Exercises in Note #2

MEAM501 Matrix Methods in Mechanical Enginnering and Applied Mechanics, September 16, 1998

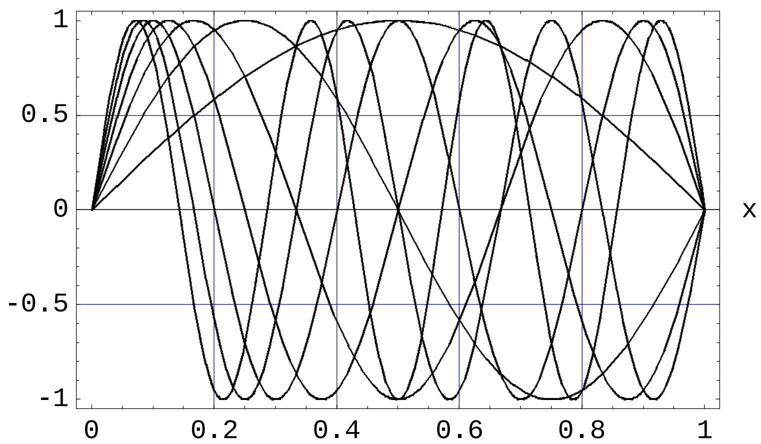
■ Exercise 1 : Trigonometric and Polynomial Basis

Ÿ setting up the basis functions and functions given

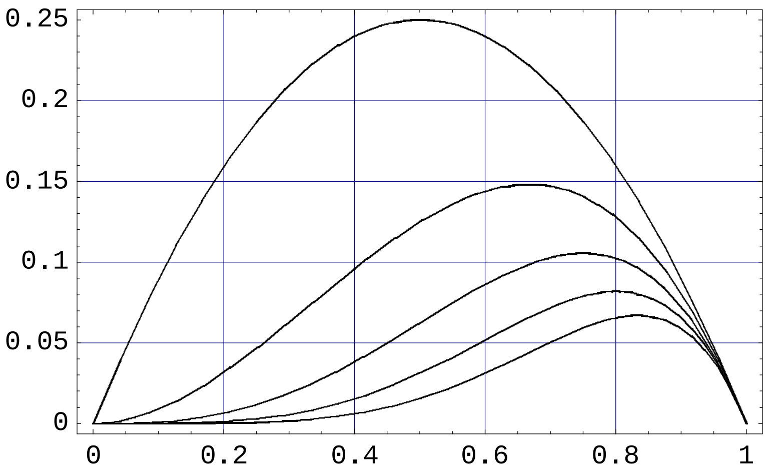
As an example, we set up m=5 and n=7, and we shall consider only the case of (a) which involves both trigonometric and polynomial basis functions. Because of the choice of the basis functions, the boundary conditions are automatically satisfies, and then we need not consider them in this example.

First we define Fj and Yj using Table, and then these functions are plotted by Plot. Then we plot the right hand side f(x,t) and the initial displacement condition u0.

