CONSTITUTIVE MODELING AND OPTIMAL DESIGN OF POLYMERIC FOAMS FOR CRASHWORTHINESS

Jun Zhang

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Computational Mechanics Laboratory
Department of Mechanical Engineering and Applied Mechanics
The University of Michigan

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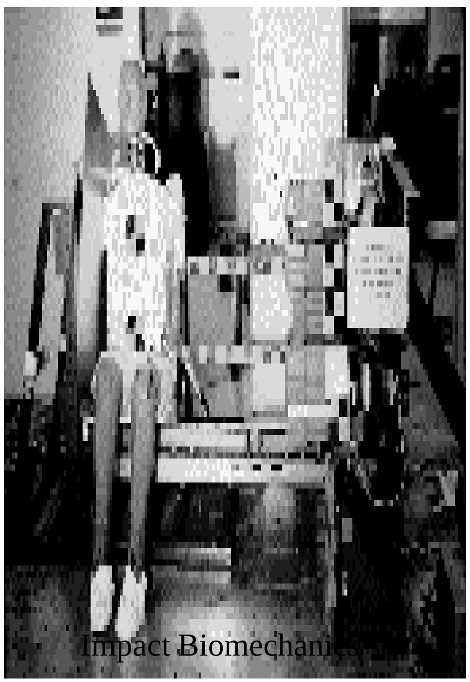
Outline

- Introduction
- Experimental Investigation and Result
- Constitutive Modeling
- Numerical Implementation Procedures
- Image-based Fixed-grid Homogenization Method
- Foam Design Optimization
- Conclusion and Future Work

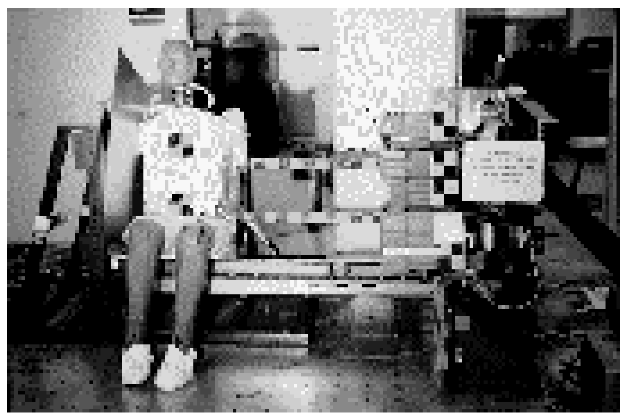
Introduction

- **Background**

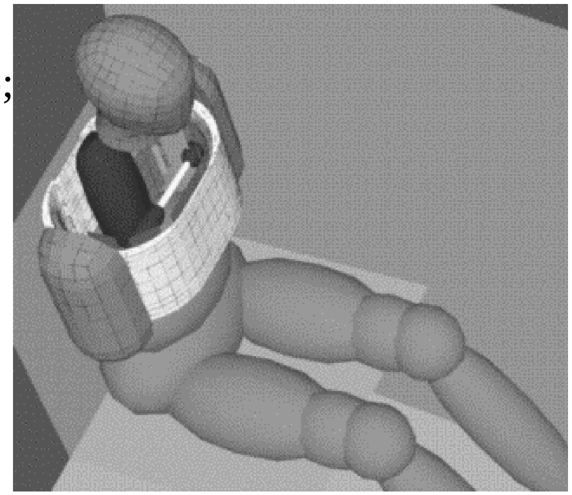
  * In 1991, 56,000 people died in auto accident in the US (NHTSA);
  * New federal motor vehicle safety standards (FMVSS);
  * Usage of polymeric foam for cushion purpose;
  * Mathematical Modeling of Transportation Safety.

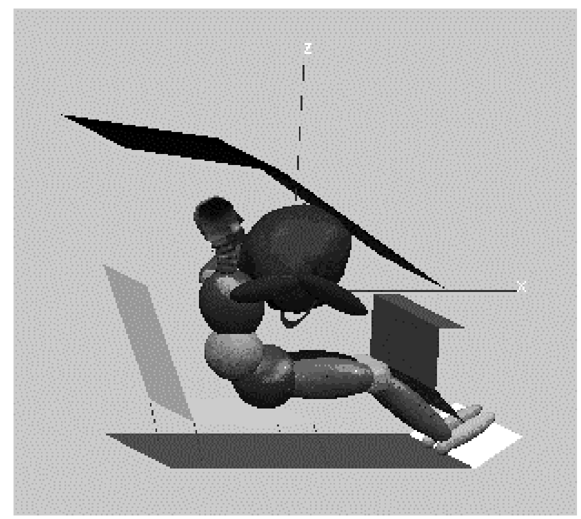


Impact Biomechanics



Hybrid III Dummy and Honeycomb Padding





Computational Model

Objective and Tasks

- **Tasks**

  * Phenomenological modeling of PU, PS and PP foams;

  * Numerical implementation as user defined material subroutine in LS-DYNA3D;

  *Model validation: simple loading and structural test;

  *Microscopic constitutive modeling by image-based fixed-grid; representative volume element analysis using homogenization method

  *Optimization of polymeric foam structure.

- **Foam specific cushion character**

  * Limited compressive stress by long plateau regime

  * Compression and shear properties

  * Large deformation (80% volumetric strain) and low bulk modulus

  * Rate sensitive:High strain rate (35 mph)

  * Temperature sensitive:-20^{o} C to 80^{o} C

Polymer Material Properties

Types of polymer foams
(at room temperature 20° C) :

Flexible(elastomeric) foam: Polyurethane foam
Rigid (elastic-plastic) foam: Polystyrene foam
Semi rigid foam: Polypropylene foam

Time-Temperature Correspondence

$$E_s(t, T_0) = E_s\left(\frac{t}{a_t}, T_1\right)$$

$$\log a_T = \frac{-C_1(T_1 - T_g)}{C_2 + T_1 - T_g}$$

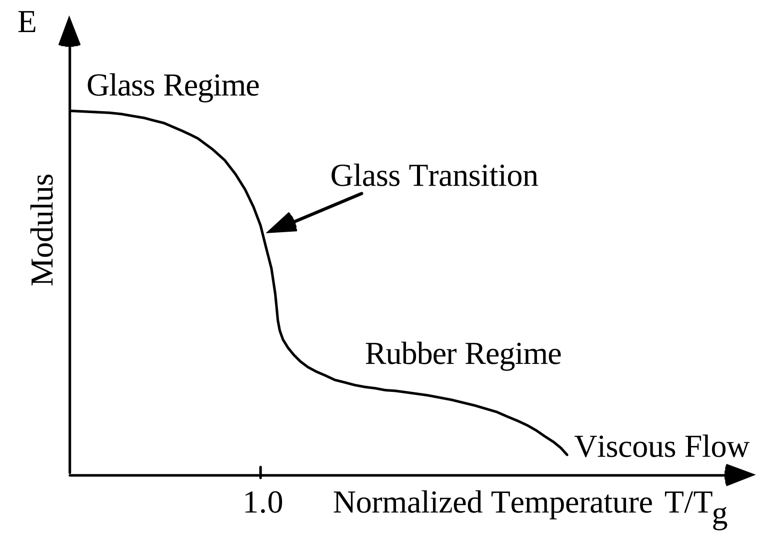


Table 1.1 Properties of Solid Polymers (at 20 °C)

Material	Density (Mg/m^3)	Glass Temperature (K)	Young's Modulus E_s (GN/m^2)	Yield Strength σ_{ys}(MN/m^2)	Fracture Strength (MN/m^2)	Fracture Toughness K_{IC}(MN/m$^{1.5}$)
Polyurethane	1.2	-	1.6	127	130	-
Polystyrene	1.05	373	1.2-1.7	30-70	40-80	-
Polypropylene	0.91	253	1.2-1.7	30-70	35-90	2

Related Work

- Dimensional mechanism model

  Gibson and Ashby (1988);

  Gibson et al (1989) and Triantafillou et al (1989);

  Puso and Govindjee (1995).

- Simple loading phenomenological model

  Rush, 1969;

  Ramon et al, 1990;

  Sherwood and Frost, 1992.

- Continuum model

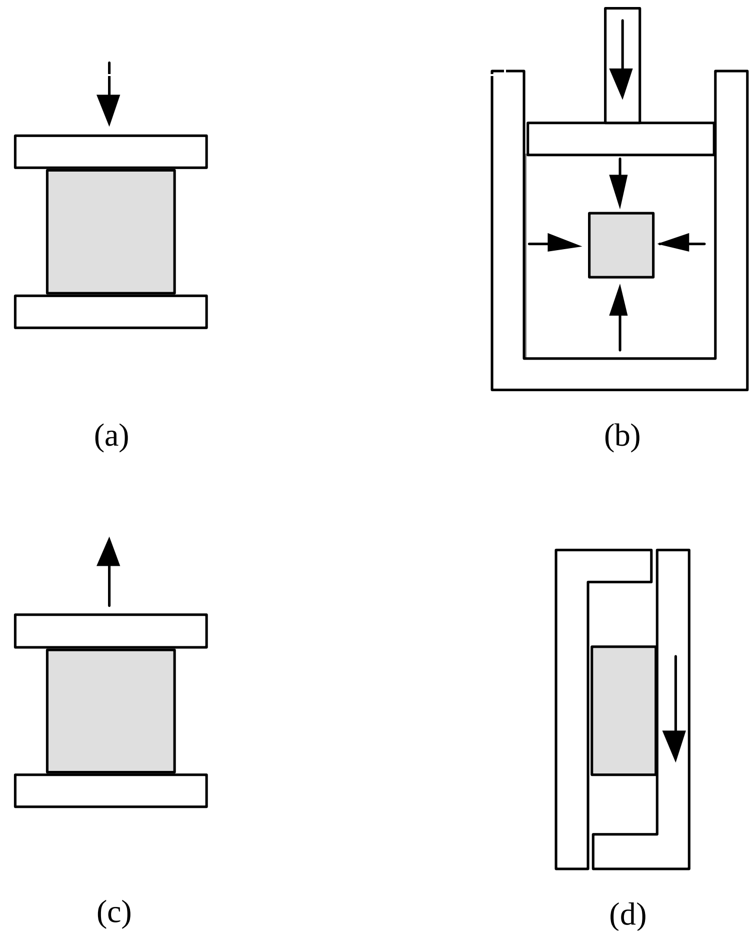
  Roscoe's critical state theory (Schofield and Worth, 1968);

  Krieg (1972);

  Neilsen et al (1995).

