Homework #5, 1998 Fall

MEAM 501 Analytical Methods in Mechanics and Mechanical Engineering

1. Consider a constrained minimization problem

$$\min_{\mathbf{v}\in K} F(\mathbf{v}) \quad , \quad F(\mathbf{v}) = \frac{1}{2}\mathbf{v}^T\mathbf{A}\mathbf{v} - \mathbf{v}^T\mathbf{b} \quad , \quad \mathbf{v}\in\mathbf{R}^n \ , \mathbf{A}\in\mathbf{R}^{n\times n} \ , \mathbf{b}\in\mathbf{R}^n$$

$$K = \{\mathbf{v}\in\mathbf{R}^n : \mathbf{B}\mathbf{v} - \mathbf{g} \le \mathbf{0}\} \quad , \quad \mathbf{B}\in\mathbf{R}^{m\times n} \ , \mathbf{g}\in\mathbf{R}^m$$

where a matrix $\mathbf{A}$ is symmetric, that is, $\mathbf{A}^T = \mathbf{A}$, and the constrained set K is non-empty.

(1) Find the necessary condition that an element $\mathbf{u}\in K$ is a minimizer of the functional F on the constrained set K.

(2) Obtain the Lagrangian L to this constrained minimization problem, and set up the "equivalent" unconstrained problem on the primal variable v by considering the necessary condition of the problem obtained by the Lagrange multiplier method.

(3) Solve the problem for $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{g}$ defined as follows :

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 1 & 2 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 2 & 1 \end{bmatrix} \quad , \quad \mathbf{g} = \begin{Bmatrix} 5 \\ 10 \\ 8 \end{Bmatrix}$$

2. Consider a functional

$$F(v)=\frac{1}{2}\int_0^L\left(1+\frac{1}{2}\sin\left(m\pi\frac{x}{L}\right)\right)\left(\frac{dv}{dx}\right)^2dx-\int_0^L v\,dx$$

on an admissible space V such that its element is continuous and piecewise continuously differentiable in the interval (0, L), and its value at x = 0 is zero.

(1) Find the first variation of the functional F, and the necessary condition of the minimizer u of the functional F on the admissible space V.

(2) Solve the minimization problem by the Ritz method by using the polynomial basis functions for m = 10. Number of terms must be determined appropriately so as to yield sufficient accuracy in Ritz's approximation.

(3) Solve the minimization problem by the Ritz method by using the trigonometric basis functions for m = 10.

(4) Solve the minimization problem by the Ritz method by using the basis functions :

$$\phi_1(x)=\frac{x}{L}$$

$$\phi_2(x)=\psi\left(\frac{x}{L}\right)\quad,\quad \psi(s)=\begin{cases}2s & if \quad 0\le s\le\frac{1}{2}\\ 2-2s & if \quad \frac{1}{2}\le s\le 1\\ 0 & if \quad s<0 \quad or \quad s>1\end{cases}$$

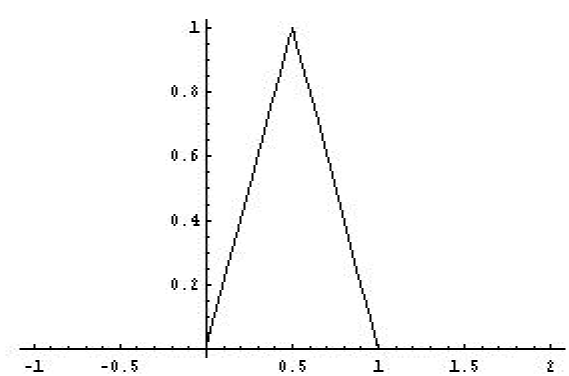

Figure X Function Profile of $\psi(s)$

y[s_]:=If[s<0,0,If[s<1/2,2*s,If[s<1,2-2*s,0]]]

$$\varphi_3(x)=\psi\left(2\frac{x}{L}\right)$$

$$\varphi_4(x)=\psi\left(2\frac{x}{L}-1\right)$$

$$\varphi_5(x)=\psi\left(4\frac{x}{L}\right)$$

$$\varphi_6(x)=\psi\left(4\frac{x}{L}-1\right)$$

.....

$$\varphi_{2^j+k}(x)=\psi\left(2^j\frac{x}{L}-k+1\right) \quad , \quad k=1,\dots,2^j \ , \ j=0,1,2,\dots..$$

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for m = 10. That is, find the coefficients c_i , $i=1,2,\dots.,n$ of an approximation of the minimizer u : $u(x)\approx\sum_{i=1}^{n}c_i\varphi_i(x)$.