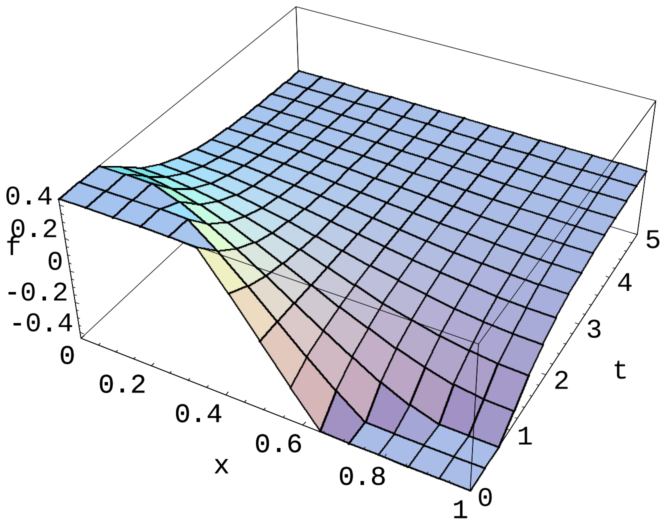
```
m = 5;
n = 7;
L = 1;
Fj = Table@Sin@j Pi x • LD, 8j, 1, n<D;
Yi = Table@x HL - xL x^Hi - 1L, 8i, 1, m<D;
f = Exp@-tD * Cos@Pi x • LD;
u0 = Sin@Pi x • LD;
Plot@Release@FjD, 8x, 0, L<,
 Frame -> True, GridLines -> Automatic, AxesLabel -> 8"x", "F"<D
Plot@Release@YiD, 8x, 0, L<,
 Frame -> True, GridLines -> Automatic, AxesLabel -> 8"x", "Y"<D
Plot3D@f, 8x, 0, L<, 8t, 0, 5<, AxesLabel -> 8"x", "t", "f"<D
Plot@u0, 8x, 0, L<, Frame -> True, GridLines -> Automatic, AxesLabel -> 8"x", "u0"<D
```



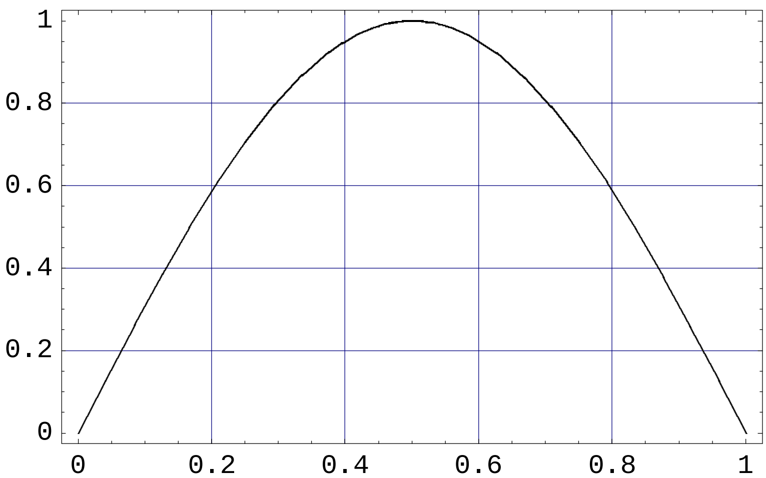
… Graphics …



… Graphics …



… SurfaceGraphics …



… Graphics …

Ÿ Case (a)

We evaluate the components of the mass matrix M and the stiffness matrix K by using numerical integration methods, NIntegral in *Mathematica*. Here we introduce Block and Do commands to make looped caculation.

Ÿ Evaluate a discrete form of the equation of motion

```
M = 1;
T = 1;
Mij = Table@0, 8i, 1, m<, 8j, 1, n<D;
Kij = Table@0, 8i, 1, m<, 8j, 1, n<D;
Block@8i, j<,
Do@Mij@@i, jDD = NIntegrate@M * Yi@@iDD * Fj@@jDD, 8x, 0, L<D;
  DFjj = D@Fj@@jDD, xD;
  DYii = D@Yi@@iDD, xD;
  Kij@@i, jDD = NIntegrate@T * DYii * DFjj, 8x, 0, L<D,
 8j, 1, n<, 8i, 1, m<DD
MatrixForm@MijD
MatrixForm@KijD
```

0.129006	6.93889 ´ 10^{-18}	0.00477801	-1.35525 ´ 10^{-20}	0.00103205	3.40239 ´ 10^{-18}	0.00037
0.0645031	-0.0241887	0.002389	-0.00302358	0.000516025	-0.000895876	0.00018
0.0366566	-0.0241887	0.00652152	-0.00302358	0.00149788	-0.000895876	0.00055
0.0227333	-0.0199974	0.00858778	-0.0036485	0.00198881	-0.00114407	0.00073
0.015027	-0.0158062	0.0089356	-0.00427342	0.00233526	-0.00139227	0.00089

