CONSTITUTIVE MODELING AND OPTIMAL DESIGN OF POLYMERIC FOAMS FOR CRASHWORTHINESS

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Outline

• Introduction
• Experimental Investigation and Result
• Constitutive Modeling
• Numerical Implementation Procedures
• Image-based Fixed-grid Homogenization Method
• Foam Design Optimization
• Conclusion and Future Work
Introduction

● **Background**
  * In 1991, 56,000 people died in auto accident in the US (NHTSA);
  * New federal motor vehicle safety standards (FMVSS);
  * Usage of polymeric foam for cushion purpose;
  * Mathematical Modeling of Transportation Safety.

Hybrid III Dummy and Honeycomb Padding

Computational Model
Objective and Tasks

- **Tasks**
  - Phenomenological modeling of PU, PS and PP foams;
  - Numerical implementation as user defined material subroutine in LS-DYNA3D;
  - Model validation: simple loading and structural test;
  - Microscopic constitutive modeling by image-based fixed-grid; representative volume element analysis using homogenization method
  - Optimization of polymeric foam structure.

- **Foam specific cushion character**
  - Limited compressive stress by long plateau regime
  - Compression and shear properties
  - Large deformation (80% volumetric strain) and low bulk modulus
  - Rate sensitive: High strain rate (35 mph)
  - Temperature sensitive: -20º C to 80º C
Polymer Material Properties

Types of polymer foams
(at room temperature 20° C):

Flexible(elastomeric) foam: Polyurethane foam
Rigid (elastic-plastic) foam: Polystyrene foam
Semi rigid foam: Polypropylene foam

Time-Temperature Correspondence

\[ E_s(t, T_0) = E_s \left( \frac{t}{a_t}, T_1 \right) \]

\[ \log a_T = -\frac{C_1(T_1 - T_g)}{C_2 + T_1 - T_g} \]

Table 1.1 Properties of Solid Polymers (at 20 °C)

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (Mg/m³)</th>
<th>Glass Temperature (K)</th>
<th>Young's Modulus ( E_s ) (GN/m²)</th>
<th>Yield Strength ( \sigma_y ) (MN/m²)</th>
<th>Fracture Strength (MN/m²)</th>
<th>Fracture Toughness ( K_{IC} ) (MN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polyurethane</td>
<td>1.2</td>
<td>-</td>
<td>1.6</td>
<td>127</td>
<td>130</td>
<td>-</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>1.05</td>
<td>373</td>
<td>1.2-1.7</td>
<td>30-70</td>
<td>40-80</td>
<td>-</td>
</tr>
<tr>
<td>Polypropylene</td>
<td>0.91</td>
<td>253</td>
<td>1.2-1.7</td>
<td>30-70</td>
<td>35-90</td>
<td>2</td>
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</table>
Related Work

- **Dimensional mechanism model**
  - Gibson and Ashby (1988);
  - Gibson et al (1989) and Triantafillou et al (1989);

- **Simple loading phenomenological model**
  - Rush, 1969;
  - Ramon et al, 1990;

- **Continuum model**
  - Roscoe's critical state theory (Schofield and Worth, 1968);
  - Krieg (1972);
## Experiment Program

<table>
<thead>
<tr>
<th>Test mode</th>
<th>Foam type</th>
<th>PP foam</th>
<th>PS foam</th>
<th>PU foam</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Density (pcf)</td>
<td>1.89</td>
<td>3.06</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>Strain rate (sec (^{-1}))</td>
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<td>✔</td>
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<tr>
<td><strong>1.60 \times 10^{-3}</strong></td>
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<tr>
<td><strong>8.00 \times 10^{-1}</strong></td>
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<td><strong>8.80 \times 10^{-1}</strong></td>
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<td><strong>Hydrostatic Compression</strong></td>
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<td>✔</td>
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<tr>
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<td><strong>Simple Shear</strong></td>
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</tbody>
</table>

