Homework #3_1998 Fall

MEAM 501 Analytical Methods in Mechanics and Mechanical Engineering

(Due day : November 10)

1. Consider a rectangular matrix and a vector

$$\mathbf{A} = \begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ 1 & -2 & 3 & -4 & 1 \\ -5 & 4 & 0 & -2 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{f} = \begin{Bmatrix} 3 \\ 1 \\ 2 \end{Bmatrix}$$

(1) Find the rank of $\boldsymbol{A}$*, the null space of* $\boldsymbol{A}$*, the range of* $\boldsymbol{A}$*, and the 2-norm of* $\boldsymbol{A}$ *by using the singular value decomposition of* $\boldsymbol{A}$*.*

```
A =
     5     4     3     2     1
     1    -2     3    -4     1
    -5     4     0    -2     2

f =
     3
     1
     2


Rank =      3


U =
    0.7917    0.5939    0.1429
    0.0320   -0.2740    0.9612
   -0.6100    0.7565    0.2359
```

```
S =

    7.9646         0         0         0         0
         0    6.4382         0         0         0
         0         0    5.4877         0         0



V =

    0.8840   -0.1688    0.0904    0.4264         0
    0.0832    0.9241   -0.0742    0.2090   -0.3000
    0.3103    0.1491    0.6036   -0.7122   -0.1000
    0.3359    0.1197   -0.7345   -0.4933    0.3000
   -0.0498    0.2847    0.2872    0.1549    0.9000
```

Null Space of the matrix **A** is spanned by the two vectors of the matrix V associated with the zero singular values, that is, the fourth and fifth column vectors :

$$\mathbf{v}_4 = \begin{Bmatrix} 0.4264 \\ 0.2090 \\ -0.7122 \\ -0.4933 \\ 0.1549 \end{Bmatrix} \quad and \quad \mathbf{v}_5 = \begin{Bmatrix} 0 \\ -0.3000 \\ -0.1000 \\ 0.3000 \\ 0.9000 \end{Bmatrix}$$

Range of **A** is spanned by the three column vectors of the matrix U.

2 norm of the matrix A is the first singular value $s_1 = 7.9646$.

(2) Compare the norm of the solution $\mathbf{x}^* = \mathbf{A} \backslash \mathbf{f}$ *with the one for* $\mathbf{x} = pinv(\mathbf{A}) * \mathbf{f}$.

```
xstar =

    0.2955
    0.6250
```

```
         0
   -0.4886
         0

norm =    0.8466

x =
    0.0832
    0.4208
    0.3210
   -0.1430
    0.2236

norm = 0.5979
```

They are different, but both are solutions of the matrix equation. The solution by pinv is the solution with the least norm.

(3) *Find the eigenvalues and eigenvectors of* $\mathbf{A}^T\mathbf{A}$ *and* $\mathbf{A}\mathbf{A}^T$ *, and make comparison with the result of the singular value decomposition of* $\boldsymbol{A}$.

```
X1 =
    0.3525   -0.2400    0.0904    0.1688   -0.8840
    0.3416    0.1304   -0.0742   -0.9241   -0.0832
   -0.5324    0.4835    0.6036   -0.1491   -0.3103
   -0.5766    0.0296   -0.7345   -0.1197   -0.3359
   -0.3785   -0.8311    0.2872   -0.2847    0.0498
```

```
R1 =
    0.0000         0         0         0         0
         0    0.0000         0         0         0
         0         0   30.1149         0         0
         0         0         0   41.4504         0
         0         0         0         0   63.4346


X2 =
   -0.1429   -0.5939   -0.7917
   -0.9612    0.2740   -0.0320
   -0.2359   -0.7565    0.6100


R2 =
   30.1149         0         0
         0   41.4504         0
         0         0   63.4346
```

Non zero eigenvalues are the same, and are the same with singular values of **A**. The eigenvectors of $\mathbf{A}\mathbf{A}^{\mathrm{T}}$ are the same with the matrix U of the singular value decomposition, while the eigenvectors associated with non zero eigenvalues of $\mathbf{A}^{\mathrm{T}}\mathbf{A}$ are the same with V of the singular value decomposition associated with the non zero singular values.