1.27324	1.66533 ´ 10^{-16}	0.424413	3.05311 ´ 10^{-16}	0.254648	2.22045 ´ 10^{-16}	0.181891
0.63662	-0.95493	0.212207	-0.477465	0.127324	-0.31831	0.0909457
0.361786	-0.95493	0.579284	-0.477465	0.369587	-0.31831	0.268324
0.224369	-0.789467	0.762822	-0.576148	0.490719	-0.406496	0.357013
0.14831	-0.624003	0.793717	-0.674831	0.576203	-0.494681	0.432442

We evaluate the right hand side that is a function of time, and then it cannot be evaluated by NIntegrl, since it contains symbolics too. Here we apply analytical integration because of integration in only one variable. Similarly, we evaluate the initial displacement vector.

Ÿ Evaluate the right hand side of the equation of motion and the initial condition

```
fi = Table@0, 8i, 1, m<D;
u0i = Table@0, 8i, 1, m<D;
Block@8i<,
Do@fi@@iDD = Integrate@f * Yi@@iDD, 8x, 0, L<D;
  u0i@@iDD = NIntegrate@u0 * Yi@@iDD, 8x, 0, L<D;
  Print@"fH", i, "L = ", fi@@iDDD;
  Print@"u0H", i, "L = ", u0i@@iDDD,
 8i, 1, m<DD
fi
u0i
```

fH1L = 0

u0H1L = 0.129006

fH2L = $-\frac{12\,E^{-t}}{p^4}+\frac{E^{-t}}{p^2}$

u0H2L = 0.0645031

fH3L = $-\frac{12\,E^{-t}}{p^4}+\frac{E^{-t}}{p^2}$

u0H3L = 0.0366566

fH4L = $\frac{120\,E^{-t}}{p^6}+\frac{E^{-t}\,H120-36\,p^2+p^4L}{p^6}$

u0H4L = 0.0227333

fH5L = $\frac{480\,E^{-t}}{p^6}-\frac{60\,E^{-t}}{p^4}+\frac{E^{-t}}{p^2}$

u0H5L = 0.015027

$90,\ -\frac{12\,E^{-t}}{p^4}+\frac{E^{-t}}{p^2},\ -\frac{12\,E^{-t}}{p^4}+\frac{E^{-t}}{p^2},\ \frac{120\,E^{-t}}{p^6}+\frac{E^{-t}\,H120-36\,p^2+p^4L}{p^6},\ \frac{480\,E^{-t}}{p^6}-\frac{60\,E^{-t}}{p^4}+\frac{E^{-t}}{p^2}=$

80.129006, 0.0645031, 0.0366566, 0.0227333, 0.015027<

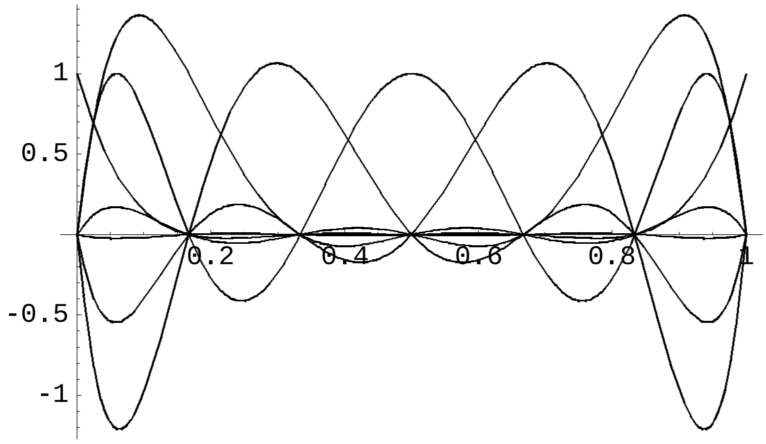
Ÿ Exercise 2 : Lagrange Polynomials

Here we are taking the Lagrange polynomials with the points including the both boundary points. Thus the basis functions need not satisfy the zero boundary condition a priori. To construct the Lagrange polynomials, we have applied If command in *Mathematica*.

