

Review Problems for the Test on March 31, 1999
MEAM 502 Differential Equation Methods in Mechanics

1. What is a general expression of the second order partial differential equations defined on a domain in \mathbf{R}^n ?
2. Suppose that two coordinates (x, y) are obtained by a mapping from another coordinate system (ξ, η) . Transform the differential equation in the system (x, y) :
$$-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$$
 to the one in the coordinate system (ξ, η) . Furthermore, if the mapping
$$\begin{cases} x = x(\xi, \eta) \\ y = y(\xi, \eta) \end{cases}$$
 and the first derivatives of a function g are calculated in the coordinate system (ξ, η) , find the way to compute the first derivatives of g in the coordinate system (x, y) .
3. What is the steepest descent method to solve a system of linear equations $\mathbf{Ax} = \mathbf{b}$? Here we shall assume symmetry of the coefficient matrix \mathbf{A} .
4. What is the conjugate gradient method to solve a system of linear equations $\mathbf{Ax} = \mathbf{b}$? Here we shall assume symmetry of the coefficient matrix \mathbf{A} .
5. What is the Newton's method to solve a system of nonlinear equations $\mathbf{f}(\mathbf{u}) = \mathbf{0}$?
6. State the fixed point theorem.
7. What is a norm?
8. What is a scalar product (or inner product)?
9. What is a normed linear space?
10. What is the strong convergence of a sequence $\{f_n\}$ in a normed linear space?
11. What is the weak convergence of a sequence $\{f_n\}$ in a scalar product (or inner product) space?
12. Define the best approximation of an arbitrary element $f \in V$, where V is a Hilbert space with an inner product (f, g) , $f, g \in V$.
13. What is a convex set in a linear space V ?
14. Find the best approximation of a continuous function $f(x) = \sin\left(\frac{\pi}{2}x\right)$ defined on an interval $(0,1)$ as an element of $L^2(0,1)$, onto the closed linear subspace $K = \left\{v \in L^2(0,1) \mid v(x) \text{ is spanned by } \{1, x\}, \text{ that is, } v(x) = c + dx \text{ for some } c \text{ and } d\right\}$.
15. What is the Lagrange interpolation?
16. If the function $f(x) = \sin\left(\frac{\pi}{2}x\right)$ is interpolated by a linear polynomial by using the nodal points $x = 0$ and 1 , how can we estimate the interpolation error?
17. State briefly the element free Galerkin method?
18. What is the Haar scaling and mother wavelet functions?
19. State difference between Wavelet and Fourier transformations.
20. Define the wavelet functions.