* ASTM Standard D1621
* 50 x 50 x 50 mm\(^3\) for uniaxial and hydrostatic tests
* 100 x 50 x 50 mm\(^3\) for shear tests
Compressive Response of Polymeric Foam

* Negligible size effect
* Uniform deformation
* Near zero Poisson’s ratio

Quasi-static Response (BASF Polypropylene foam, 1.89 pcf)
Compressive Responses of Polypropylene Foams

Polypropylene foam (1.89 pcf)  Polypropylene foam (3.06 pcf)
Hydrostatic Compression Response of Polypropylene Foam

BASF Polypropylene Foam (Density 1.89 pcf)

Polypropylene foam (3.06 pcf)
Shear Response of Rigid Polystyrene Foam

Polystyrene Foam (1.0 pcf) under shear loading
Tensile Response of Polyurethane Foams

[Graph showing stress-strain curves for PU Foam (4.3 pcf) and PU Foam (6.0 pcf) at different strain rates.]

PU Foam (4.3 pcf)  
PU Foam (6.0 pcf)
Temperature Effect on Polypropylene Foam

Polypropylene Foam (3.06 pcf) under Uniaxial Compression
Rigid Polymeric Foam Elasticity

**Foam Elasticity**

\[ \dot{\sigma} = C : \dot{\varepsilon} \]

where objective stress rate \( \dot{\sigma}^J \) is the Jaumman stress rate in a corotational frame.

**Isotropic Foam**

\[ \dot{\sigma}^J = 2G(\dot{\varepsilon}_d - \dot{\varepsilon}_{dp}) - K(\dot{\varepsilon}_v - \dot{\varepsilon}_{vp}) \]

**Anisotropy Foam**

\[ \dot{\sigma} = S : \dot{\varepsilon} \]

\[ S = \begin{bmatrix}
\frac{1}{E_1} & -\frac{v_{12}}{E_2} & -\frac{v_{13}}{E_3} & 0 & 0 & 0 \\
-\frac{v_{12}}{E_2} & \frac{1}{E_2} & -\frac{v_{23}}{E_3} & 0 & 0 & 0 \\
-\frac{v_{13}}{E_3} & -\frac{v_{23}}{E_3} & \frac{1}{E_3} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}}
\end{bmatrix} \]
Yield Locus for Rigid Polymeric Foam

**Foam Yield Locus**

**Isotropic elasto-plastic foam**

Dimensional Argument (Gibson and Ashby, 1988)

\[ M = \frac{\sigma_{ys} b t^2}{4} \left[ 1 - \left( \frac{\sigma_a}{\sigma_{ys}} \right)^2 \right] \]

\[ \sigma_{vm} = \kappa \left[ 1 - \left( \frac{p}{\beta} \right)^2 \right] \]

\[ P_{crit} = \frac{n^2 \pi^2 E_s I}{h^2} \]

**Proposed yield locus**

\[ F(\sigma) - F_0 = \left[ p - x_0 (\epsilon_{vp}) \right]^2 + \frac{\sigma_{vm}^2}{a(\epsilon_{vp})} - 1 = 0 \]

Anisotropy elasto-plastic foam

\[ F(\sigma) = \sqrt{j} + a\tilde{i} - 1 \]

\[ \tilde{j}(\sigma) = \frac{1}{2} \left[ \frac{(\sigma_{11} - \sigma_{22})^2}{k_{11}} + \frac{(\sigma_{22} - \sigma_{33})^2}{k_{22}} + \frac{(\sigma_{33} - \sigma_{11})^2}{k_{33}} \right] + 3 \left[ \frac{(\sigma_{12})^2}{k_{12}} + \frac{(\sigma_{23})^2}{k_{23}} + \frac{(\sigma_{31})^2}{k_{31}} \right] \]

\[ \tilde{i}(\sigma) = \frac{\sigma_{11}}{k_{11}} + \frac{\sigma_{22}}{k_{22}} + \frac{\sigma_{33}}{k_{33}} \]
Temperature Sensitivity