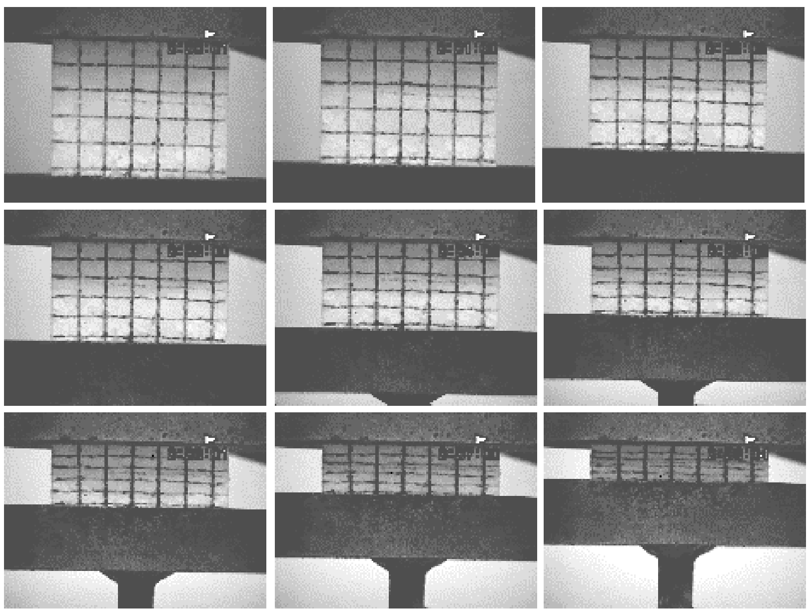
Experiment Program

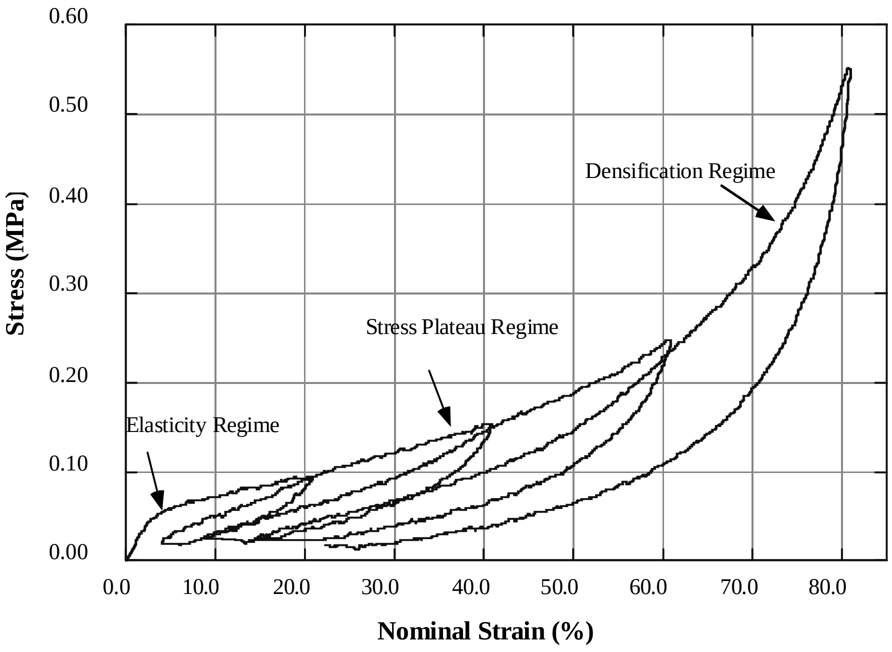
Test mode	Foam type	PP foam		PS foam		PU foam	
	Density (pcf)	1.89	3.06	1.0	4.0	4.3	6.0
	Strain rate (sec $^{-1}$)						
Uniaxial Compression	1.60×10^{-3}	•	•	•	•	•	†
	8.00×10^{-1}	•	•	•	•	•	•
	4.60	•	•	•	•	•	•
	8.80×10^{1}	•	•	•	•		•
Hydrostatic Compression	4.00×10^{-3}	•	•			•	•
	2.00×10^{-1}	•	•			•	•
	1.15×10^{1}	•	•			•	•
Uniaxial Tension	1.60×10^{-3}					•	•
	8.00×10^{-1}					•	•
	4.60					•	•
	8.80×10^{1}						
Simple Shear	1.60×10^{-3}			•	•	•	•
	8.00×10^{-1}			•	•	•	•
	4.60			•	•	•	•
	8.80×10^{1}						



* ASTM Standard D1621
* 50 x 50 x 50 mm^3 for uniaxial and hydrostatic tests
* 100 x 50 x 50 mm^3 for shear tests

Compressive Response of Polymeric Foam

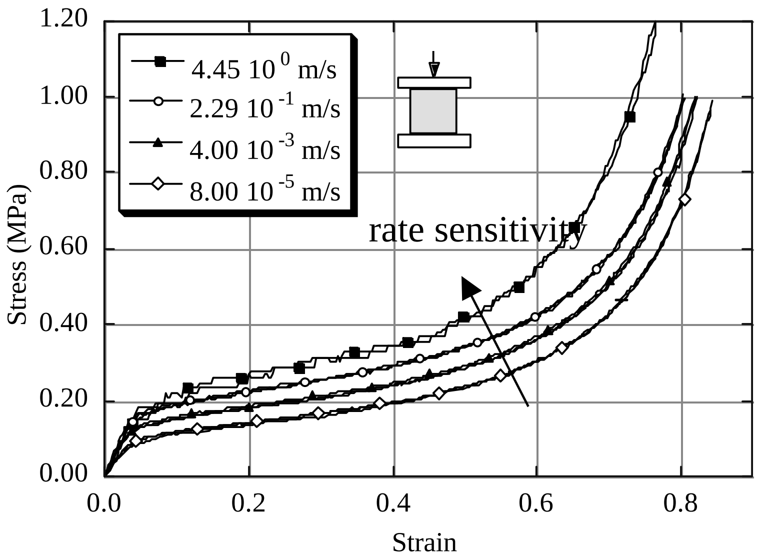




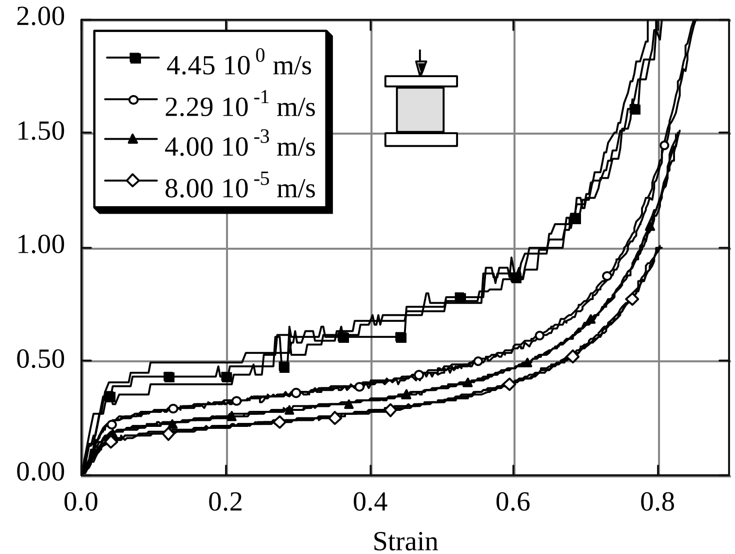
* Negligible size effect
* Uniform deformation
* Near zero Poisson's ratio

Quasi-static Response (BASF Polypropylene foam,1.89 pcf)

