Homework #3_1998 Fall

MEAM 501 Analytical Methods in Mechanics and Mechanical Engineering

(Due day : November 10)

1. Consider a rectangular matrix and a vector

$$\mathbf{A} = \begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ 1 & -2 & 3 & -4 & 1 \\ -5 & 4 & 0 & -2 & 2 \end{bmatrix} \quad and \quad \mathbf{f} = \begin{Bmatrix} 3 \\ 1 \\ 2 \end{Bmatrix}$$

(1) Find the rank of **A**, the null space of **A**, the range of **A**, and the 2-norm of **A** by using the singular value decomposition of **A**.

(2) Compare the norm of the solution $\mathbf{x}^* = \mathbf{A} \backslash \mathbf{f}$ with the one for $\mathbf{x} = pinv(\mathbf{A}) * \mathbf{f}$.

(3) Find the eigenvalues and eigenvectors of $\mathbf{A}^T\mathbf{A}$ and $\mathbf{A}\mathbf{A}^T$, and make comparison with the result of the singular value decomposition of **A**.

(4) Find the eigenvalues and eigenvectors of $\mathbf{A}^T\mathbf{A} + \varepsilon\mathbf{I}$ for ε = 10, 1, 0.1, 0.01, 0.001, and 0.0001.

(5) Similarly, solve the problem $\left(\mathbf{A}^T\mathbf{A} + \varepsilon\mathbf{I}\right)\mathbf{x}_\varepsilon = \mathbf{A}^T\mathbf{f}$ for ε = 10, 1, 0.1, 0.01, 0.001, and 0.0001, and find the limit of $\left\|\mathbf{x}_\varepsilon\right\|_2 = \sqrt{\mathbf{x}_\varepsilon{}^T\mathbf{x}_\varepsilon}$ as ε going to zero.

2. We have data sampled at time $t_i = (i-1)/20$, $i = 1,2,.....,21$,

{3.85315 ,4.32357 ,4.46511 ,3.80561 ,3.18799 ,2.29519 ,1.86286 ,0.416273, 0.0741709,-0.548387 ,-1.36595 ,-1.02851 ,-0.770808 ,-0.240976 ,0.712875 ,1.95097 , 3.78372 ,3.26727 ,3.45551 ,3.71947 ,3.04153}

(1) Curve fit this data by using the functions

$$f_1 = \sin(3\pi t), f_2 = e^{-t}, \ f_3 = t, f_4 = t^2, f_5 = \cos(10\pi t)$$

(2) Apply the (discrete) Fourier Transformation to the data in above, and chopped off the transformed data by enforcing to be zero for the transformed data (complex number) whose absolute value is less than 10% of the largest absolute number of the transformed data. Then apply the Inverse Fourier Transformation to the chopped transformed data, and compare it with the original data.

(3) Make the least squares curve fitting using the first 5 Legendre polynomials, and find the error of the approximation to the original data.