Dynamical Developments: a conference in complex dynamics and Teichmüller theory
(aka JHH70)

Abstracts of Talks

All of these talks will take place in the building Research II

Misha Lyubich, Stony Brook
Title: On critical points and wandering domains for dissipative complex Henon maps
Abstract: The absence of "critical points" for invertible maps can be understood in terms of a "dominated splitting", i.e., existence of an invariant cone field. We will show that a (moderately dissipative) complex Hènon map without critical and parabolic points on the Julia set is hyperbolic. (The main step is to prove absence of wandering domains.) We will also discuss how "critical points" are associated to periodic points of different types (attracting, parabolic, Cremer, and Siegel).

Based on a joint work (in progress) with Han Peters.

Kathryn Lindsey, U Chicago
Title: Convex subsets of $\mathbb{R}^3$ and measures of maximal entropy
Abstract: I will show how any connected "shape" in the plane can be embedded isometrically as part of the boundary of a convex subset of $\mathbb{R}^3$. In particular, if the "shape" is a connected Julia set, this yields a 3-dimensional object associated to a complex polynomial dynamical system. This is joint work with Laura DeMarco.

Carsten Petersen, Roskilde
Title: On quasi-conformal (in-)compatibility of satellite copies of the Mandelbrot set.
Abstract: In the groundbreaking paper *On the dynamics of polynomial-like mappings*, Douady and Hubbard introduced the notion of polynomial-like maps and they proved the so-called straightening theorem for polynomial-like mappings. To illustrate the power of their theory they used it to identify via straigthening, two types of homeomorphic copies M′, primitive and satellite of the Mandelbrot set M inside the Mandelbrot set.

They conjectured that the primitive copies, which are characterized by having a cusp and a root for which the parabolic multiplier is equal to 1, are quasi-conformally homeomorphic to M. This is now a theorem due to Lyubich. The satellite copies M′ however, which are characterized by having a smooth round main component and a root for which the parabolic multiplier is a q-th root of unity for some $q > 1$, Douady and Hubbard conjectured and Lyubich proved to be locally q.c. homeomorphic to M away from the root. The natural question is then, if this is an artefact of the proof or if the q.c.-dilatation of the Douady-Hubbard homeomorphism is unbounded near the root. This gives rise to two questions: is the induced Douady-Hubbard homeomorphism onto half of the logistic Mandelbrot set q.c.? Are the induced homeomorphisms between different satellite copies q.c.?

In this talk I will present a proof that two satellite copies M′ and M″ are not q.c. homeomorphic, if the root multipliers are q- and q'- roots of unity with $q \neq q'$ and in particular no satellite copy is qc-homeomorphic to half of the logistic Mandelbrot set. Joint work with Luna Lomonaco, Univ of São Paolo.

Guizhen Cui, Chinese Academy of Sciences
Title: Twist deformation and parabolic implosion surgery for rational maps
Abstract: Consider the Riemann surfaces of a geometrically finite rational map. A repeated Dehn twist will produce a sequence of rational maps in its moduli space. We will prove that this sequence

is convergent under certain condition. The proof involves iteration on Teichmüller spaces and a surgery perturbation which realize a parabolic implosion.

Xavier Buff, Toulouse
Title: Wandering domains for polynomial endomorphisms of $\mathbb{C}^2$
Abstract: We prove the existence of polynomial endomorphisms of $\mathbb{C}^2$ having a wandering Fatou component. The proof relies on techniques of parabolic implosion and is based on an original idea of Lyubich. Joint work with Astorg, Dujardin, Peters and Raissy.

Giulio Tiozzo, Yale University
Title: Core entropy and the infinite clique polynomial
Abstract: The core entropy of quadratic polynomials, recently introduced by W. Thurston, is a dynamical invariant which can be defined purely in combinatorial terms, and provides a useful tool to study parameter spaces of polynomials.

A classical tool to compute the entropy of a dynamical system is the clique polynomial (recently used by McMullen to study the entropy of pseudo-Anosov maps). We will develop an infinite version of the clique polynomial for infinite graphs, and use it to study the symbolic dynamics of Hubbard trees.

Using these methods we will prove that the core entropy of quadratic polynomials varies continuously as a function of the external angle, answering a question of Thurston and Hubbard.

