Homework #3

MEAM 501 Winter 2000

Due Date March 6, 2000

One-dimensional Element Free Galerkin Method and Related Eigenvalue Problems Let us consider axial vibration of an elastic bar, whose length is *L* while the axial rigidity is *EA*, shown in Fig. 1:

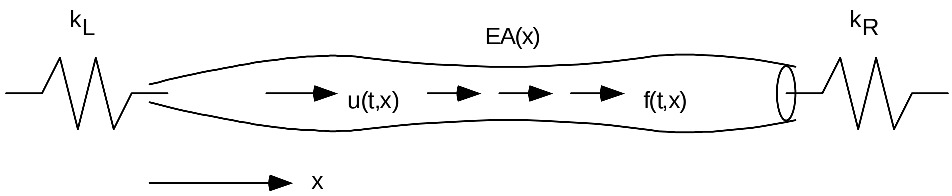

Figure 1 Vibration of an Elastic Bar in the Axial Direction

Suppose that the left and right end points are supported by two discrete springs whose spring constant is given by k_L and k_R. The equation of motion of this elastic bar is written as

$$\rho A \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}\left(EA \frac{\partial u}{\partial x} \right) + f \quad in \quad (0, L)$$

where ρ is the mass density, and the boundary condition is written by

$$-EA\frac{\partial u}{\partial x} = -k_L u \quad at \quad x = 0 \qquad and \qquad EA\frac{\partial u}{\partial x} = -k_R u \quad at \quad x = L.$$

We shall apply the weighted residual method using the basis functions constructed by the SPH method to derive a discrete system of the axial vibration problem. To this end, we shall place 11 nodes equally distributed in the domain $(0, L)$, $L = 1$, and let *u* and an arbitrary weight function *w* be approximated by

$$u(x,t)=\sum_{i=1}^{n+1} u_i(t)\phi_i(x) \quad , \quad w(x,t)=\sum_{i=1}^{n+1} w_i(t)\phi_i(x) \quad , \quad n=10$$

$$\phi_i(x)=\frac{w_i(x)}{\sum_{j=1}^{n+1} w_j(x)} \quad , \quad w_i(x)=\exp\left(-\alpha(x-x_i)^2\right) \quad , \quad \alpha=50$$

Noting that the weighted residual formulation of the equation of the motion and the boundary condition may be represented by the integral form

$$\int_0^L \left\{ \rho A \frac{\partial^2 u}{\partial t^2} w + EA \frac{\partial u}{\partial x}\frac{\partial w}{\partial x} \right\} dx + k_L u(t,0) w(t,0) + k_R u(t,L) w(t,L) = \int_0^L f w dx \quad , \quad \forall w$$

Applying the approximation, we can derive the following discrete problem

$$\mathbf{w}^T \left(\mathbf{M}\frac{d^2\mathbf{u}}{dt^2} + \mathbf{K}\mathbf{u} \right) = \mathbf{w}^T \mathbf{f} \qquad \forall \mathbf{w}$$

that is

$$\mathbf{M}\frac{d^2\mathbf{u}}{dt^2} + \mathbf{K}\mathbf{u} = \mathbf{f}$$

where **M** and **K** are the 11-by-11 mass and stiffness matrices, respectively. Here we assume $EA = L = \rho A = 1$.

1. For the case that $k_L = k_R = 0$, compute det(**K**), rank(**K**), and find the null space N(**K**) as well as the range of **K**, i.e., R(**K**).
2. For the case that $k_L = k_R = 10000$, find the eigenvalues and eigenvectors of the global stiffness matrix.

3. For the force $f(x)=\begin{cases}+1 & if \quad x\in(0,L/2)\\ -1 & if \quad x\in(L/2,L)\end{cases}$, find its finite element approximation **f** corresponding to the choice of M and N_E in above. Then compute $\mathbf{x}_i^T\mathbf{f}$, $i=1,\ldots,neq$, where $\mathbf{x}_i$ are the eigenvectors obtained in 2.
4. Orthonormalize the eigenvectors $\mathbf{x}_i$ obtained in 2 with respect to the mass matrix ***M***.
5. Diagonalize the mass matrix **M** by using the Householder transformation **P**.
6. Make **QR** decomposition of **M**.
7. Solve the generalized eigenvalue problem $\mathbf{Kx}=\lambda\mathbf{Mx}$ for the case that $k_L = k_R = 0$.
8. Find the dynamical response of the bar at $x = 0.5$ for the case that

$$k_L = k_R = 10000,\ f(t,x)=e^{-2t}\sin\left(2\pi\frac{x}{L}\right),\ u_0(x)=0.1\sin\left(\pi\frac{x}{L}\right),\ v_0(0)=0 .$$

Reference in Some Sense

Look at the web site at Northwestern University
http://tam6.mech.nwu.edu/mfleming/efg.html
where you can find the latest EFG methods