 | 81671 | 30923 | 920 | 7755 | 8037 | 4365 | 7986 | 6058 | 22446 |
| 27. | 20647 | 33253 | 16486 | 6523 | 618 | 2903 | 1671 | 1276 | 2050 | 8728 | 6749 |
| 28. | 4722 | 4836 | 97144 | 10982 | 1146 | 8450 | 2517 | 2165 | 4460 | 9334 | 20655 |
| 29. | 1635 | 886 | 6517 | 16499 | 4075 | 2371 | 1181 | 1049 | 2291 | 4671 | 13466 |
| 30. | 2459 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 31. | 182 | 126 | 50 | 308 | 30 | 231 | 194 | 74 | 100 | 186 | 433 |
| 32. | 164 | 30 | 1069 | 743 | 85 | 198 | 97 | 123 | 336 | 832 | 472 |
| 33. | 66 | 1872 | 6 | 1033 | 31 | 175 | 15 | 62 | 980 | 480 | 6900 |
| TGO | 283291 | 147738 | 1006179 | 493690 | 46203 | 163259 | 118243 | 65889 | 161487 | 212238 | 385970 |

Table 3: Exchange of Goods and Services in the U.S. for 1992 (in millions of 1992 dollars)

| Sect# | 12. | 13. | 14. | 15. | 16. | 17. | 18. | 19. | 20. | 21. | 22. |
|-------|--------|--------|------|-------|--------|--------|--------|--------|--------|--------|--------|
| 1. | 37 | 1090 | 2 | 65 | 120 | 99 | 122 | 379 | 49 | 192 | 221 |
| 2. | 59472 | 32 | 2 | 4438 | 6687 | 113 | 55 | 119 | 20 | 370 | 79 |
| 3. | 1457 | 1258 | 63 | 713 | 1435 | 1656 | 1334 | 5276 | 869 | 2400 | 1468 |
| 4. | 172 | 18 | 851 | 25 | 0 | 0 | 0 | 0 | 0 | 4 | 98 |
| 5. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6. | 0 | 3042 | 484 | 201 | 16 | 19 | 289 | 9 | 7526 | 666 | 1967 |
| 7. | 5 | 286 | 0 | 627 | 586 | 157 | 397 | 124 | 7 | 1138 | 822 |
| 8. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 748 | 6857 | 217 | 88 |
| 9. | 324 | 3142 | 48 | 1732 | 198 | 1434 | 725 | 3330 | 87 | 748 | 3067 |
| 10. | 9 | 51 | 1 | 11 | 101 | 96 | 35 | 119 | 31 | 82 | 113 |
| 11. | 3005 | 35163 | 374 | 3396 | 3295 | 3459 | 674 | 5849 | 2754 | 2750 | 3908 |
| 12. | 15345 | 256 | 21 | 460 | 714 | 364 | 327 | 534 | 166 | 361 | 288 |
| 13. | 668 | 9098 | 197 | 1032 | 1028 | 3355 | 4287 | 12528 | 14478 | 6724 | 5371 |
| 14. | 1 | 0 | 2370 | 0 | 0 | 7 | 0 | 0 | 3 | 2 | 51 |
| 15. | 751 | 1079 | 5 | 10872 | 2509 | 779 | 713 | 2997 | 2555 | 1112 | 1046 |
| 16. | 12 | 802 | 0 | 735 | 46771 | 57336 | 23615 | 22167 | 497 | 29688 | 6663 |
| 17. | 299 | 1582 | 85 | 634 | 3493 | 21819 | 15286 | 19384 | 26169 | 18577 | 7202 |
| 18. | 132 | 2000 | 10 | 565 | 4830 | 6176 | 21673 | 4790 | 1494 | 12658 | 1865 |
| 19. | 16 | 197 | 0 | 148 | 1288 | 1953 | 5133 | 127957 | 15180 | 13315 | 18918 |
| 20. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 75 | 2299 | 0 |
| 21. | 24 | 18 | 0 | 73 | 31 | 59 | 104 | 40 | 64858 | 37315 | 2255 |
| 22. | 30 | 104 | 1 | 75 | 147 | 191 | 256 | 3217 | 1875 | 8595 | 8566 |
| 23. | 6906 | 6453 | 205 | 6056 | 9053 | 5217 | 3634 | 6243 | 5597 | 5513 | 2780 |
| 24. | 216 | 473 | 24 | 276 | 376 | 695 | 808 | 2213 | 262 | 627 | 1060 |
| 25. | 3937 | 3246 | 139 | 3267 | 7972 | 3871 | 2193 | 4901 | 885 | 2951 | 1729 |
| 26. | 5523 | 6802 | 410 | 3925 | 15350 | 13594 | 13512 | 39140 | 17021 | 15412 | 11494 |
| 27. | 3779 | 3243 | 128 | 1394 | 2613 | 4102 | 3475 | 10514 | 2568 | 4537 | 4306 |
| 28. | 3774 | 5154 | 179 | 2883 | 6319 | 8512 | 6481 | 20459 | 4810 | 9741 | 9129 |
| 29. | 1562 | 2237 | 218 | 1237 | 2443 | 3273 | 3060 | 9695 | 17918 | 5663 | 5427 |
| 30. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 31. | 109 | 553 | 3 | 48 | 111 | 325 | 130 | 632 | 348 | 341 | 522 |
| 32. | 75 | 146 | 23 | 71 | 224 | 266 | 214 | 506 | 285 | 471 | 308 |
| 33. | 549 | 406 | 5 | 340 | 7575 | 481 | 671 | 9146 | 790 | 820 | 831 |
| TGO | 133536 | 160122 | 8266 | 90486 | 189931 | 250513 | 222904 | 482730 | 229740 | 308128 | 230249 |

