Homework #3
MEAM 502   Differential Equation Methods in Mechanics

1.   Using the Haar wavelet function or other available wavelet functions, recreate the following image (522 x 522 pixels image in Photoshop 4.0, you can download the image file HW3_1_98W from the homepage of this course. This file is prepared in MAC and stored as jpeg so that you can get both in PC and MAC as well as UNIX through internet.) by 1%, 3%, 5%, and 10%, and 25% non-zero data on the 128 x 128 pixels and 512 x 512 pixel after trimming the original to 512 x 512 pixels image by deleting 10 pixels in the both directions.

2. Solve the following boundary value problem

$$-\frac{d}{dx}\left(a(x)\frac{du}{dx}\right) + a_0(x)u = f(x) \quad \text{in} \quad (0,10)$$

$$u(0) = 1 \quad \text{and} \quad \left.a(x)\frac{du}{dx}\right|_{x=10} = 1$$
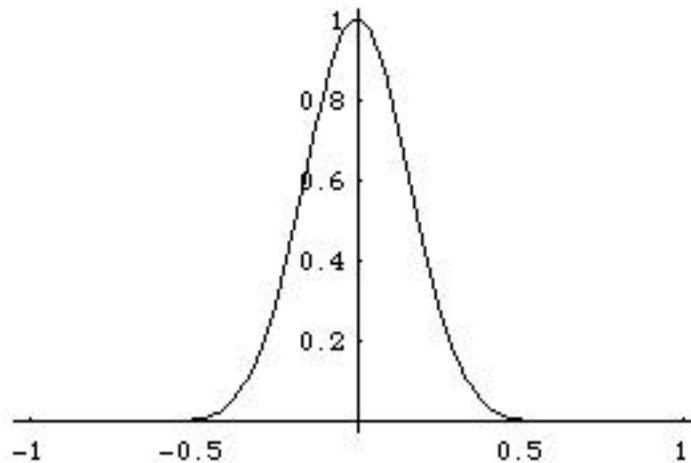
for

$$a(x) = 1 - 0.5*\cos\left(\frac{1}{2}x\right)$$

$$a_0(x) = 1 + 0.2*\sin\left(\frac{1}{5}x\right)$$

$$f(x) = \cos\left(\frac{1}{4}x\right)$$

by using

(1) the element free Galerkin method with the influence functions

$$w_j(x) = \exp\left(-20(x - x_j)^2\right)$$

while an arbitrary quadratic polynomial can be reproduced. The total number of the nodal points placed on the domain (0,10) can be chosen as you wish. But I expect the total number of nodal points is around 21.

(2) the Galerkin method together with the wavelet like basis functions by approximating the trial and test functions :

$$u(x) \approx c_0 + c_1\left(1 - \frac{x}{10}\right) + c_2\left(\frac{x}{10}\right) + \sum_{j=0}^{j_{max}}\sum_{k=0}^{2^j-1} c_{jk}2^{(j+1)/2}\left(2\frac{x}{10} + k\right)^j$$

where

$$\phi_1(x) = 1 - \frac{x}{10} \quad , \quad \phi_2(x) = \frac{x}{10}$$

$$\phi(x) = \begin{cases} x & , & x \in [0,0.5) \\ 1 - x & , & x \in [0.5,1) \\ 0 & , & \text{otherwise} \end{cases}$$

and c_0, c_1, c_2, and c_{jk} are constants to be determined or arbitrary.