Homework #5
MEAM 501 Analytical Methods in Mechanical Engineering

1997 Fall

1. Suppose that the following is the coordinates of the eleven control points of the Bezier spline curve C in the three dimensional space $\boldsymbol{R}^3$.

{0,0,0}
{0.1, 0.870501, 0.938693}
{0.2, -0.968673, 1.13874}
{0.3, -1.71508, 0.323311}
{0.4, -0.202442, -0.385068}
{0.5, 1.25, 0}
{0.6, -0.278835, 0.530376}
{0.7, -3.25672, -0.613928}
{0.8, -2.53949, -2.98535}
{0.9, 3.15378, -3.40084}
{1., 7.5, 0}

(1) Compute the total length L of the curve C by applying an appropriate quadrature.

(2) Obtain the tangent, normal, and bi-normal vectors at the point P on the curve, where P is the 20% point of the total length L of the curve C. To identify the location of the 20% point of the total length, you may use either the bisection or Newton method.

2. Approximate the function

$$f(x)=\begin{cases} -x^2+1 \quad , \quad x\in(-1,0) \\ 1-x \quad , \quad x\in(0,1) \end{cases}$$

defined on the interval (-1,1) by using

(1) Legendre polynomials and the least squares method,

(2) trigonometric functions (i.e. Fourier series approximation),

and

(3) the Lagrange interpolation. Plot the approximation and the original function, and also plot the amount of approximation error with respect to the number of terms used in approximation.

3. Consider the minimization of a functional

$$J(w)=\frac{1}{2}\int_0^1\left\{(1+\exp(-x))\left(\frac{dw}{dx}\right)^2+\cos\left(\frac{\pi}{2}x\right)w^2\right\}dx-\int_0^1\sin(\pi x)w(x)dx-w(1)$$

on the admissible space

$$K=\{w : w(0)=0 \text{ , } w \text{ is piecewise continuously differentiable functions in } (0,1)\}.$$

(1) Find the first variation of the functional J, and obtain the necessary condition of the minimum of the functional J on K.

(2) Find Euler's equation of the functional J, that is, find out the corresponding boundary value problem to the above minimization problem.

(3) Solve the minimization problem by using the Ritz method. You may use polynomials or trigonometric functions as the basis functions.

(4) Solve the minimization problem by the finite difference or finite element method.