(4) Find the eigenvalues and eigenvectors of $\mathbf{A}^T\mathbf{A} + \varepsilon\mathbf{I}$ *for* ε *= 10, 1, 0.1, 0.01, 0.001, and 0.0001.*

(5) Similarly, solve the problem $\left(\mathbf{A}^T\mathbf{A} + \varepsilon\mathbf{I}\right)\mathbf{x}_\varepsilon = \mathbf{A}^T\mathbf{f}$ *for* ε *= 10, 1, 0.1, 0.01, 0.001, and 0.0001, and find the limit of* $\left\|\mathbf{x}_\varepsilon\right\|_2 = \sqrt{\mathbf{x}_\varepsilon{}^T\mathbf{x}_\varepsilon}$ *as* ε *going to zero.*

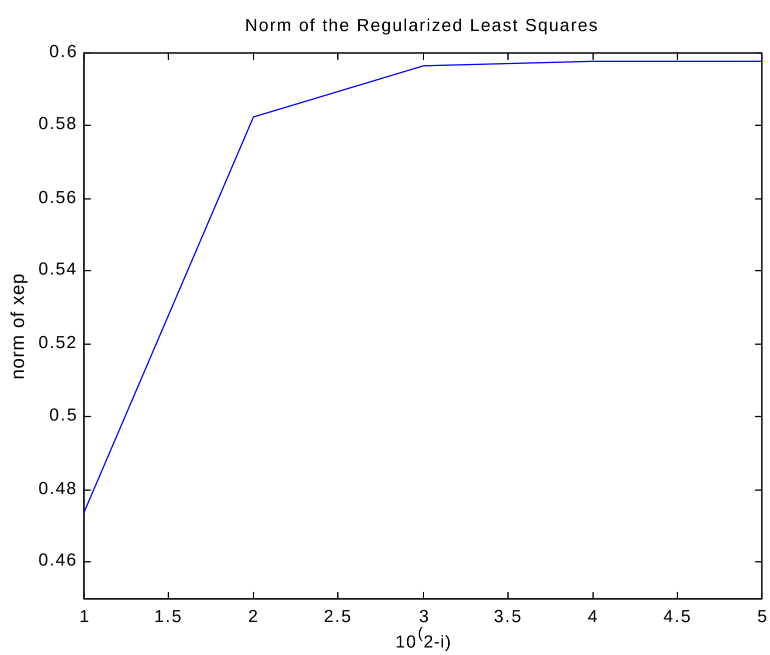
```
ep =    10


X =
   -0.4200    0.0740    0.0904    0.1688   -0.8840
   -0.2579   -0.2592   -0.0742   -0.9241   -0.0832
    0.6840   -0.2221    0.6036   -0.1491   -0.3103
    0.5378    0.2098   -0.7345   -0.1197   -0.3359
    0.0037    0.9132    0.2872   -0.2847    0.0498


R =
   10.0000         0         0         0         0
         0   10.0000         0         0         0
         0         0   40.1149         0         0
         0         0         0   51.4504         0
         0         0         0         0   73.4346


xep =
    0.0731
    0.3411
    0.2500
   -0.0986
    0.1743


normxep =    0.4736


ep =     1
```

```
X =

    0.4084    0.1228    0.0904    0.1688   -0.8840
    0.1137    0.3475   -0.0742   -0.9241   -0.0832
   -0.7108   -0.1094    0.6036   -0.1491   -0.3103
   -0.3859   -0.4294   -0.7345   -0.1197   -0.3359
    0.4076   -0.8172    0.2872   -0.2847    0.0498


R =

    1.0000         0         0         0         0
         0    1.0000         0         0         0
         0         0   31.1149         0         0
         0         0         0   42.4504         0
         0         0         0         0   64.4346


xep =

    0.0821
    0.4112
    0.3120
   -0.1370
    0.2174


normxep =    0.4736    0.5824


ep =    0.1000


X =

    0.4103   -0.1161    0.0904    0.1688   -0.8840
    0.2828    0.2318   -0.0742   -0.9241   -0.0832
```

```
  -0.6580    0.2902    0.6036   -0.1491   -0.3103
  -0.5563   -0.1543   -0.7345   -0.1197   -0.3359
  -0.0960   -0.9082    0.2872   -0.2847    0.0498


R =
    0.1000         0         0         0         0
         0    0.1000         0         0         0
         0         0   30.2149         0         0
         0         0         0   41.5504         0
         0         0         0         0   63.5346


xep =
    0.0831
    0.4198
    0.3201
   -0.1423
    0.2229


normxep =    0.4736    0.5824    0.5963


ep =    0.0100


X =
    0.3561   -0.2346    0.0904    0.1688   -0.8840
    0.3395    0.1355   -0.0742   -0.9241   -0.0832
   -0.5397    0.4753    0.6036   -0.1491   -0.3103
   -0.5769    0.0209   -0.7345   -0.1197   -0.3359
   -0.3658   -0.8368    0.2872   -0.2847    0.0498
```

```
R =
    0.0100         0         0         0         0
         0    0.0100         0         0         0
         0         0   30.1249         0         0
         0         0         0   41.4604         0
         0         0         0         0   63.4446


xep =
    0.0832
    0.4207
    0.3209
   -0.1429
    0.2235


normxep =    0.4736    0.5824    0.5963    0.5977


ep =  1.0000e-003


X =
    0.4071   -0.1271    0.0904    0.1688   -0.8840
    0.2889    0.2241   -0.0742   -0.9241   -0.0832
   -0.6500    0.3077    0.6036   -0.1491   -0.3103
   -0.5603   -0.1393   -0.7345   -0.1197   -0.3359
   -0.1204   -0.9053    0.2872   -0.2847    0.0498


R =
```

```
   0.0010         0         0         0         0
        0    0.0010         0         0         0
        0         0   30.1159         0         0
        0         0         0   41.4514         0
        0         0         0         0   63.4356


xep =
    0.0832
    0.4208
    0.3210
   -0.1430
    0.2236


normxep =    0.4736    0.5824    0.5963    0.5977    0.5979
```