Compressive Responses of Polypropylene Foams

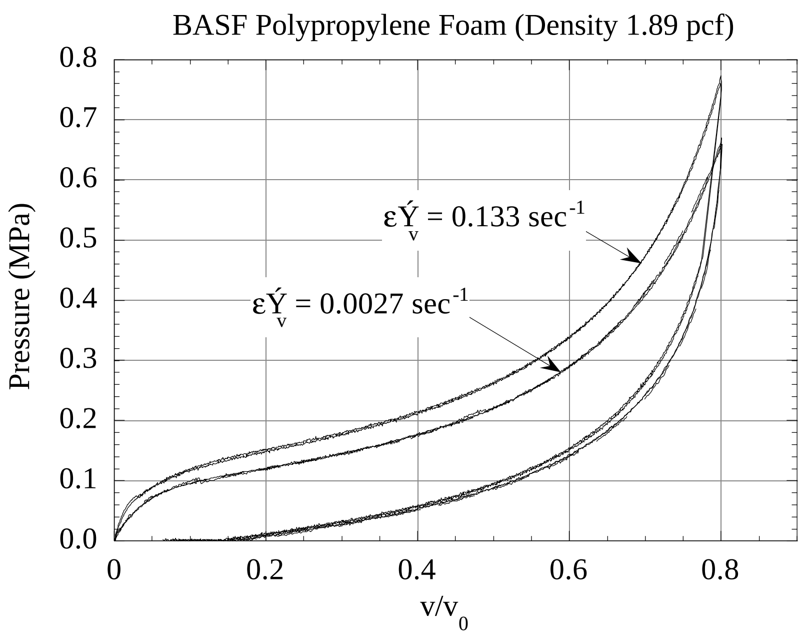


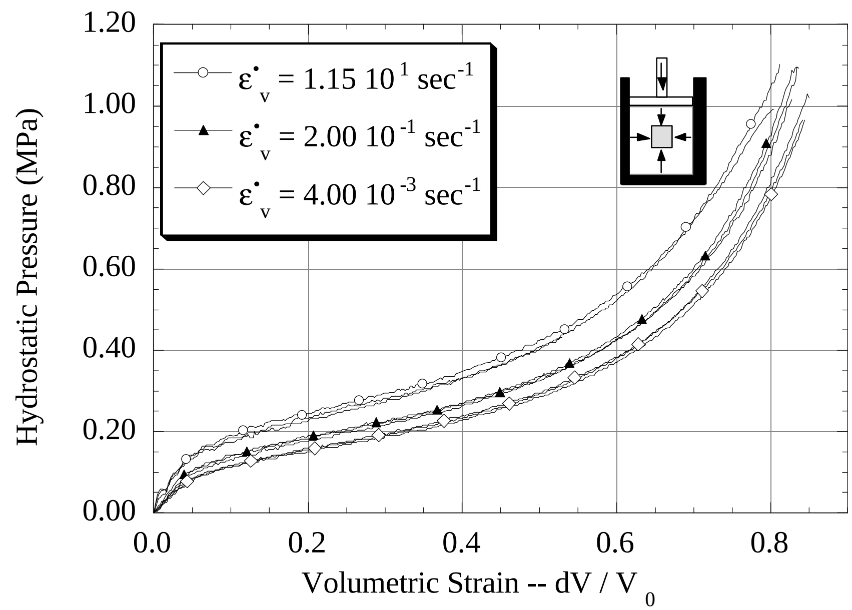
Polypropylene foam (1.89 pcf)



Polypropylene foam (3.06 pcf)

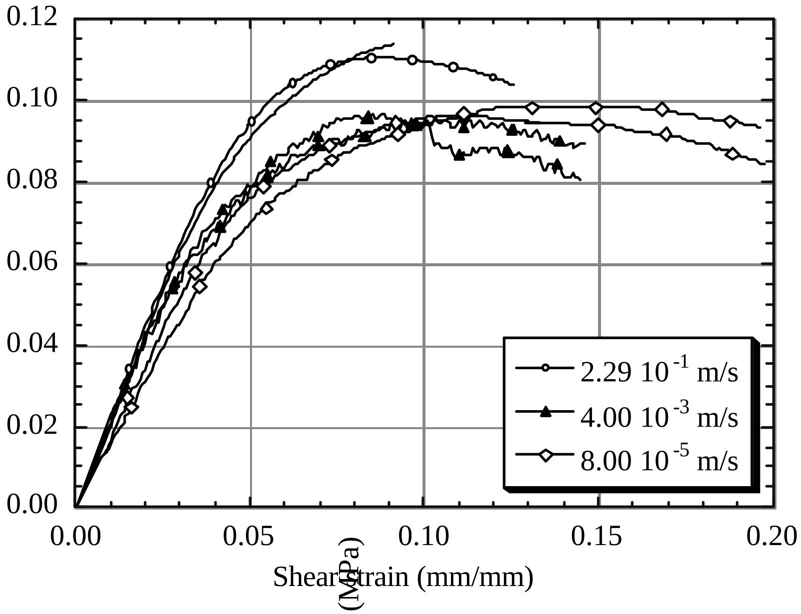
Hydrostatic Compression Response of Polypropylene Foam

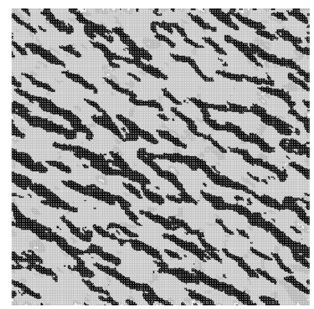


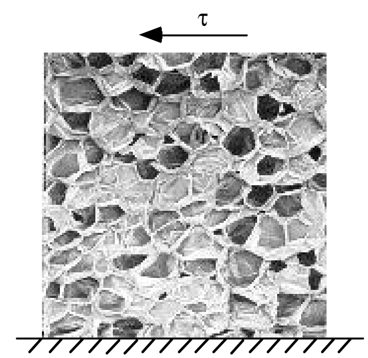


Polypropylene foam (3.06 pcf)

Shear Response of Rigid Polystyrene Foam

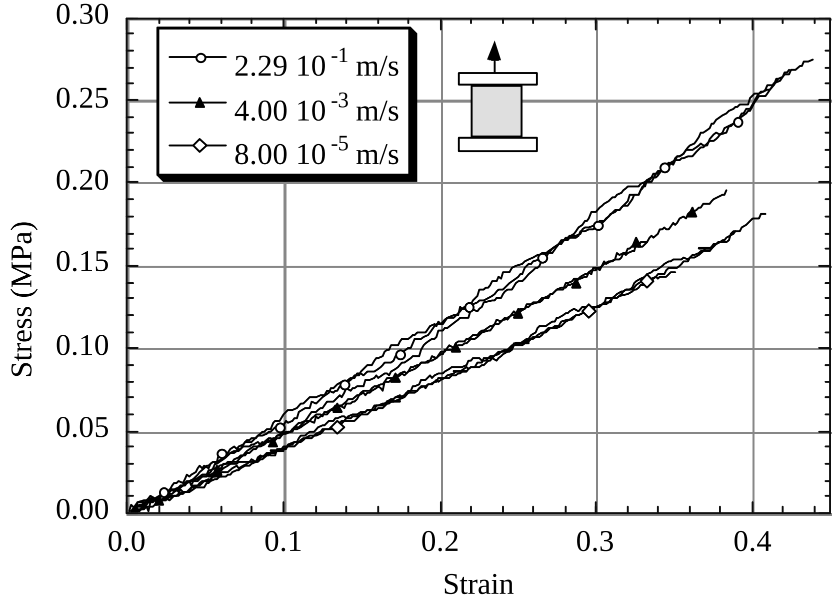


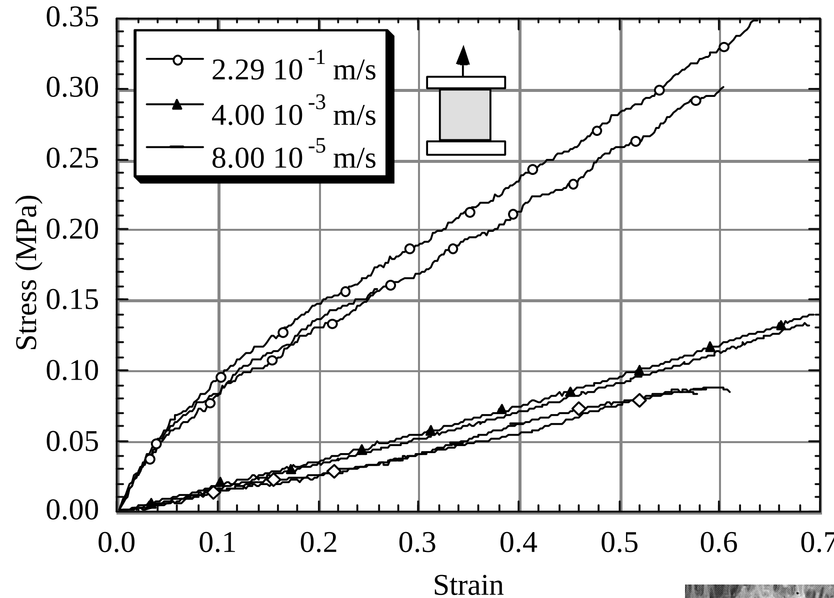




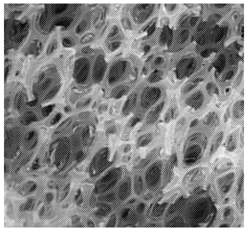
Polystyrene Foam (1.0 pcf) under shear loading

