Emerging Technology in Optimization
An Image Based Approach for CAE

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Major Collaborators

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QUINT Corporation
Graduate Students in CML
current

Emilio Silva
Shinji Nishiwaki & Susumu Ejima
J.H. Yoo
Bing-Chung Chen
Daichi Fujii
Minako Sekiguchi
CAE at Present

An Introduction to Image Based CAE
Current Approach in CAE

- Parametric (Geometry Based) CAD / CAE
  - Standard CAD Software is based on computational geometry by using parametric spline representation to define shape of a structure/domain
  - All of the existing CAD software are geometry based: Pro-E, UNIGRAPHICS, I-DEAS, CATIA, ....
  - In FEA, automatic mesh generation methods are also based on parametric representation of geometry
Lots of Sophistication and Big Success (2D, 3D?)

Realization of importance and profitability of Parametric Geometry Based CAD and CAE
Industry Standard in CAD

● Automotive Industry
  – UNIGRAPHICS in GM
  – I-DEAS in FORD
  – CATIA in CHRYSLER

● Leading companies have given up In-House CAD/CAE software

Paradigm Shift in 90s

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CAD/ CAE Acceptance
Not Yet

● 2D CAD is widely accepted, but 3D CAD is too sophisticated for majority of designers and manufacturers

● CAE becomes an accepted tool for single disciplinary analysis, but not sufficient to create new value except few areas ( crash, forming, etc )
MCAE + FCAE = CAE

- MCAE (Mechanical CAE)
- FCAE (Fluid CAE)

- Two separated CAE, Two separated Preprocessing Software, Two separated CAE analysis specialists ..... Difficulty of Integration for Design and Manufacturing
Trend in (M)CAE

- Major Software Houses
  - MSC/NASTRAN, PATRAN, ABAQUS (US, Europe, Japan)
  - ESI/PAMCRASH, PAMSTAMP, COMPOSIC (Europe, Japan, US)
  - Others: Swanson/ANSYS, LS/DYNA, ALGOR, .....MDI/ADAMS,

- Consolidation
  - Linear
  - Nonlinear
  - Impact
  - (Multi-Body)
  - Design Optimization

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Two Paths for Survival

- **Total Consolidated MCAE/FCAE**
  - Analysis (Linear, Nonlinear, Impact, Multi-body), Design Optimization, Simulation of Manufacturing Processes: *Total CAE*
  - ESI is a typical example: European’s Approach
  - MSC may follow: US for survival

- **Integration with (Imbedding to) CAD**
  - CAD software absorb linear CAE for *Design*
  - MCAE is a part of major CAD software
CAD Imbedded MCAE

- CAD absorbs CAE software
- Simulation of Design Feasibility
  - Based on only Linear Analysis
  - users are Designers rather than Analysts
  - Less Accuracy but user oriented
  - possibly Design Optimization capability
  - **DESIGN ORIENTED**
- Short Turn Around Time

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Effort in MCAE

● For Shortening of Turn Around Time by Simplifying FE Modeling Methods
  – CAD Linked Automatic Mesh Generation
  – Adaptive FE Methods (h and p elements)
  – Meshless FE Methods (ANALYSIS)

● Integration with Design Optimization
  – Design Sensitivity Analysis
  – Size, Shape, and Topology Optimization
Importance

- Shortening of *Modeling Time*
- Integration of MCAE and FCAE for
  - Design and Simulation of Manufacturing Process
  - Automatic Mesh Generation?  How?

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Image Based CAE

Originated From/Based On OPTISHAPE

Topology Optimization

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Topology Design Method

- Shape and Topology Design of Structures is transferred to Material Distribution Design (Bendsoe and Kikuchi, 1986)
TDM : 3D Shaping

Truly Three-dimensional shaping of a structure for optimum

Without parametric shape definition by splines

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Closely Related to Rapid Prototype
Layer by Layer Operation
Link with CAD for pixel operation
Utility of STL (SLC ) file
Typical Layerd Manufacturing Processes

Stereolithography

Selective Laser Sintering

Fused Deposition Modeling

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What we have done at University of Michigan in a DARPA Project?