Converged to the solution by pinv(A)*b.

2. We have data sampled at time $t_i = (i-1)/20$, $i = 1,2,.....,21$,

{3.85315 ,4.32357 ,4.46511 ,3.80561 ,3.18799 ,2.29519 ,1.86286 ,0.416273, 0.0741709,-0.548387 ,-1.36595 ,-1.02851 ,-0.770808 ,-0.240976 ,0.712875 ,1.95097 , 3.78372 ,3.26727 ,3.45551 ,3.71947 ,3.04153}

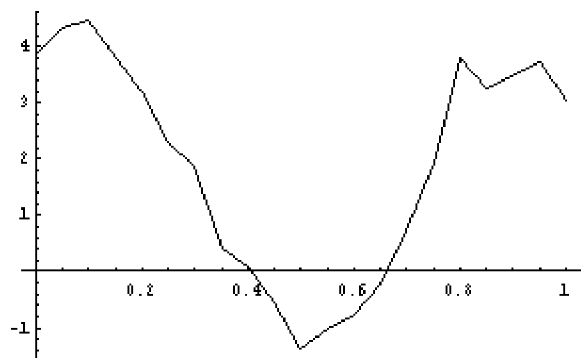


Figure X Plot of the Given Data with Respect to Time t

(1) Curve fit this data by using the functions

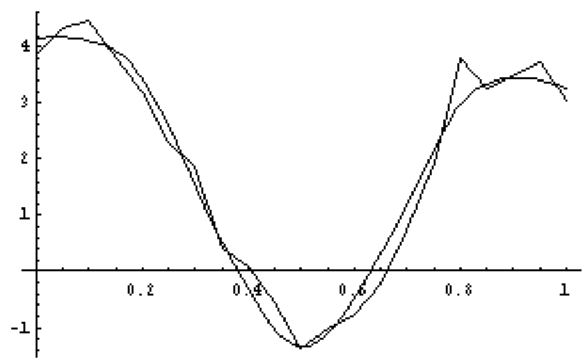
$$\phi_1 = \sin(3\pi t), \phi_2 = e^{-t}, \ \phi_3 = t, \phi_4 = t^2, \phi_5 = \cos(10\pi t)$$