Tensile Response of Polyurethane Foams

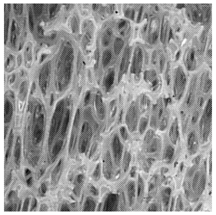




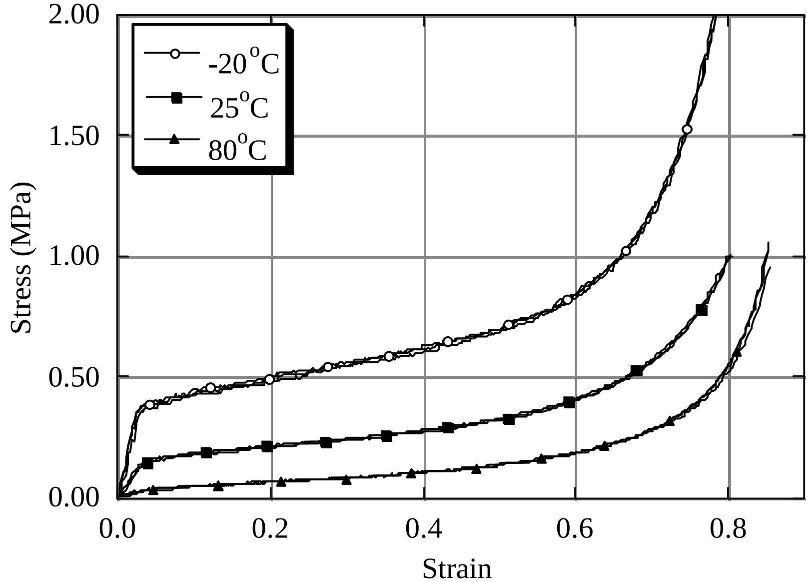
PU Foam (4.3 pcf)



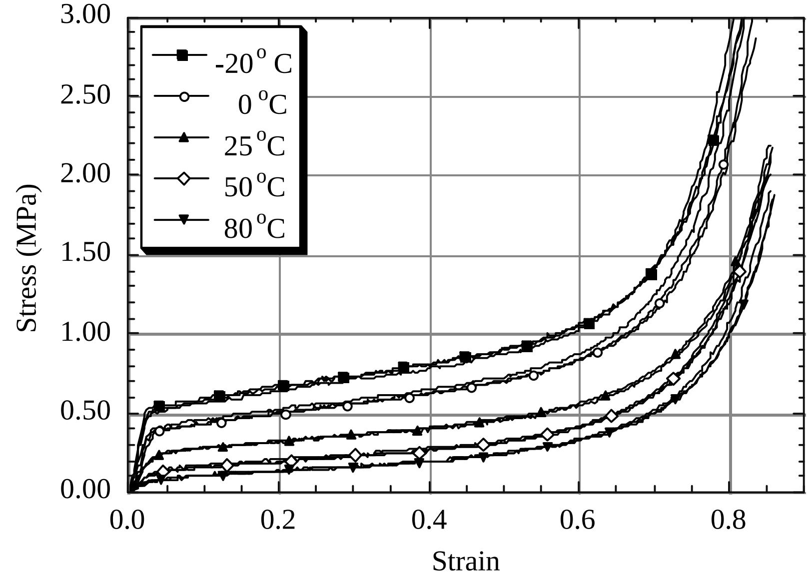
PU Foam (6.0 pcf)



Temperature Effect on Polypropylene Foam



Strain rate 0.0016 1/sec



Strain rate 4.6 1/sec

Polypropylene Foam (3.06 pcf) under Uniaxial Compression

Rigid Polymeric Foam Elasticity

Foam Elasticity

$\dot{\sigma} = \boldsymbol{C} : \dot{\varepsilon}$ where objective stress rate $\overset{\nabla}{\dot{\sigma}}_J$ is the Jaumman stress rate in a corotational frame

Isotropic Foam $\dot{\sigma} = 2G\left(\dot{\varepsilon}_d - \dot{\varepsilon}_{dp}\right) - K\left(\dot{\varepsilon}_v - \dot{\varepsilon}_{vp}\right)$

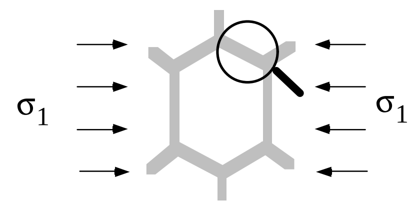
Anisotropy Foam

$$\dot{\varepsilon} = \boldsymbol{S} : \dot{\sigma}$$

$$S = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$$

Yield Locus for Rigid Polymeric Foam

Foam Yield Locus

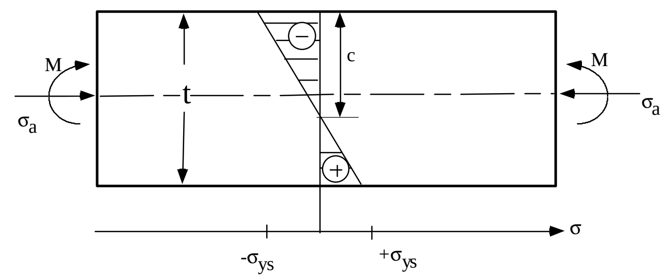


Isotropic elasto-plastic foam

Dimensional Argument(Gibson and Ashby, 1988)

$$M = \frac{\sigma_{ys} bt^2}{4}\left[1 - \left(\frac{\sigma_a}{\sigma_{ys}}\right)^2\right] \Rightarrow \sigma_{vm} = \kappa\left[1 - \left(\frac{p}{\beta}\right)^2\right]$$

$$P_{crit} = \frac{n^2 p^2 E_s I}{h^2}$$



Proposed yield locus

$$F(s) - F_0 = \frac{\left[p - x_0(\varepsilon_{vp})\right]^2}{a(\varepsilon_{vp})} + \frac{\sigma_{vm}^2}{b(\varepsilon_{vp})} - 1 = 0$$

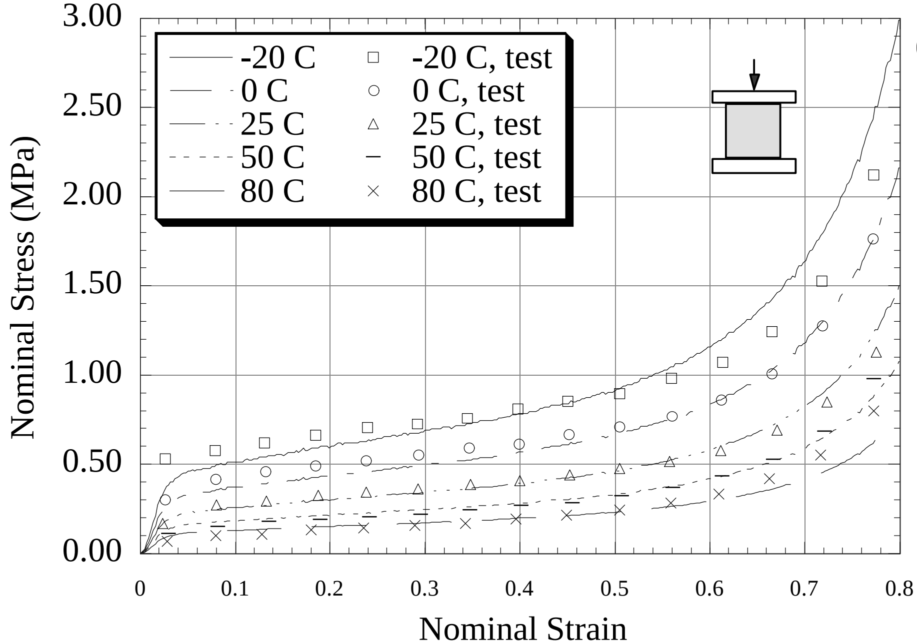
Anisotropy elasto-plastic foam

$$F(s) = \sqrt{\tilde{J}} + a\tilde{I} - 1$$

$$\tilde{J}(s) = \frac{1}{2}\left[\left(\frac{\sigma_{11}}{k_{11}} - \frac{\sigma_{22}}{k_{22}}\right)^2 + \left(\frac{\sigma_{22}}{k_{22}} - \frac{\sigma_{33}}{k_{33}}\right)^2 + \left(\frac{\sigma_{33}}{k_{33}} - \frac{\sigma_{11}}{k_{11}}\right)^2\right] + 3\left[\left(\frac{\sigma_{12}}{k_{12}}\right)^2 + \left(\frac{\sigma_{23}}{k_{23}}\right)^2 + \left(\frac{\sigma_{31}}{k_{31}}\right)^2\right]$$

$$\tilde{I}(s) = \frac{\sigma_{11}}{k_{11}} + \frac{\sigma_{22}}{k_{22}} + \frac{\sigma_{33}}{k_{33}}$$