Project MAXWELL
Two way communication between image and CAD data for Topology Optimization
OPTISHAPE : Material Design

A Homogenization Design Method for Topology of Structures and Materials

Poisson’s Ratio - 0.5
Image Manipulation

Gray Scale Image
Adjust Level of Gray Scale

Mosaic Filtering
pixel/voxel mesh
Pinching Filtering

Filtering Operation makes design change

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Image Algebra for Modeling

- $\chi^{isa} = 253$ or $252$, $\chi^{TRR} = 255$, $0 < \chi^{anat} < 252$
- Initial Scaffold defined by
- Accomplished in PV-Wave using Where mask
Resulted Finite Element Model

Scaffold/Bone Image

Scaffold/Bone Mesh

Done by Dr. Scott Hollister using Voxelcon2.0

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Image Based CAE

- Voxelcon : a Derivative of OPTISHAPE
  - CAD/CT/MRI Image Scan or Equivalent Ways
  - **Image Based** Automated CAE
    - Mesh Generation
    - Construction of Common Model for Multiple Analyses
    - Load/Support Condition
    - FE Analysis

- **Image Based Design & Optimization**
  - **OPTISHAPE** for topology.layout design
  - Rapid Prototype by Layered Manufacturing
  - Simulation of Material Processing (Casting etc)
Database : Image

- Rather than STL files, SLC files are considered
- SLC files are stored as IMAGES
- Images are then compressed
- 25K/slice x 500 = 7.5M
Image Regenerated
FEM Model

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Virtual Femur with Nail - Rendering

- 3D Surface Rendering of femur with nail
- Only screws show through femur
- Data ready for mesh generation
VOXELCON byproduct of OPTISHAPE
VOXELCON for I-DEAS
Quint Corporation

CAD Model by I-DEAS

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VOXELCON for I-DEAS (2)

75M Voxel Elements          9.4 M Voxel Elements

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OPTISHAPE
Quint Corporation

Topology (NK, A. Diaz)
Compliant Mechanisms (NK, S. Nishiwaki)
Shape (H. Azekami)
Size (H. Miura)
Extension of OPTISHAPE

● Structural Design
  – Static and Dynamic Stiffness Design
  – Control Eigen-Frequencies
  – Design Impact Loading
  – Elastic-Plastic Design

● Material Microstructure Design
  – Young’s and Shear Moduli, Poisson’s Ratios
  – Thermal Expansion Coefficients

● Flexible Body Design

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New Extension of OPTISHAPE

Piezocomposite and Piezoelectric Actuator Design

For Creation of New Value

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Introduction

Force
Mechanical Energy
Displacement

Piezoelectric Material

Electric potential
Electrical Energy
Electric charge

Examples: Quartz (natural)
Ceramic (PZT5A, PMN, etc…)
Polymer (PVDF)
Applications

Pressure sensors
accelerometers
actuators,
acoustic wave generation
    ultrasonic transducers, sonar, hydrophones
etc...
Constitutive Equations of Piezoelectric Medium

\[
\begin{align*}
T_{ij} &= c_{ijkl}^E S_{kl} - e_{kij} E_k \\
D_i &= \varepsilon_{ik}^S E_k + e_{ikl} S_{kl}
\end{align*}
\]

- \( T_{ij} \) - stress
- \( S_{kl} \) - strain
- \( E_k \) - electric field
- \( D_i \) - electric displacement

\( c_{ijkl}^E \) - stiffness property
\( e_{ikl} \) - piezoelectric strain property
\( \varepsilon_{ik}^S \) - dielectric property
Topology Design

- Change the topology of microstructure (material) or structure (transducer)
- Improvement in the performance of piezocomposite materials; design of new kinds of transducers for different applications

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Many Approaches: MDM

Simple: Density Method

Material Design

\[ E_{ijkl} = x^p E_{ijkl}^0 \]

fraction of material in each point

General: Homogenization Method

Structure Design

A point with material

A point with no material

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Performance Characteristics 1