Table 3: Exchange of Goods and Services in the U.S. for 1992 (in millions of 1992 dollars)

| Sect# | 23. | 24. | 25. | 26. | 27. | 28. | 29. | 30. | 31. | 32. | 33. | Open |
|-------|--------|--------|--------|---------|---------|---------|--------|--------|--------|--------|--------|---------|
| 1. | 15 | 110 | 28 | 1817 | 11476 | 1340 | 7879 | 2170 | 923 | 567 | 0 | 34940 |
| 2. | 133 | 0 | 52220 | 31 | 6 | 11 | 8 | 6 | 6 | 3062 | 0 | -39241 |
| 3. | 6985 | 13603 | 26781 | 12695 | 66515 | 6458 | 5746 | 4802 | 11780 | 25894 | 0 | 787208 |
| 4. | 337 | 0 | 0 | 847 | 0 | 206 | 77652 | 5322 | 4234 | 631 | 0 | 300587 |
| 5. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 42698 |
| 6. | 343 | 159 | 3 | 1209 | 216 | 2001 | 1017 | 1911 | 273 | 356 | 0 | 74717 |
| 7. | 330 | 50 | 217 | 3615 | 318 | 478 | 219 | 252 | 568 | 0 | 0 | 5783 |
| 8. | 0 | 0 | 0 | 1 | 2 | 8 | 454 | 3 | 1 | 741 | 0 | 53226 |
| 9. | 447 | 513 | 128 | 14851 | 2780 | 7607 | 3033 | 3532 | 2083 | 526 | 0 | 19585 |
| 10. | 1307 | 1484 | 118 | 9165 | 6855 | 10074 | 3911 | 3836 | 13527 | 1568 | 0 | 54556 |
| 11. | 589 | 234 | 814 | 1931 | 876 | 4225 | 2152 | 32674 | 834 | 2835 | 0 | 130924 |
| 12. | 14900 | 293 | 1881 | 5598 | 1042 | 2491 | 2387 | 1050 | 988 | 3187 | 0 | 62301 |
| 13. | 3414 | 796 | 423 | 6661 | 1023 | 3516 | 7689 | 8724 | 1056 | 186 | 0 | 16991 |
| 14. | 5 | 2 | 2 | 263 | 24 | 486 | 105 | 31 | 117 | 92 | 0 | 4022 |
| 15. | 125 | 41 | 87 | 441 | 643 | 1210 | 2347 | 2059 | 322 | 883 | 0 | -360 |
| 16. | 508 | 26 | 125 | 347 | 31 | 276 | 32 | 11 | 39 | 73 | 0 | -23318 |
| 17. | 2573 | 1445 | 488 | 3112 | 986 | 1878 | 7329 | 1009 | 528 | 560 | 0 | 26431 |
| 18. | 1962 | 767 | 500 | 3149 | 367 | 3930 | 1170 | 182 | 142 | 1653 | 0 | 131867 |
| 19. | 1569 | 14513 | 691 | 10873 | 1919 | 42443 | 5393 | 3681 | 1077 | 1810 | 0 | 171722 |
| 20. | 0 | 0 | 0 | 0 | 0 | 0 | 103 | 0 | 0 | 0 | 0 | 228200 |
| 21. | 13449 | 66 | 1347 | 3944 | 256 | 1639 | 25749 | 503 | 434 | 2340 | 0 | 151044 |
| 22. | 347 | 342 | 737 | 2000 | 1989 | 6917 | 1904 | 15868 | 2069 | 153 | 0 | 160536 |
| 23. | 93427 | 2376 | 8489 | 15621 | 13947 | 15053 | 9724 | 7356 | 3014 | 10731 | 0 | 214819 |
| 24. | 6241 | 53623 | 924 | 26371 | 22228 | 23399 | 5965 | 8370 | 3495 | 1066 | 0 | 181216 |
| 25. | 5702 | 1999 | 28155 | 26223 | 16452 | 12020 | 15368 | 12574 | 4425 | 11078 | 0 | 190215 |
| 26. | 9516 | 3551 | 2012 | 32685 | 4925 | 17999 | 29519 | 15050 | 5467 | 2647 | 0 | 1103110 |
| 27. | 19668 | 13119 | 7498 | 108418 | 445681 | 89791 | 52151 | 61523 | 40287 | 7945 | 0 | 1520718 |
| 28. | 35168 | 33860 | 12864 | 133305 | 147224 | 225101 | 46800 | 74654 | 37098 | 11787 | 0 | 541132 |
| 29. | 22658 | 35852 | 1497 | 83370 | 42024 | 29184 | 63808 | 15237 | 13932 | 1640 | 0 | 626758 |
| 30. | 83 | 0 | 0 | 0 | 0 | 4 | 56 | 15181 | 0 | 0 | 0 | 746044 |
| 31. | 910 | 552 | 1055 | 2551 | 2117 | 4855 | 1057 | 2259 | 1751 | 159 | 0 | 300450 |
| 32. | 1195 | 1406 | 701 | 11225 | 28194 | 10351 | 3177 | 7628 | 3355 | 3035 | 0 | 47974 |
| 33. | 13528 | 8276 | 141 | 7725 | 8555 | 4429 | 887 | 48 | 30 | 1144 | 0 | 914665 |
| TGO | 534111 | 393893 | 322303 | 1552311 | 2547553 | 1559162 | 832112 | 764606 | 323751 | 221648 | 990060 | |

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