Homework #2
MEAM 502 Differential Equation Methods in Mechanical Engineering

1998 Winter

Consider a functional F defined by

$$F(w) = \frac{1}{2}\int_{\Omega} D\left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\nu)\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right)\right\} dydx$$

$$-Pw\left(x_0, y_0\right) - \frac{1}{2}\int_{\Omega} N_x\left(\frac{\partial w}{\partial x}\right)^2 dydx$$

where D = $Eh^3/(12(1-\nu^2))$ is the bending rigidity, E is Young's modulus, h is the thickness of the plate, ν is Poisson's ratio, P is an applied point force, N_X is applied compressive traction, and (x_0,y_0) is a specified point in a quadrilateral domain Ω that is the one shown in the following figure.

Solve the minimization problem

$$\min_{v \in V_0} F(v)$$

where $V_0 = \left\{v \in H^2(\Omega) \middle| \; v = 0 \quad \text{on} \quad \partial\Omega\right\}$, $H^2(\Omega)$ is the linear space of all functions whose upto second derivatives are square integrable on W in the generalized sense, using the Ritz-Galerkin method.

Verify your results by any finite element methods available in CAEN, for example, by using MSC/NASTRAN or ABAQUS, by setting up appropriate constants and geometrical dimensions.

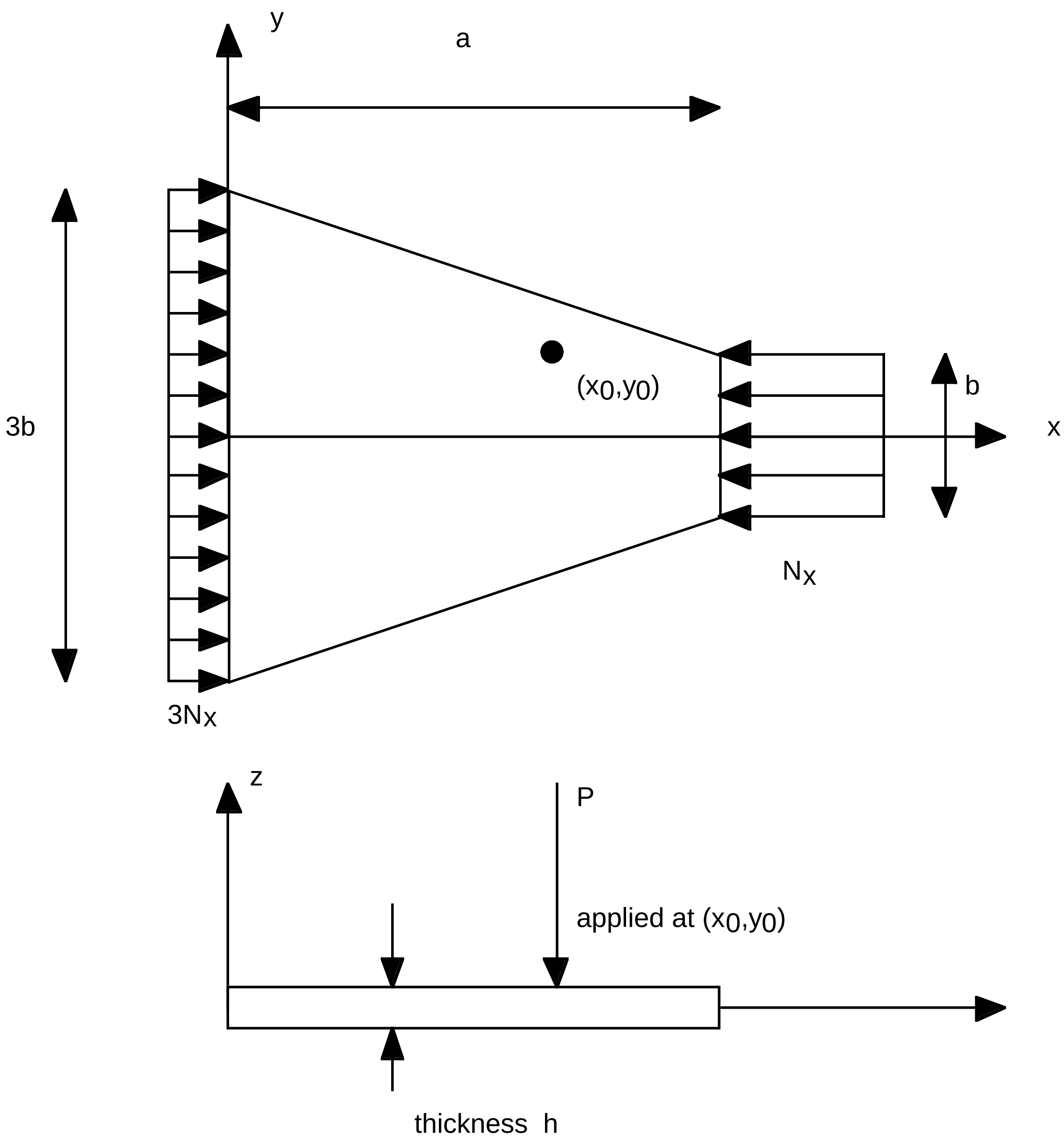

Figure 1 A Quadrilateral Plate Subjected to a Point Force P and Compressive In-plane Traction