Roland Roeder, IUPUI
Title: Rational maps of $\mathbb{CP}^2$ with equal dynamical degrees, no invariant foliation, and two different measures of maximal entropy.
Abstract: The ergodic properties of a rational map $f : \mathbb{CP}^2 \to \mathbb{CP}^2$ are tied to its dynamical degrees $\lambda_1(f)$ and $\lambda_2(f)$. Maps with $\lambda_1(f) > \lambda_2(f)$ share many properties of the Hènon maps, having a measure of maximal entropy of saddle type. Maps with $\lambda_2(f) > \lambda_1(f)$ share many properties of holomorphic endomorphisms, having a measure of maximal entropy that is repelling. In both cases it is believed (and often proved) that f has a unique measure of maximal entropy.

Early examples of maps with $\lambda_1(f) = \lambda_2(f)$ were skew products, having an invariant fibration. Guedj asked whether this happens in general. We show that there is a simple way to produce rational maps of $\mathbb{CP}^2$ with equal dynamical degrees, no invariant foliation, and two measures of maximal entropy, one of saddle-type and one repelling. This is joint work with Jeff Diller, Scott Kaschner, and Rodrigo Perez.

Dylan Thurston, Indiana University
Title: Detecting rational maps using elastic graphs
Abstract: Given a topological branched covering $f : (S^2, P) \to (S^2, P)$ of the sphere by itself, with branch values contained in P, can f be realised as a rational map? William Thurston gave a positive criterion in 1982, based on annular obstructions: If f has a hyperbolic orbifold, it can be realised as a rational map iff there is not an invariant multi-curve satisfying certain conditions. This condition is hard to apply in practice, since it involves checking infinitely many multi-curves. We give a complete positive combinatorial condition, using the notion of *domination* of graphs, a stricter condition than being 1-Lipshitz. In more physical terms, f is realised as a rational map iff, for some (or equivalently any) rubber-band spine G for $S^2 \setminus P$, a sufficiently large inverse image $f^{-n}(G)$ of G is looser than G. I have written a program that finds such spines and maps between them, if they exist.

Portions of this work are joint with Kevin Pilgrim and Jeremy Khan.

Anton Zorich, Paris
Title: On the Density of Strebel Differentials: forty years after
Abstract: A. Douady and J. Hubbard have solved in 1975 the conjecture of Strebel on the density of Strebel differentials. Versions of this result with extra constraints on combinatorics of the horizontal foliation were elaborated later by many authors in various contexts.

We prove that square-tiled surfaces (correspondingly pillowcase covers) tiled with tiny squares sharing a fixed combinatorics of cylinder gluings are asymptotically equidistributed in the ambient stratum in the moduli space of Abelian (correspondingly quadratic) differentials. We prove similar equidistribution results for rational interval exchange transformation.

We compute explicitly the contribution of square-tiled surfaces (correspondingly pillowcase covers) having a single horizontal cylinder to the volume of the corresponding stratum. The resulting count is particularly simple and efficient in large genus asymptotics. We conjecture that this contribution is asymptotically of the order $1/d$ where d is the dimension of the stratum and prove that this conjecture is equivalent to the long-standing conjecture on asymptotics for the volumes of the moduli spaces of Abelian differentials. In certain particular cases the conjecture was recently proved by D. Chen, M. Moller, and D. Zagier.

This is joint work with V. Delecroix, E. Goujard, and P. Zograf.

Raluca Tanase, Stony Brook
Title: Stability and continuity of Julia sets in $\mathbb{C}^2$ *Abstract:* We discuss the dynamics of a complex Hénon map (which is a polynomial automorphism of $\mathbb{C}^2$) near a semi-parabolic parameter. In particular, we look at the parameter space of strongly dissipative Hénon maps which have a fixed point with one eigenvalue $(1+t)\lambda$, where λ is a root of unity and t is real and small. These maps have a semi-parabolic fixed point when t is 0, and we use techniques that we have developed for the semi-parabolic case to describe nearby perturbations. We show that for small nonzero t, the Hénon map is hyperbolic and has connected Julia set. We prove a two-dimensional analogue of radial convergence for polynomial Julia sets and show that the Hénon map is stable on J and J^+ when t is nonnegative. This is joint work with R. Radu.

Volodia Nekrashevych, Texas A & M
Title: Paper folding curves, mating, and an endomorphism of $\mathbb{CP}^2$
Abstract: We will discuss topological structure of the Julia set of a post-critically finite endomorphism of $\mathbb{CP}^2$. Relations with mating of non-autonomous polynomial iteration and an uncountable family of "dragon curves" defined using paper folding will be shown. We will also discuss an example of "rotating tuning" appearing naturally in connection with the Julia set of the endomorphism, and some algebraic aspects of its iterated monodromy group.