Ÿ Problem (a), (b), and (c)

```
n = 7;
L = 1;
Fj = Table@1, 8j, 1, n<D;
xj = Table@Hj - 1L * L • Hn - 1L, 8j, 1, n<D
Block@8j, k<,
Do@Fjj = 1;
Do@Fjj = Fjj * If@k != j, Hx - xj@@kDDL • Hxj@@jDD - xj@@kDDL, 1D,
  8k, 1, n<D;
Fj@@jDD = Fjj,
8j, 1, n<DD
Plot@Release@FjD, 8x, 0, L<D
g = Exp@-xD + Sin@2 * Pi * x • LD
gj = Table@g •. x -> xj@@jDD, 8j, 1, n<D
gn = gj . Fj;
Plot@8g, gn<, 8x, 0, L<, AxesLabel -> 8"x", "g & gn"<D
eI = Sqrt@NIntegrate@Hg - gnL^2, 8x, 0, L<DD
```

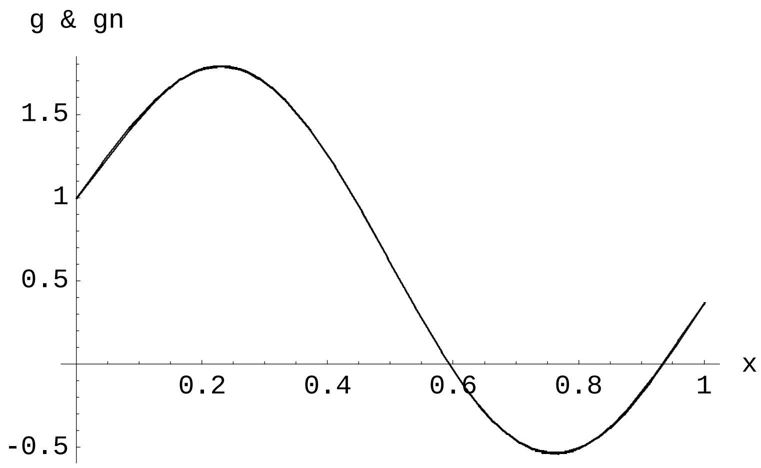
$9 0, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1 =$



… Graphics …

E^{-x} + Sin@2 p xD

$9 1, \frac{\sqrt{3}}{2} + \frac{1}{E^{1/6}}, \frac{\sqrt{3}}{2} + \frac{1}{E^{1/3}}, \frac{1}{\sqrt{E}}, -\frac{\sqrt{3}}{2} + \frac{1}{E^{2/3}}, -\frac{\sqrt{3}}{2} + \frac{1}{E^{5/6}}, \frac{1}{E} =$



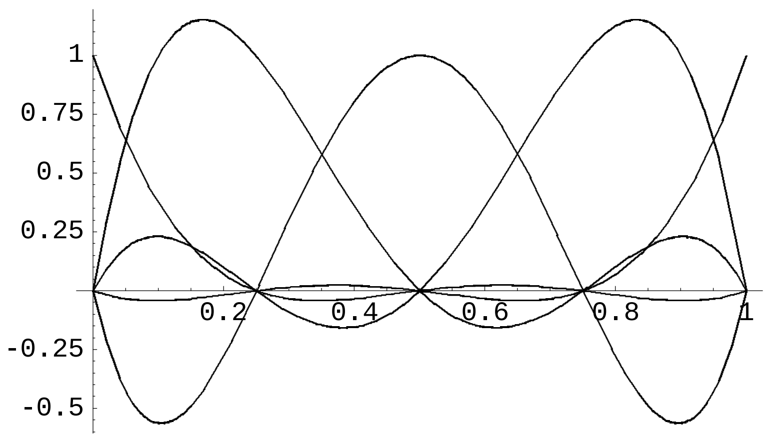

```
… Graphics …
```

```
0.0077784
```

