Homework #1

MEAM502 Differential Equation Methods in Mechanical Engineering

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Consider a functional

$$F(v)=\int_0^1\sqrt{1+\left(\frac{dv}{dx}(x)\right)^2}\,dx-\int_0^1\left(x-\frac{1}{2}\right)v(x)dx+v\left(\frac{3}{4}\right)$$

and a minimization problem

$$\min_v F(v)$$

on the admissible space

$$K=\left\{\ v\in V\ |\quad v(0)=v(1)=0\ \right\}$$

and V is a linear space of all the functions defined on the interval (0,1) whose generalized first derivatives are square integrable, while they are square integrable on the interval (0,1).

(1) Find Euler's equation of this minimization problem.

(2) Using the Ritz method, solve this minimization problem. Here note that the discrete problem is nonlinear. You may apply the successive iteration method, Newton's method, bi-section method, and other appropriate methods to solve a system of nonlinear equations.

(3) Based on Euler's equation, derive a finite difference approximation to this problem.

(4) Using a finite element method, solve this problem.

(5) How can we assure convergence of the approximation method ? Give your convergence study on one of Ritz, FDM, and FEM methods.