Temperature Sensitivity



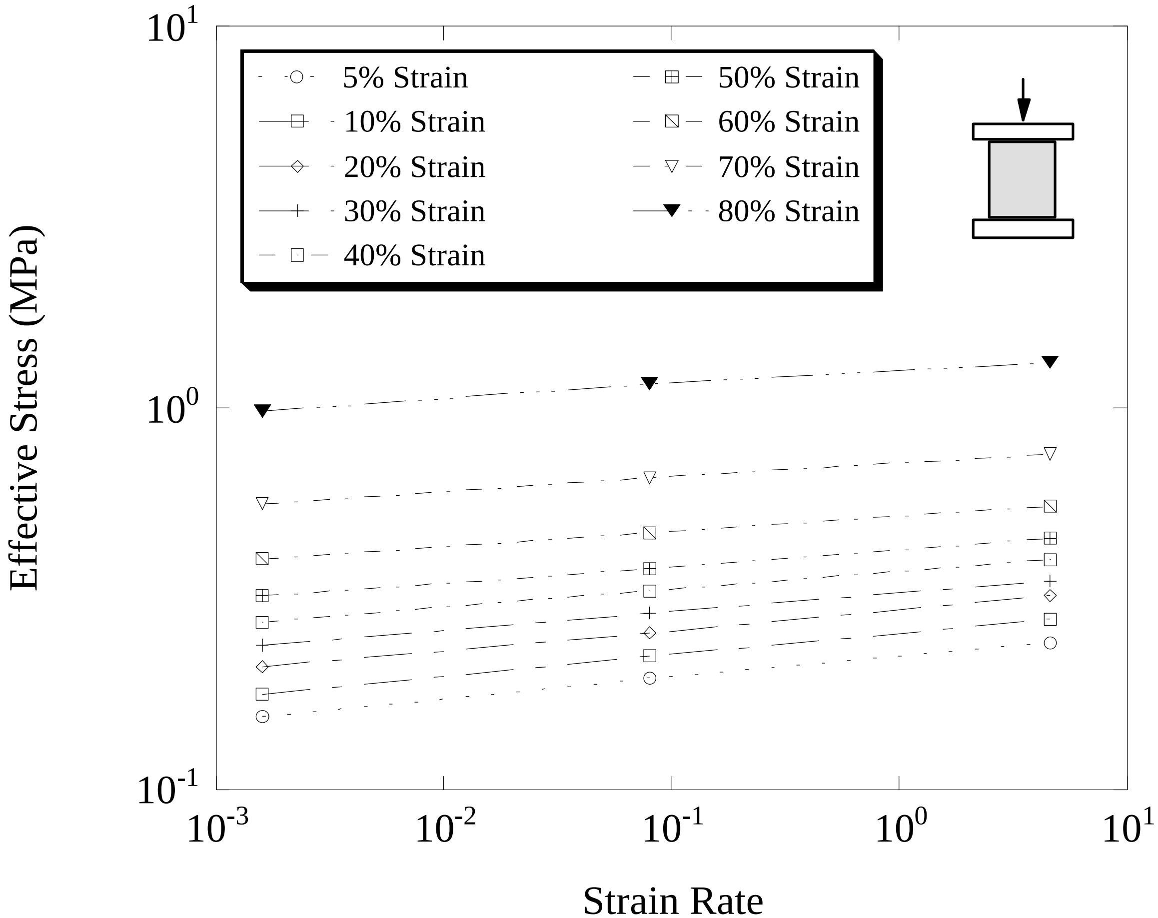
Williams-Landel-Ferry (WLF) Equation
(Williams et al, 1955)

$$L(T) = \exp\left[-\frac{C_1(T-T_r)}{C_2+T-T_r}\right]$$

PP foam (3.06 pcf)

C1=6.52ÞC, C2=468.7ÞC

Rate Dependency of PP foam (3.06 pcf)



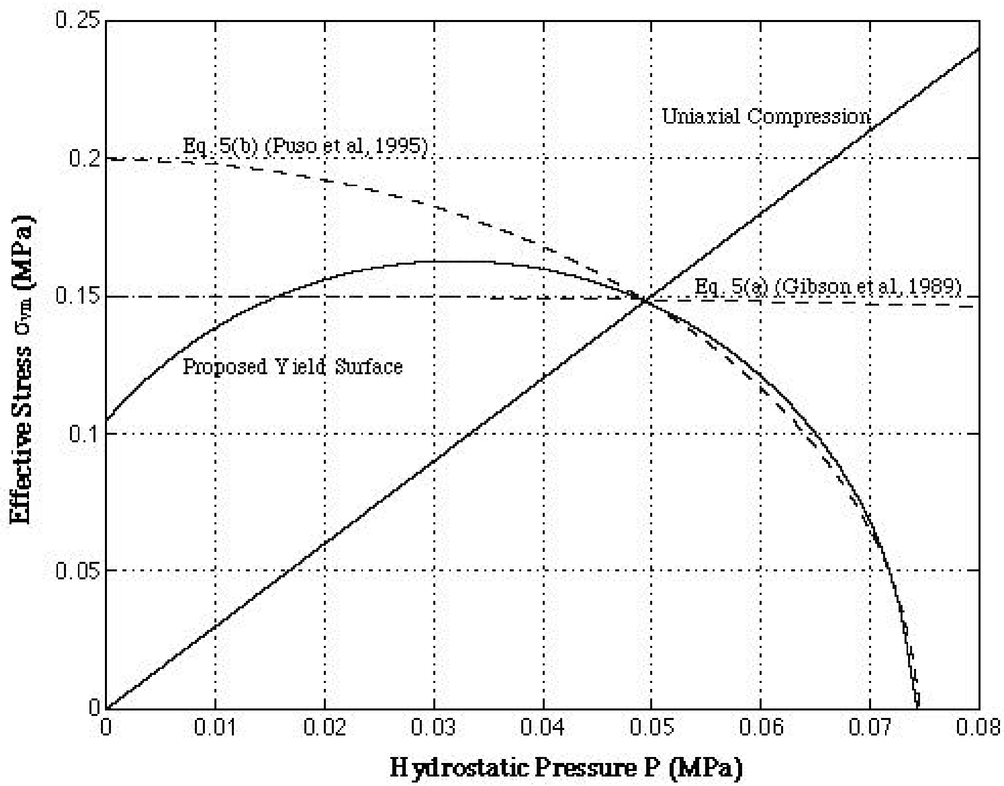
Nagy et al,1964

$$\dot{\bar{\varepsilon}}_p = D\left(\frac{f}{f_0}\right)^{\frac{1}{n}} \qquad n = a + b\bar{\varepsilon}_p$$

Combined temperature and rate effect

$$\sigma(\varepsilon) = \sigma_0(\varepsilon)L(T)\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{a+b\varepsilon}$$

Comparison of Yield Criterion



Plastic yield envelop (Gibson et al, 1989)

$$\frac{\sigma_{vm}}{\sigma_{ys}} = -\gamma\left(\frac{\rho^*}{\rho_s}\right)^{3/2}\left\{1 - \left[\frac{3p}{\sigma_{ys}\left(\rho^*/\rho_s\right)}\right]^2\right\}$$

Buckling surface (Puso and Govindjee 1995)

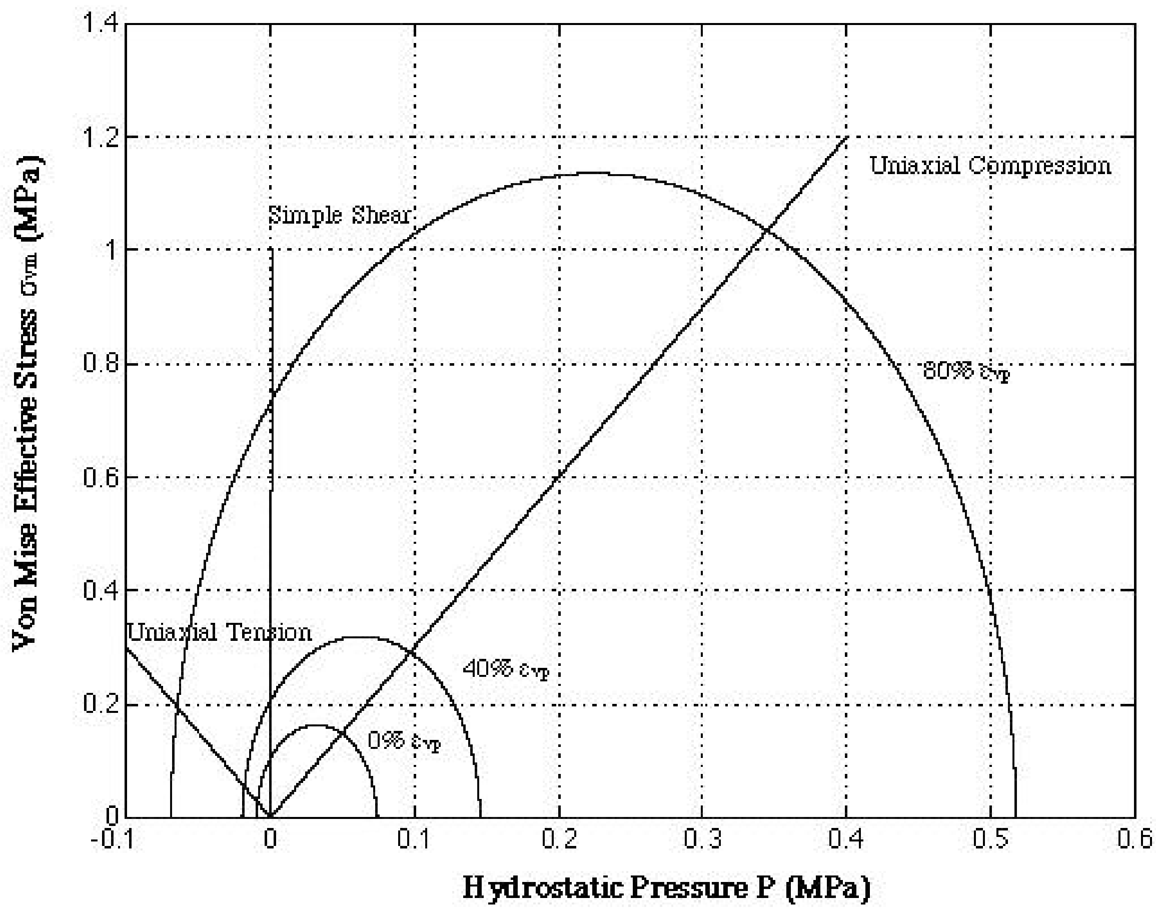
$$\sigma_{vm}^{2} + \frac{1}{R^2}\left(p^2 - h^2\right) = 0$$