Hydrophones (Hydrostatic Mode)

• Hydrostatic Coupling Coefficient ($|d_h|$):

$$|d_h| = d_{13} + d_{23} + d_{33}$$

• Figure of Merit ($d_h g_h$):

$$d_h g_h = \frac{d_h^2}{\epsilon_{33}^T}$$

• Hydrostatic Electromechanical Coupling Factor ($k_h$):

$$k_h = \sqrt{\frac{d_h^2}{\epsilon_{33}^T s_h^E}}$$
Performance Characteristics 2

Ultrasonic Transducers (Thickness Mode)

• Electromechanical Coupling Factor ($k_t$):

$$k_t = \sqrt{\frac{e_{33}^2}{c_{33}^D \varepsilon_{33}^S}}$$

• Impedance ($Z$):

$$Z = \sqrt{\rho c_{33}^D}$$

• Longitudinal Velocity ($v_t$):

$$v_t = \sqrt{\frac{c_{33}^D}{\rho}}$$
Reference unit cell for comparison: 2-2 piezocomposite

(Poled in the 3 direction)
2D Piezocomposite Unit Cell
ultrasonic transducer

Initially

Optimized Microstructure

Piezocomposite

$2\times2$ piezocomposite

$\begin{array}{c}
\text{Initially} \\
\text{Optimized Microstructure} \\
\text{Piezocomposite}
\end{array}$

$k_t = \sqrt{\frac{e_{33}^2}{c_{33}^D \varepsilon_{33}^S}}$

Suggested Transducer

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Improvement

Improvement in relation to the 2-2 piezocomposite unit cell:

- $|d_h|$: 2.5 times
- $d_{hg}$: 4.2 times
- $k_t$: 1.13 times

\[ \rho \downarrow \Rightarrow Z \downarrow \quad v_t \ (\approx \text{same}) \]

stiffness constraint: $c_{11}^E > 8.10^8 \text{ N/m}^2$
2D Piezocomposite Unit Cell

hydrophone

Initially

Optimized Microstructure

Piezocomposite

polymer

air

polymer

Suggested Transducer

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Improvement in relation to the 2-2 piezocomposite unit cell:

- $|d_h|$: 2.8 times
- $d_h g_h$: 7.1 times
- $k_t$: 1.13 times

$\rho \downarrow \Rightarrow Z \downarrow$ $v_t (\cong \text{same})$

stiffness constraint: $c_{11}^E > 8.10^8 \text{N/m}^2$
Experimental Verification

• Rapid Prototyping: Stereolithography Technique

Optimized Transducer

Reference Transducer

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## Experimental Result

![Diagram of polymer part and bar of PZT5A](image)

**Measured Performances**

<table>
<thead>
<tr>
<th></th>
<th>$d_h (pC/N)$</th>
<th>$d_h g_h (fPa^{-1})$</th>
<th>$k_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>9.1</td>
<td>13.2</td>
<td>0.69</td>
</tr>
<tr>
<td>Optimized</td>
<td>246.</td>
<td>10400.</td>
<td>0.70</td>
</tr>
<tr>
<td>(Simulation)</td>
<td>(229.)</td>
<td>(10556.)</td>
<td>(0.66)</td>
</tr>
</tbody>
</table>
2D Piezocomposite Unit Cell
hydrophone

Initially

Optimized Microstructure

Piezocomposite

PZT5A

“optimized porous ceramic”
Improvement in relation to the 2-2 piezocomposite unit cell:

\[ |d_h|: \ 3. \ \text{times} \]
\[ d_h g_h: \ 9.22 \text{ times} \]
\[ k_h: \ 3.6 \text{ times} \]

stiffness constraint: \( c_{33}^E > 1.10 \times 10^{10} \text{N/m}^2 \)
Piezocomposite Manufacturing

Microfabrication by coextrusion technique

Theoretical unit cell

Fugitive

Ceramic
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Ceramic Feedrod Reduction Zone Extrudate