Ÿ Problem (d)

```
m = 5;
L = 1;
Yi = Table@1, 8i, 1, m<D;
xi = Table@Hi - 1L * L • Hm - 1L, 8i, 1, m<D
Block@8i, k<,
Do@Yii = 1;
Do@Yii = Yii * If@k != i, Hx - xi@@kDDL • Hxi@@iDD - xi@@kDDL, 1D,
  8k, 1, m<D;
Yi@@iDD = Yii,
8i, 1, m<DD
Plot@Release@YiD, 8x, 0, L<D
```

90, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1=




```
… Graphics …
```

Ÿ Matrices M & K for the State Equation

```
M = 1;
T = 1;
Mij = Table@0, 8i, 1, m<, 8j, 1, n<D;
Kij = Table@0, 8i, 1, m<, 8j, 1, n<D;
Block@8i, j<,
Do@Mij@@i, jDD = NIntegrate@M * Yi@@iDD * Fj@@jDD, 8x, 0, L<D;
  DFjj = D@Fj@@jDD, xD;
  DYii = D@Yi@@iDD, xD;
  Kij@@i, jDD = NIntegrate@T * DYii * DFjj, 8x, 0, L<D,
 8j, 1, n<, 8i, 1, m<DD
MatrixForm@MijD
MatrixForm@KijD
```

0.0373954	0.0818182	-0.0688312	0.0505051	-0.0224026	0.00181818	-0.00252525
0.0234343	0.232727	0.0467532	0.0646465	-0.0675325	0.0498701	0.00565657
-0.0151515	-0.109091	0.144156	0.0935065	0.144156	-0.109091	-0.0151515
0.00565657	0.0498701	-0.0675325	0.0646465	0.0467532	0.232727	0.0234343
-0.00252525	0.00181818	-0.0224026	0.0505051	-0.0688312	0.0818182	0.0373954

6.26889	-7.33714	2.95714	-3.83492	3.3	-1.85143	0.49746
-10.0851	19.7486	-12.6857	10.1587	-13.3714	8.77714	-2.54222
5.86095	-19.3371	19.8	-12.6476	19.8	-19.3371	5.86095
-2.54222	8.77714	-13.3714	10.1587	-12.6857	19.7486	-10.0851
0.49746	-1.85143	3.3	-3.83492	2.95714	-7.33714	6.26889

■ Exercise 3 : Finite Element Like Piecewise Polynomials

The piecewise linear polynomial functions are constructed by using nested If statements. It is also noted that these basis functions does satisfy the zero boundary condition at the end points. Thus, we have large interpolation error in the vicinity of the two boundary points, since the original function is not vanished at these points.

