Homework 2 Posted on Feb 18, 1999

MEAM 502 Differential Equation Methods in Mechanics

1. Solve the following matrix equation

$$\mathbf{Ax} = \mathbf{b}$$

by

(1) Steepest Descent Method and/or Preconditioned SD Method

(2) Conjugate Gradient Method and/or Pre-conditioned CG Method

(3) Gauss Elimination Method such as LU decomposition or using MATLAB

Where

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 3 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 4 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 4 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 2 \end{bmatrix}, \quad \mathbf{b} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -0.1 \\ -0.2 \\ -0.2 \\ -0.2 \\ -0.2 \\ -0.1 \end{Bmatrix}$$

2. Solve the following nonlinear differential equation

$$-\frac{d}{dx}\left\{\left(\frac{1}{\sqrt{1+(du/dx)^2}}\frac{du}{dx}\right)\right\}+u^4=\sin(\pi x) \quad in \quad (0,1)$$

with the boundary condition $u(0)=u(1)=0$, by using the Newton method or modified Newton method, after it would be approximated by FDM, FEM, or weighted residual methods.

3. Find the best approximation $h_0 \in H$ of a given function $f(x)=(1-x^2)\exp(x)$ in the Sobolev space $V=H^1(-1,1)=\{v \mid v, \partial v \in L^2(-1,1)\}$ with the inner product

$$(u,v)=\int_{-1}^{1}\left\{u(x)v(x)+\left(1+\frac{1}{2}\sin(\pi x)\right)\partial u(x)\partial v(x)\right\}dx$$

where $H=\{v \in V \mid v(x)=c_0+c_1x+c_2x^2+c_3x^3+c_4x^4 \text{ , } c_i \in R \text{ , } i=0,1,2,3,4\}$.