Kinematic Hardening of Polypropylene Foam (3.06 pcf)

Kinematic hardening

$F = F(\sigma, \varepsilon_{vp}, \dot{\varepsilon})$

$g = g(\sigma, \varepsilon_{vp}, \dot{\varepsilon})$



Evolution of Yield Ellipse with Plastic Volumetric Strain

Stress Integration Procedure for Elastic-plastic Materials

Deformation decomposition $d\varepsilon = d\varepsilon^{e} + d\varepsilon^{p}$

Plasticity consistency condition

$$d\varepsilon = \boldsymbol{D}^{-1} d\sigma + \frac{\partial g}{\partial \sigma} d\lambda$$

$$\left\{\frac{\partial F}{\partial \sigma}\right\} d\sigma - A\, d\lambda = 0$$

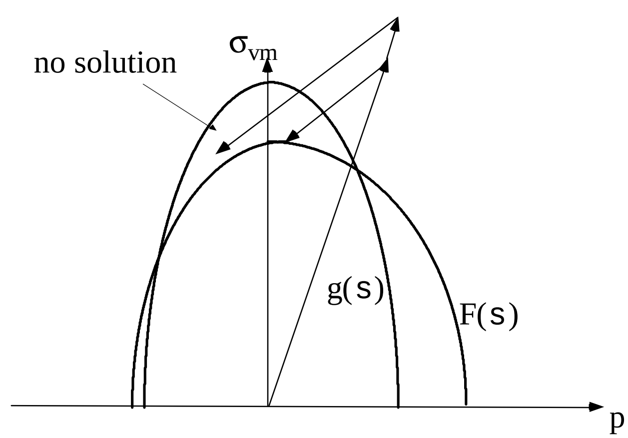
$$\Rightarrow \quad \begin{Bmatrix} d\varepsilon \\ 0 \end{Bmatrix} = \begin{bmatrix} \mathbf{D}^{-1} & \left\{\frac{\partial g}{\partial \sigma}\right\} \\ \left\{\frac{\partial F}{\partial \sigma}\right\}^{T} & -\mathbf{A} \end{bmatrix} \begin{Bmatrix} d\sigma \\ d\lambda \end{Bmatrix}$$

$$d\sigma = \mathbf{D}_{ep}^{*} d\varepsilon$$

$$\mathbf{D}_{ep}^{*} = \mathbf{D} - \mathbf{D}\left\{\frac{\partial g}{\partial \sigma}\right\}\left\{\frac{\partial F}{\partial \sigma}\right\}^{T} \mathbf{D}\left[A + \left\{\frac{\partial F}{\partial \sigma}\right\}^{T} \mathbf{D}\left\{\frac{\partial g}{\partial \sigma}\right\}\right]^{-1}$$

if $F \neq g$ $\quad \mathbf{D}_{ep}^{*}$ is a non-symmetric matrix

Non-unique solution for non-associative plastic flow
The stress return is not radial



Non-smooth Multisurface Plasticity

Plastic potential variation (assuming associative plastic potential)

$$\dot{F}_i = \frac{\partial F_i}{\partial \sigma} : \boldsymbol{C} : \dot{\varepsilon} - \dot{\lambda} \frac{\partial F_i}{\partial \sigma} : \boldsymbol{C} : \sigma = 0 \qquad \text{(i=1,2,...)}$$

If plastic yield and loading condition active

$$F_i(\sigma) = 0 \quad \text{and} \quad \frac{\partial F_i}{\partial \sigma} : \boldsymbol{C} : \dot{\varepsilon} > 0$$

Plasticity consistency condition

$$\dot{\lambda} = \frac{\partial_\sigma F_i : \boldsymbol{C} : \dot{\varepsilon}}{\partial_\sigma F_i : \boldsymbol{C} : \sigma}$$

(1) $\dot{\lambda}_i = 0$ (i=1,2,...), loading is not active;

(2) $\dot{\lambda}_i > 0$ $\quad \dot{\lambda}_i = \partial_\sigma F_i : \boldsymbol{C} : \dot{\varepsilon} / \partial_\sigma F_i : \boldsymbol{C} : \sigma$

(3) $\dot{\lambda}_i > 0$ for multiple surfaces $\dot{\lambda} = \max\left(\partial_\sigma F_i : C : \dot{\varepsilon} / \partial_\sigma F_i : C : \sigma, i = 1, 2, \ldots\right)$

Closest-point-projection (Simo et al, 1988)

In summary

$$C^{ep} = \begin{cases} \boldsymbol{C} & \text{if } \dot{\lambda} = 0 \\ \boldsymbol{C} - \dfrac{[\boldsymbol{C} : \sigma] \otimes [\boldsymbol{C} : \partial_\sigma F_i]}{\partial_\sigma F_i : \boldsymbol{C} : \sigma} (i = 1, 2, \ldots) & \text{if } \dot{\lambda} > 0 \end{cases}$$

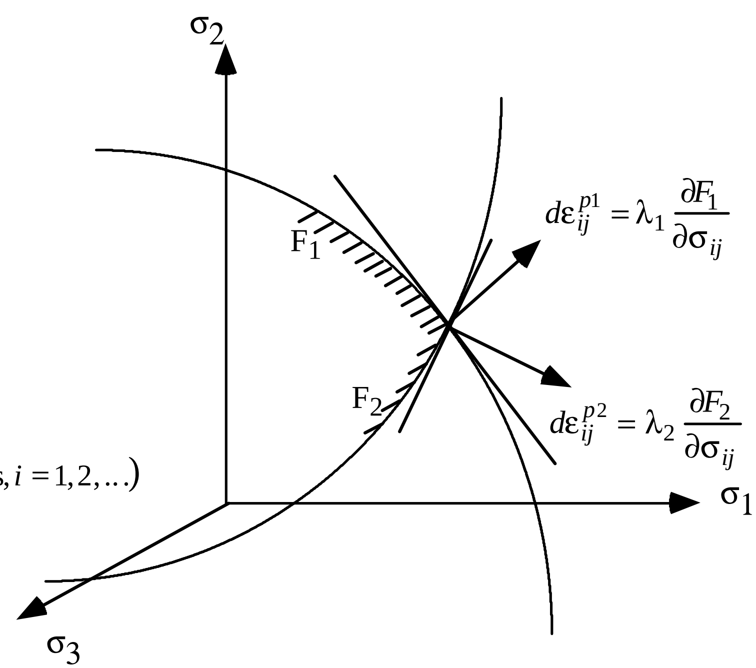


Illustration of a sigular point in yield Surface

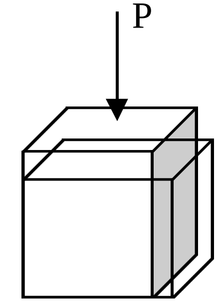
Polymeric Rigid Foam Plastic Flow Law

- Non-associative plastic potential $g(\hat{s},\phi) = \sqrt{\alpha p^2 + \sigma_{vm}^2}$

Plasticity consistency condition $\dot{\hat{\varepsilon}}_p = \dot{\lambda}\dfrac{\partial g(\hat{s},\phi)}{\partial \hat{s}}$

Plastic Poisson's Ratio $\dot{\varepsilon}_{xxp} = \dot{\varepsilon}_{yyp} = -\nu_p \dot{\varepsilon}_{zzp}$

Under uniaxial compression $\dot{\varepsilon}_{vp} = (1 - 2\nu_p)\dot{\varepsilon}_{zzp}$



$$\dot{\hat{\varepsilon}}_p = \dot{\lambda}\frac{1}{2g}\left[2\sigma_{vm}\frac{\partial \sigma_{vm}}{\partial \hat{s}} + 2\alpha p \frac{\partial p}{\partial \hat{s}}\right] \quad \text{or} \quad \dot{\hat{\varepsilon}}_p = \dot{\lambda}\frac{3}{2g}\left[\hat{\mathbf{s}} - \frac{2\alpha}{9} p \hat{\mathbf{I}}\right]$$