Williams-Landel-Ferry (WLF) Equation
(Williams et al, 1955)

\[ L(T) = \exp \left( -\frac{C_1(T-T_r)}{C_2+T-T_r} \right) \]

PP foam (3.06 pcf)

\( C_1 = 6.52 \) pC, \( C_2 = 468.7 \) pC
Rate Dependency of PP foam (3.06 pcf)

Effective Stress (MPa)

Strain Rate

Nagy et al, 1964

\[ \ddot{\gamma} = D \left( \frac{f}{f_0} \right)^{\frac{1}{n}} \]

\[ n = a + b\dot{\varepsilon}_p \]

Combined temperature and rate effect

\[ \sigma(\varepsilon) = \sigma_s(\varepsilon)L(T) \left( \frac{\dot{\gamma}}{\dot{\gamma}_s} \right)^{n_{\text{se}}} \]
Comparison of Yield Criterion

Plastic yield envelop (Gibson et al, 1989)

\[ \frac{\sigma_{vm}}{\sigma_{ys}} = \pm \gamma \left( \frac{\rho^*}{\rho_s} \right)^{3/2} \left\{ 1 - \left[ \frac{3p}{\sigma_{ys} \left( \rho^*/\rho_s \right)} \right]^2 \right\} \]

Buckling surface (Puso and Govindjee 1995)

\[ \sigma_{vm}^2 + \frac{1}{R^2} (p^2 - h^2) = 0 \]
Kinematic hardening

\[ F = F(\sigma, \epsilon_{vp}, \bar{\epsilon}) \]

\[ g = g(\sigma, \epsilon_{vp}, \bar{\epsilon}) \]

Evolution of Yield Ellipse with Plastic Volumetric Strain
Stress Integration Procedure for Elastic-plastic Materials

Deformation decomposition

\[ \varepsilon = \varepsilon^e + \varepsilon^p \]

Plasticity consistency condition

\[ \varepsilon = D^{-1} d\sigma + \frac{\partial g}{\partial \sigma} d\lambda \]

\[ \begin{bmatrix} \frac{\partial F}{\partial \sigma} \end{bmatrix} d\sigma - A d\lambda = 0 \]

\[ d\sigma = D_{ep}^* d\varepsilon \]

\[ D_{ep}^* = D - D \left( \begin{bmatrix} \frac{\partial g}{\partial \sigma} \end{bmatrix} \left[ \frac{\partial F}{\partial \sigma} \right]^T D \right) \left( \begin{bmatrix} \frac{\partial F}{\partial \sigma} \end{bmatrix} D \begin{bmatrix} \frac{\partial g}{\partial \sigma} \end{bmatrix} \right)^{-1} \]

if \( F \neq g \), \( D_{ep}^* \) is a non-symmetric matrix

Non-unique solution for non-associative plastic flow
The stress return is not radial
Non-smooth Multisurface Plasticity

Plastic potential variation (assuming associative plastic potential)

\[
\dot{\lambda}_i = \frac{\partial F_i}{\partial \sigma} : C \varepsilon - \dot{\lambda}_i \frac{\partial F_i}{\partial \sigma} : C \sigma = 0 \quad (i = 1, 2, \ldots)
\]

If plastic yield and loading condition active

\[F_i(\sigma) = 0 \quad \text{and} \quad \frac{\partial F_i}{\partial \sigma} : C \varepsilon > 0\]

Plasticity consistency condition

\[
\dot{\lambda}_i = \frac{\partial F_i}{\partial \sigma} : C \varepsilon
\]

(1) \(\dot{\lambda}_i = 0 \quad (i = 1, 2, \ldots),\) loading is not active;

(2) \(\dot{\lambda}_i > 0 \quad \dot{\lambda}_i = \frac{\partial F_i}{\partial \sigma} : C \varepsilon\)