Using the following MATHEMATICA program

```
basis={Sin[3*Pi*t],Exp[-t],t,t^2,Cos[10*Pi*t]}
Plot[Release[basis],{t,0,1}]
A=Table[0,{i,1,21},{j,1,5}];
b=Table[0,{i,1,21}];
Block[{i,j},
      Do[A[[i,j]]=N[basis[[j]]/.{t->time[[i]]}];
            b[[i]]=data[[i]],{i,1,21},{j,1,5}]]
```

```
MatrixForm[A]
b
ATA=Transpose[A].A
ATb=Transpose[A].b
c=Inverse[ATA].ATb
fapp=Sum[c.basis];
g2=Plot[fapp,{t,0,1},PlotRange->All]
Show[{g1,g2}]
```

We have the following results :



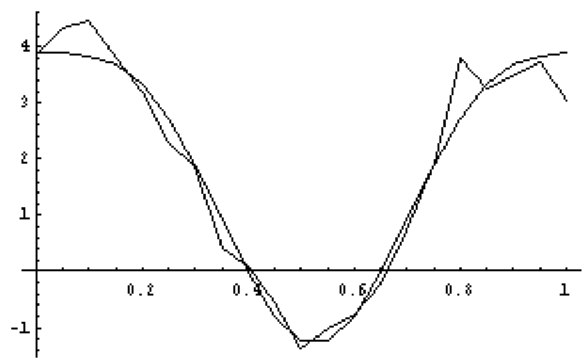
(2) Apply the (discrete) Fourier Transformation to the data in above, and chopped off the transformed data by enforcing to be zero for the transformed data (complex number) whose absolute value is less than 10% of the largest absolute number of the transformed data. Then apply the Inverse Fourier Transformation to the chopped transformed data, and compare it with the original data.

Using the following MATHEMATICA program :

```
fdata=Fourier[data]
afdata=Max[Abs[fdata]]
cfdata=Chop[fdata,afdata*0.1]
fadata=Chop[InverseFourier[cfdata]]
```

```
fadatatime=Table[{time[[i]],fadata[[i]]},{i,1,n}];
g3=ListPlot[fadatatime,PlotJoined->True]
Show[{g3,g1}]
```

we have



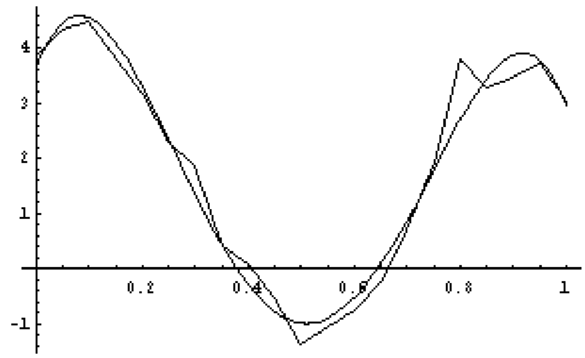
(3) Make the least squares curve fitting using the first 5 Legendre polynomials, and find the error of the approximation to the original data.

Using the following MATHEMATICA program:

```
basis=Table[LegendreP[i,2*t-1],{i,0,4}]
Plot[Release[basis],{t,0,1}]
A=Table[0,{i,1,21},{j,1,5}];
b=Table[0,{i,1,21}];
Block[{i,j},
      Do[A[[i,j]]=N[basis[[j]]/.{t->time[[i]]}];
            b[[i]]=data[[i]],{i,1,21},{j,1,5}]]
MatrixForm[A]
b
ATA=Transpose[A].A
ATb=Transpose[A].b
c=Inverse[ATA].ATb
```

```
fapp=Sum[c.basis];
g4=Plot[fapp,{t,0,1},PlotRange->All]
Show[{g1,g4}]
```

We have



Here we have used the following five Legendre polynomials :