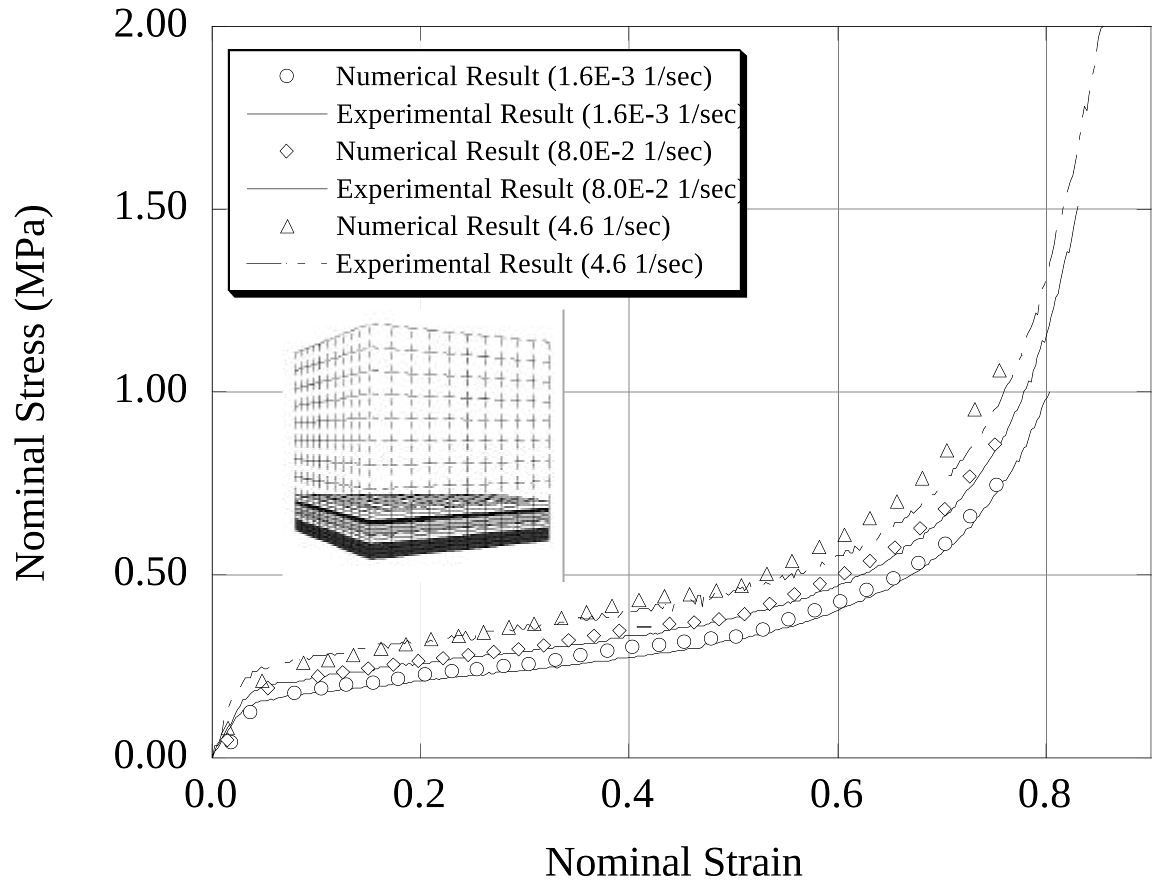
$$\dot{\varepsilon}_{zzp} = \dot{\lambda}\frac{3}{2g}\left(s_{zz} - \frac{2\alpha}{9} p\right)$$

$$\dot{\varepsilon}_{vp} = -\dot{\lambda}\frac{\alpha p}{g}$$

zero plastic Poisson's ratio

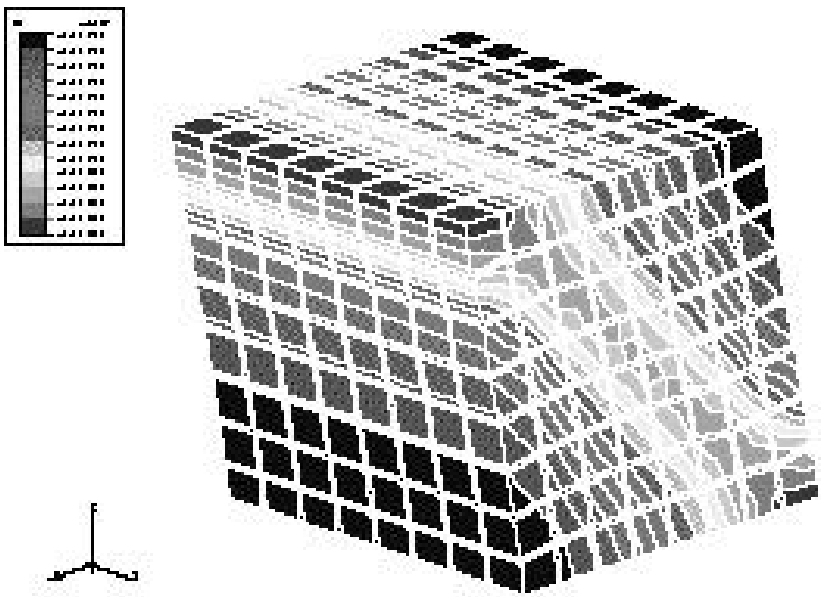
$$\alpha = \frac{9(1 - 2\nu_p)}{2(1 + \nu_p)} \quad \Rightarrow \quad g = \sqrt{\tfrac{9}{2} p^2 + \sigma_{vm}^2}$$

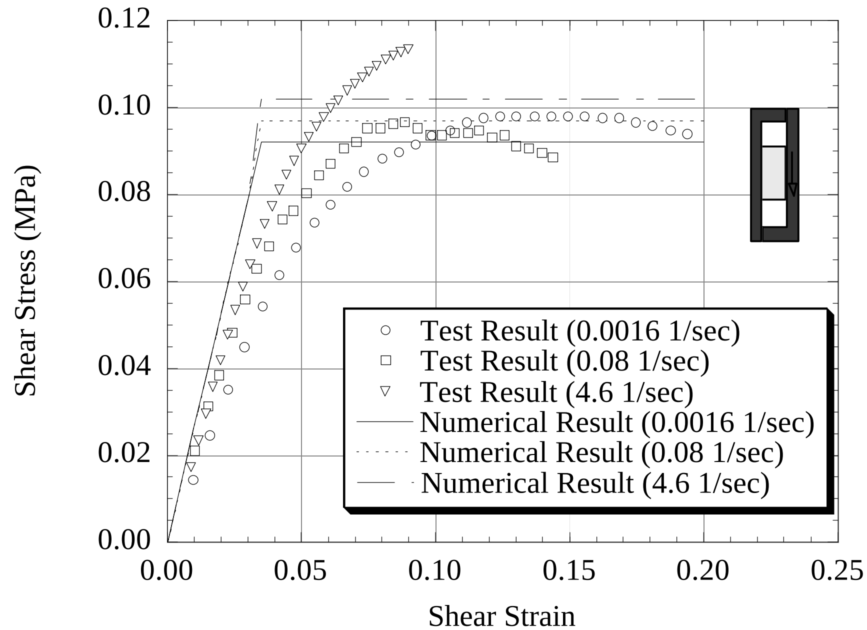
Model Validation under Uniaxial Compression



PP Foam (3.06 pcf)

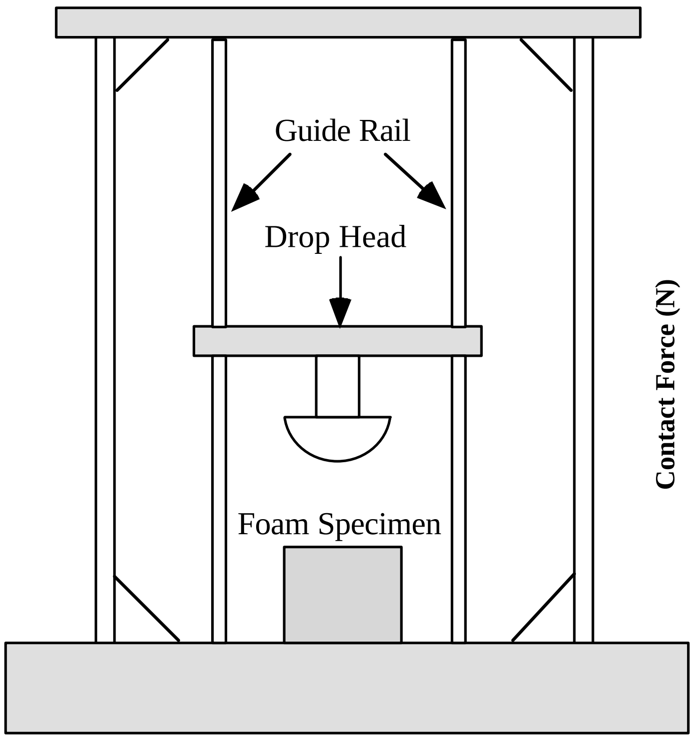
Model Validation under Simple Shear



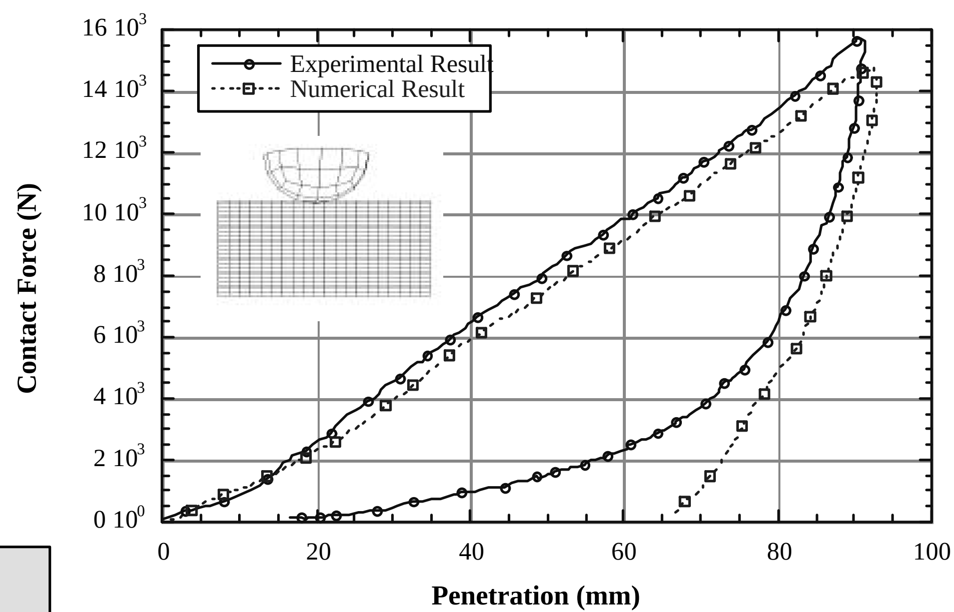


Polystyrene Foam (1.0 pcf)