(3) \(\dot{\lambda}_i > 0 \quad\) for multiple surfaces \(\dot{\lambda}_i = \max \left(\frac{\partial F_i}{\partial \sigma} : C \varepsilon, \frac{\partial F_i}{\partial \sigma} : C s, i = 1, 2, \ldots\right)\)

Closest-point-projection (Simo et al, 1988)

In summary

\[
C_{\text{ep}} = \begin{cases} 
C & \text{if } \dot{\lambda}_i = 0 \\
C - \frac{[C : \sigma] \otimes [C : \frac{\partial F_i}{\partial \sigma}]}{\frac{\partial F_i}{\partial \sigma} : C : \sigma} & (i = 1, 2, \ldots) \text{ if } \dot{\lambda}_i > 0
\end{cases}
\]

Illustration of a singular point in yield surface
Polymeric Rigid Foam Plastic Flow Law

- Non-associative plastic potential: \( g(\dot{\sigma}, \phi) = \sqrt{\alpha p^2 + \sigma_{vm}^2} \)
- Plasticity consistency condition: \( \dot{\varepsilon}_p = \dot{\varepsilon}_{\dot{\sigma}} \)
- Plastic Poisson’s Ratio: \( \tilde{\gamma}_{xp} = \tilde{\gamma}_{yyp} = -\nu_p \tilde{\gamma}_{zp} \)
- Under uniaxial compression:
  \[
  \dot{\varepsilon}_p = \dot{\varepsilon}_{\dot{\sigma}} = \frac{1}{2g} \left[ 2\sigma_{vm} \frac{\partial \sigma_{vm}}{\partial \sigma} + 2\alpha_p \frac{\partial p}{\partial \sigma} \right] \quad \text{or} \quad \dot{\varepsilon}_p = \frac{3}{2g} \left[ s - \frac{2\alpha}{9} p \right]
  \]
  \[
  \dot{\varepsilon}_{zp} = \frac{3}{2g} \left( s_{zz} - \frac{2\alpha}{9} p \right)
  \]
  \[
  \dot{\varepsilon}_{vp} = -\frac{\dot{\varepsilon}_{\dot{\sigma}p}}{g}
  \]
- Zero plastic Poisson’s ratio:
  \[
  \alpha = \frac{9(1 - 2\nu_p)}{2(1 + \nu_p)} \quad \Rightarrow \quad g = \sqrt{\frac{9}{2} p^2 + \sigma_{vm}^2}
  \]
Model Validation under Uniaxial Compression

Nominal Stress (MPa) vs. Nominal Strain

- Numerical Result (1.6E-3 1/sec)
- Experimental Result (1.6E-3 1/sec)
- Numerical Result (8.0E-2 1/sec)
- Experimental Result (8.0E-2 1/sec)
- Numerical Result (4.6 1/sec)
- Experimental Result (4.6 1/sec)

PP Foam (3.06 pcf)
Model Validation under Simple Shear

Polystyrene Foam (1.0 pcf)
Hemispherical Free Drop Test

Indenter: φ127 mm, 22.2 kgm, 4.5m/sec
PP foam, 203x 203 x101 mm³, 3.06 pcf
Hemispherical Free Drop Numerical Simulation

Original Mesh

Deformed Mesh at t=9.0 ms
Hemispherical Free Drop Numerical Simulation

Deformed Mesh at $t=16.0$ ms

Effective plastic strain $t=16.0$ ms
Conclusion

- PU foams are flexible while PS and PP foams are rigid at 20°C;
- Yield stress of polymeric foams are sensitive to strain rate, temperature and pressure;
- A phenomenological rate dependent single surface elasto-plastic yield criterion is developed and implemented in LS-DYNA3D program;
Future Work

- Experiment on polymeric foams under multiaxial loading;
- Constitutive modeling considering different failure mechanism;
- Validate rigid foam model under multiaxial loading;
- Couple homogenization constitutive modeling and LS-DYNA3D;
- Three-dimensional RVE modeling and analysis by using more powerful CT scanner.
- Three-dimensional foam design optimization.