Alex Kontorovich, Rutgers
Title: The Unreasonable Effectiveness of Thin Groups
Abstract: Thin groups are certain arithmetic subgroups of Lie groups whose quotient manifolds have infinite volume. We will describe a number of problems in pseudorandom numbers, numerical integration, and homogeneous dynamics which, it turns out, are all the same problem when viewed through the lens of thin groups.

Harry Baik, Bonn
Title: Orderability, Laminarity, and Geometry of Groups
Abstract: Following Thurston's universal circle construction, we discuss the relation among orderability, laminarity, and geometry of (3-manifold) groups. We also discuss how the following groups are related: Möbius-like groups, Möbius groups, convergence groups, hyperbolic groups, surface groups, 3-manifold groups.

Ursula Hamenstädt, Bonn
Title: Periodic Teichmüller geodesics and $\mathrm{SL}(2,\mathbb{R})$-orbit closures
Abstract: Let Q be a stratum in the moduli space of abelian differentials of differentials with at least one simple zero on a surface of genus at least 3. Then $\mathrm{SL}(2,\mathbb{R})$-orbit closures of typical periodic orbits coincide with the entire stratum. We also give a simple proof of finiteness of $\mathrm{SL}(2,\mathbb{R})$-orbit closures of rank at least two in a non-hyperelliptic component of a stratum with a single zero.

Laurent Bartholdi, Göttingen
Title: TBA

Dima Dudko, Göttingen
Title: Self-similarity of the Mandelbrot set around Siegel parameters of periodic type *Abstract:* It is indicated by pictures that the Mandelbrot set is self-similar around Siegel parameters of periodic type. Thus we can expect a renormalization operator explaining this phenomenon. In the talk we will discuss a possible candidates. As a preliminary result we will show that certain centers of hyperbolic components "behave as they should" around Siegel parameters. Joint work with Mikhail Lyubich and Nikita Selinger.

Adam Epstein, Warwick
Title: TBA

John H. Hubbard, Cornell/Marseille
Title: Complex dynamics, then and now

John Milnor, Stony Brook
Title: Remarks on Piecewise Monotone Maps
Abstract: Although piecewise monotone maps of the interval have been studied for at least 40 years, there are still interesting problems (as Thurston showed us in his last year). My talk will be largely expository, with more questions than theorems.

Remus Radu, Stony Brook
Title: Semi-indifferent dynamics
Abstract: We consider complex Hénon maps that have a semi-indifferent fixed point with eigenvalues λ and μ, where $|\lambda| = 1$ and $|\mu| < 1$. When λ is a root of unity we have a good understanding of this family: for small Jacobian, the dynamics of the Julia set of the Hénon map fibers over the dynamics of a certain polynomial Julia set (joint work with R. Tanase). The situation when $\lambda = \exp(2\pi i\alpha)$ and α is irrational is more complex as it depends on the arithmetic properties of α. When α is the golden mean, we show that the Hénon map with small Jacobian has a Siegel disk whose boundary is homeomorphic to a circle; the proof is based on renormalization of commuting pairs (joint work with D. Gaydashev and M. Yampolsky). If α is not Brjuno we prove the existence of a "hedgehog" for holomorphic germs of $(\mathbb{C}^2, 0)$ using topological techniques (joint work with T. Firsova, M. Lyubich, and R. Tanase).

Arnaud Chéritat, Bordeaux
Title: Yet another sphere eversion
Abstract: Smale proved in the 1960's that one can turn the sphere inside out in Euclidean 3-space, by going through immersions. However, he did not give an explicit description of how to realize it. Since, many people gave different ways to do it. I will briefly present what I know of the history of explicit sphere eversions, then describe a possibly new way to do it.

Mitsuhiro Shishikura, Kyoto
Title: Rempe model and Denjoy odometer for hedgehogs of complex quadratic polynomials
Abstract: When a holomorphic map has an irrationally indifferent fixed point which is not linearizable, there is an invariant set called "hedgehog" defined by Perez Marco. It has been known that such a hedgehog has a similar feature as the Julia set of exponential maps. For exponential maps, Rempe defined a straight brush model for their Julia sets or escaping sets. In this talk, we propose a topological model inspired by Rempe's for the hedgehogs for quadratic polynomials with irrational rotation number of high type.