Hemispherical Free Drop Test

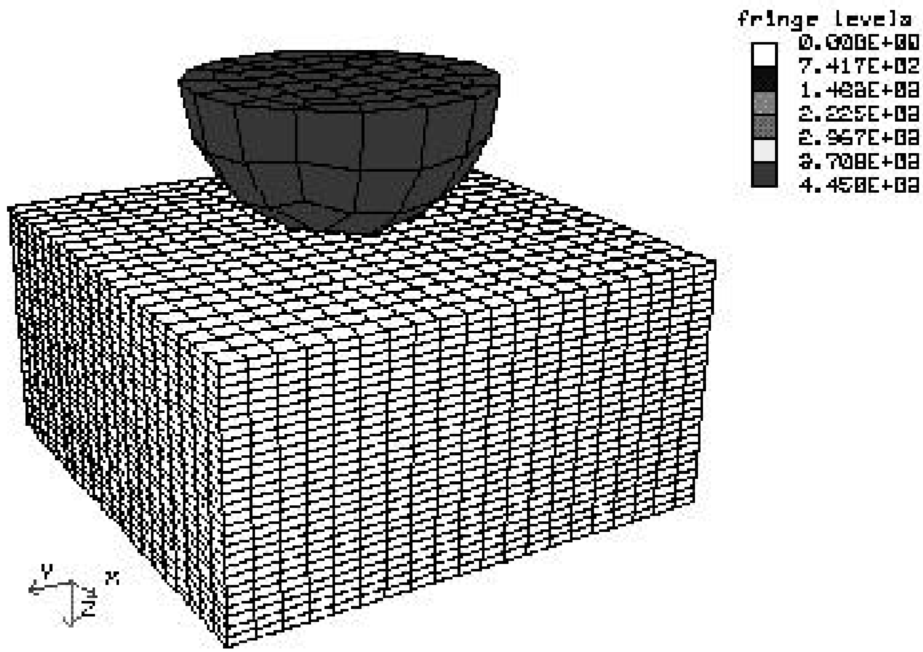


Free Drop Machine

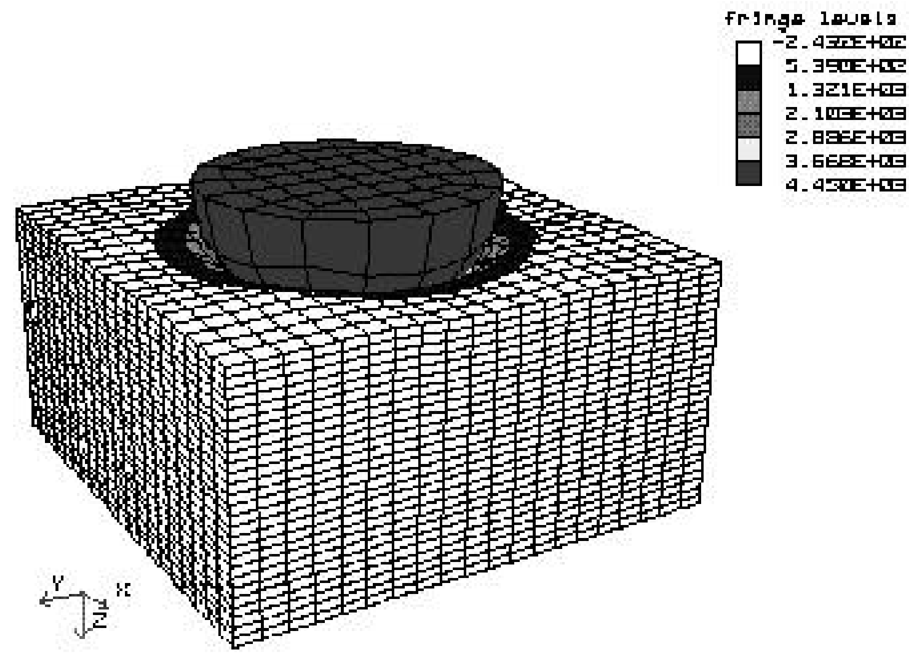


Indenter:ϕ127 mm, 22.2 kgm, 4.5m/sec
PP foam, 203x 203 x101 mm^3, 3.06 pcf

Hemispherical Free Drop Numerical Simulation

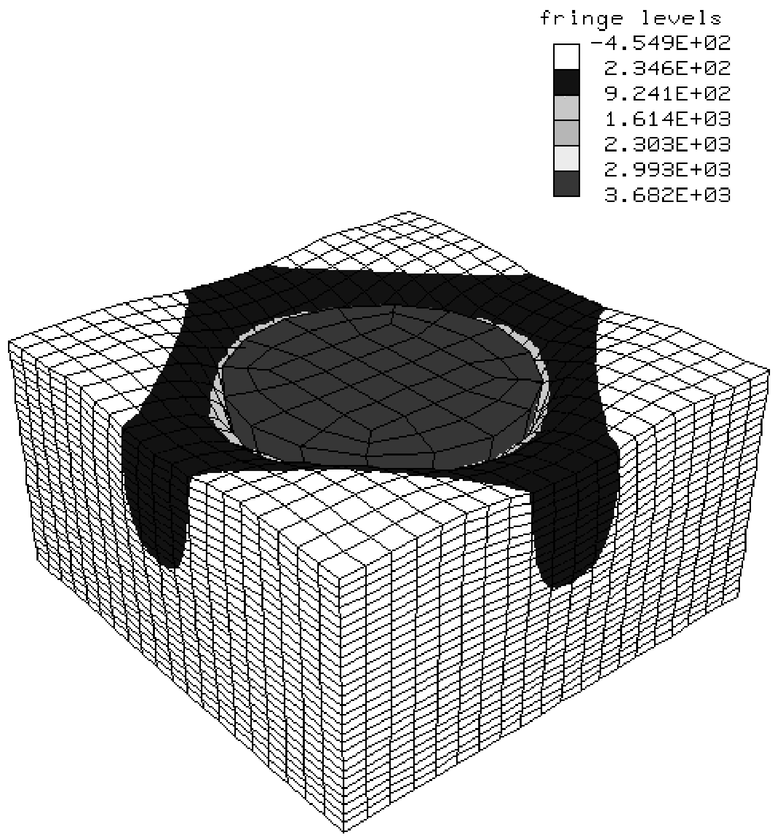


Original Mesh

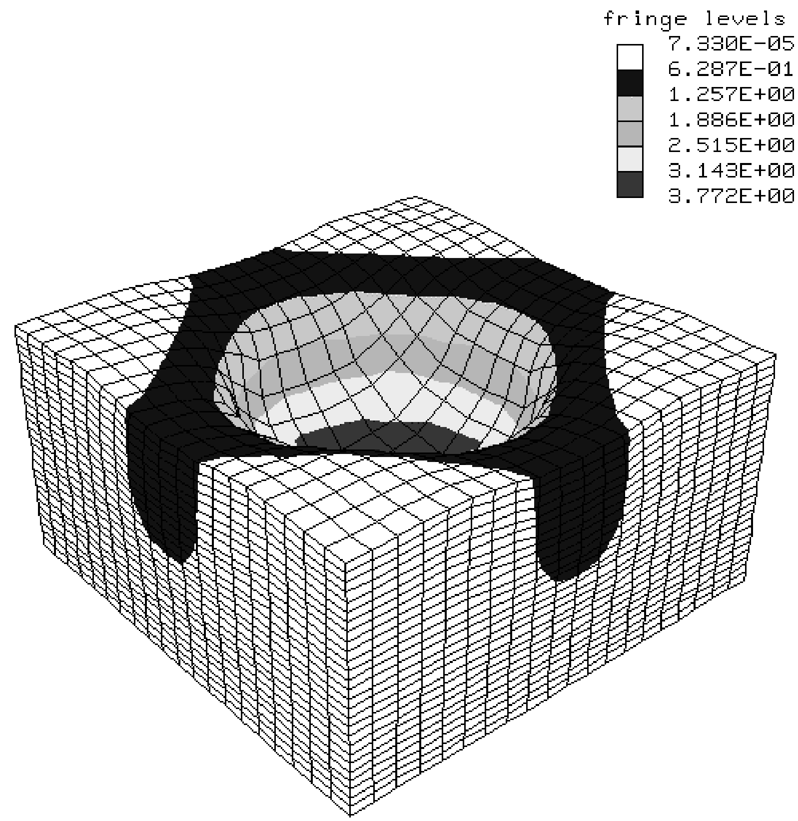


Deformed Mesh at t=9.0 ms

Hemispherical Free Drop Numerical Simulation



Deformed Mesh at t=16.0 ms



Effective plastic strain t=16.0 ms

Conclusion

- PU foams are flexible while PS and PP foams are rigid at 20^o C;
- Yield stress of polymeric foams are sensitive to strain rate, temperature and pressure;
- A phenomenological rate dependent single surface elasto-plastic yield criterion is developed and implemented in LS-DYNA3D program;

Future Work

- Experiment on polymeric foams under multiaxial loading;
- Constitutive modeling considering different failure mechanism;
- Validate rigid foam model under multiaxial loading;
- Couple homogenization constitutive modeling and LS-DYNA3D;
- Three-dimentional RVE modeling and analysis by using more powerful CT scanner.
- Three-dimentional foam design optimization.