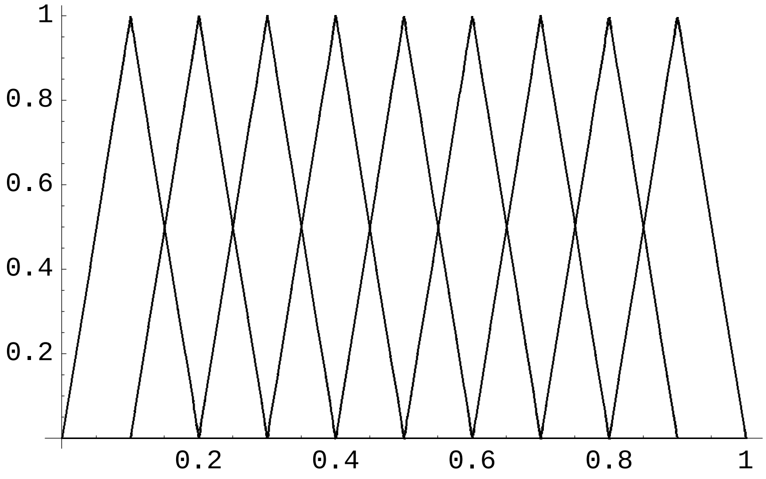
Furhermore, we have not evaluated the components of the mass and stiffness matrices M and K. Please extend the result in the previous two examples.

```
In[150]:= n = 9;
          L = 1;
          xj = Table@Hj - 1L * L • Hn + 1L, 8j, 1, n + 2<D
          fjj = If@x < xj@@jDD, 0, If@x < xj@@j + 1DD, Hx - xj@@jDDL • Hxj@@j + 1DD - xj@@jDDL,
             If@x < xj@@j + 2DD, Hxj@@j + 2DD - xL • Hxj@@j + 2DD - xj@@j + 1DDL, 0DDD
          Fj = Table@fjj •. j -> k, 8k, 1, n<D;
          Plot@Release@FjD, 8x, 0, L<D
          g = Exp@-xD + Sin@2 * Pi * x • LD
          gj = Table@g •. x -> xj@@jDD, 8j, 2, n + 1<D
          gn = gj . Fj;
          Plot@8g, gn<, 8x, 0, L<, AxesLabel -> 8"x", "g & gn"<D
          eI = Sqrt@NIntegrate@Hg - gnL ^ 2, 8x, 0, L<DD
```

Out[150]= $\{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}, 1\}$

```
Part::pspec : Part specification j is neither an integer nor a list of integers.
```

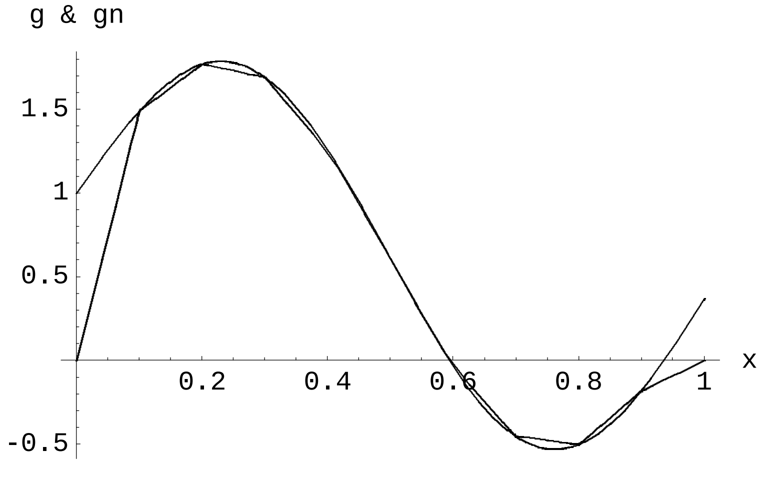
```
Out[151]= IfAx < 90, 1/10, 1/5, 3/10, 2/5, 1/2, 3/5, 7/10, 4/5, 9/10, 1=PjT, 0,
    IfAx < xjPj + 1T, (x - xjPjT)/(xjPj + 1T - xjPjT), IfAx < xjPj + 2T, (xjPj + 2T - x)/(xjPj + 2T - xjPj + 1T), 0EEE
```




```
Out[152]= … Graphics …

Out[153]= E^-x + Sin@2 p xD

Out[154]= 9 1/2 $ 1/2 I5 - 5M + 1/E^(1•10), 1/2 $ 1/2 I5 + 5M + 1/E^(1•5), 1/2 $ 1/2 I5 + 5M + 1/E^(3•10),
    1/2 $ 1/2 I5 - 5M + 1/E^(2•5), 1/E, - 1/2 $ 1/2 I5 - 5M + 1/E^(3•5), - 1/2 $ 1/2 I5 + 5M + 1/E^(7•10),
    - 1/2 $ 1/2 I5 + 5M + 1/E^(4•5), - 1/2 $ 1/2 I5 - 5M + 1/E^(9•10)=
```




```
Out[155]= … Graphics …

Out[156]= 0.197435
```