Homework #4, 1998 Fall

MEAM 501 Analytical Methods in Mechanics and Mechanical Engineering

1. Bezier and B-splines cannot reproduce conics and circles. To overcome this, we shall introduce rational curves using the homogeneous coordinates $P^h:(hx, hy, hz, h)$ for a point $P:(x, y, z)$ in a space. We first apply the Bezier splines to the homogeneous coordinates :

$$\mathbf{r}^h(s) = \sum_{i=1}^{n+1} \mathbf{r}^h{}_i B_i^n(s) \quad , \quad \mathbf{r}^h{}_i = \begin{Bmatrix} h_i x_i \\ h_i y_i \\ h_i z_i \\ h_i \end{Bmatrix}$$

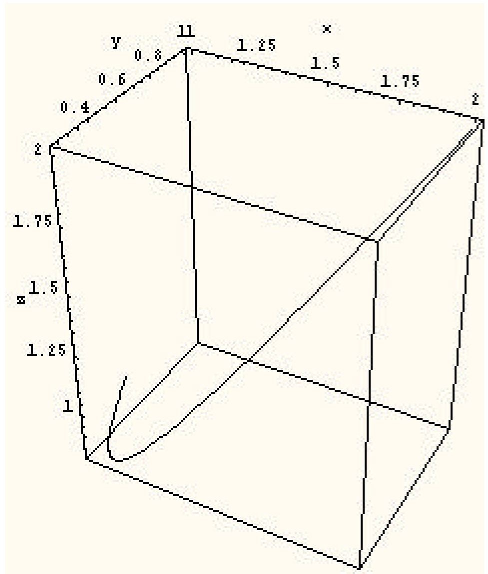
Then the corresponding curve in the three dimensional space is obtained by deviding the first three coordinates of the homogeneous ones $\mathbf{r}^h(s)$ by its homogeneous coordinate h :

$$\mathbf{r}(s) = \frac{\sum_{i=1}^{n+1} h_i \mathbf{r}_i B_i^n(s)}{\sum_{i=1}^{n+1} h_i B_i^n(s)}.$$

If we use the following MATHEMATICA script program :

```
b[n_,i_,s_]:=(n!/((i-1)! (n-i+1)!)) s^(i-1) (1-s)^(n-i+1)
n=3
cp={{2,1,2,2},{2,0,1,2},{2,1,2,1},{0,1,3,1}}
fh=Expand[Sum[cp[[i]] b[2,i,s],{i,1,n+1}]]
f={fh[[1]],fh[[2]],fh[[3]]}/fh[[4]]
ParametricPlot3D[f,{s,0,1},
   AxesLabel->{"x","y","z"}]
```

We have the curve



In this example, we have the control points :

i	$\boldsymbol{r}^h_i$	$\boldsymbol{r}_i$
1	(2,1,2,2)	(1,1/2,1)
2	(2,0,1,2)	(1,0,1/2)
3	(2,1,2,1)	(2,1,1)
4	(0,1,3,1)	(0,1,3)

(1) Find the tangent, normal, and bi-normal vectors at s = 0.5, and plot it on the curve.

(2) Find the minimum and maximum curvatures of the curve.

(3) Find the average torsion of the curve.

(4) Compute the total length of the curve.

(5) If $f(x, y, z) = x^2 - y + \cos(z)$, compute $I = \int_{\text{along the curve}} f ds$.

(6) If $\vec{f} = -\rho g \vec{e}_z$, compute $J = \int_{\text{along the curve}} \vec{f} \bullet d\vec{s}$, where ρg is a given constant.

2. Suppose that a curved surface S is given by the Bezier splines :

$$\mathbf{r}(\mathrm{x},\mathrm{h}) = \sum_{i=1}^{m+1}\sum_{j=1}^{n+1}\mathbf{r}_{ij}B_i^m(\mathrm{x})B_j^n(\mathrm{h}) \quad , \quad \mathbf{r} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \quad , \quad \mathbf{r}_{ij} = \begin{Bmatrix} x_{ij} \\ y_{ij} \\ z_{ij} \end{Bmatrix}$$

with the control points

```
[xij]={{ 0.00,-0.1, -0.1,  0,    -0.1},
      { 0.20, 0.2,  0.3,  0.25, 0.28},
      { 0.50, 0.55, 0.5,  0.45, 0.5},
      { 0.70, 0.75, 0.75, 0.7,  0.8},
      { 1.00, 0.9,  1.1,  1.2,  1}}
[yij]={{ 0   , 0.25, 0.55, 0.7,  1},
      { 0   , 0.2,  0.5,  0.75, 1.1},
      {-0.1 , 0.2,  0.45, 0.8,  0.9},
      { 0.1 , 0.3,  0.55, 0.8,  1.1},
      {-0.1 , 0.2,  0.5,  0.75, 1}}
[zij]={{ 0   , 0.2,  0.3,  0.3,  0.2},
      {-0.1 , 0.2,  0.3,  0.1,  0},
      {0    , 0.1,  0,   -0.1, -0.1},
      {0.1  , 0.1,  0,   -0.1,  0},
      {0.1   , 0  , -0.1, -0.2, -0.1}}
```

and $m = n = 4$.

(1) Draw the curved line C defined by $\mathrm{x} = \mathrm{h}$ on the surface S.

(2) Compute the length of the curve C.

(3) Draw the unit normal vector $\mathbf{n}$ and two tangent vectors $\mathbf{g}_1$ and $\mathbf{g}_2$ at $\mathrm{x} = \mathrm{h} = 0.5$.

(4) Compute the surface area S.

(5) A force field is given as $\vec{f} = -\mathrm{r}\, g\vec{e}_z$. Find the resultant pressure force applied to the surface, where $\mathrm{r}\, g$ is a given constant.