Crumm and Halloran (1997)
### Measured Performances

<table>
<thead>
<tr>
<th></th>
<th>$d_h (pC/N)$</th>
<th>$d_hg_h (fPa^{-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid PZT</td>
<td>68.</td>
<td>220.</td>
</tr>
<tr>
<td>Optimized</td>
<td>308.</td>
<td>18400.</td>
</tr>
<tr>
<td>(Simulation)</td>
<td>(257.)</td>
<td>(19000.)</td>
</tr>
</tbody>
</table>

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3D Piezocomposite Unit Cell
hydrophone

Poled in the z direction

piezoceramic
3D Piezocomposite Unit Cell

hydrophone

piezoceramic

Polarized in the z direction
OPTISHAPE

Compliant Mechanism Design

A New Release

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Structural Flexibility

Flexibility can provide higher performance or additional function if we can specify the flexible mode appropriately.
Kinematic Synthesis

Lumped compliant mechanism

Based on traditional rigid body kinematics

Lumped compliance (Pivot) $\Rightarrow$ Stress concentration

Continuum Synthesis

Distributed compliant mechanism

Based on the topology optimization method

Ananthasuresh et al. (1994, 1995), Frecker et al. (1997)
Flexibility and Stiffness

Maximize \( L^2(u^1) = \int_{\Gamma_t^2} t^2 \cdot u^1 d\Gamma \)

Mutual Mean Compliance (MMC)

Flexibility at \( \Gamma_t^2 \)

Minimize \( L^1(u^1) = \int_{\Gamma_t^1} t^1 \cdot u^1 d\Gamma \)

Mean Compliance (MC)

Stiffness at \( \Gamma_t^1 \)

Applied traction

Dummy traction

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Formulation of Mutual Stiffness

Minimize

\[ L^2 \left( u^1 \right) = \int_{\Gamma_{t^2}} t^2 \cdot u^1 \, d\Gamma \]

Slide along the line

Dummy traction

Stiffness at \( \Gamma_{t^2} \) with respect to \( t^1 \)

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Compliant Mechanism Design

- Kinematic function
  - Flexibility
- Structural function
  - Stiffness

Maximize (MMC)

Minimize (\(\sum_{MC}\))

Applied force

Reaction force

Mutual stiffness

Constrained Motion

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Multicriteria Optimization

Flexibility \rightarrow \text{Maximize (MMC)}

Trade off

Stiffness \rightarrow \text{Minimize } \sum \text{ (MC)}
Multi-objective Functions (1)

Typical methods to deal with multi-objective problems

- The weighting method
- The $\varepsilon$-constraint method
- The goal programming method

MMC ----> Infinite !

Nash’s Optimum

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Multi-objective Functions (2)

(1) Single flexibility case

(a) Maximize \[ \frac{MMC}{\sum MC} \]

(b) Maximize \[ w \log(MMC) - (1-w) \log(\sum MC) \]

\[ \text{Variation} = w \frac{\delta MMC}{MMC} - (1-w) \frac{\sum \delta MC}{\sum MC} \]
Multi-objective functions (3)

(2) Displacement single flexibility case

Minimize \( \sum MC \)

\( MMC \rightarrow Constraint \)

(3) Multi-flexibility case

Maximize \(-\frac{1}{C_f} \log(\sum \exp(-C_f^i MC)) \)

\( \frac{1}{C_s} \log(\sum \exp(C_s^j MC)) \)

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Compliant Gripper (1)

Unconstrained single flexibility

Extended Design Domain $D$

Design domain

$\Omega_s = 20\%$

$\Omega_s = 30\%$

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Torsional Compliant Mechanism

Design domain

Extracted image design

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Constrained Compliant Gripper

Constrained single flexibility

Extended Design Domain

Design domain

Constrained case

Optimal configurations ($\Omega_s=20\%$)

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Unified Design of Structures and Mechanisms

Current design approach

Unified design approach

Sub frame

Large Friction force

Large change of chamber angle

Unified parts

Small change of chamber angle

Small Friction force

Strut-type suspension

Unified parts

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Multi-flexibility Compliant Mechanism (1)

Extended Design Domain $D$

Optimal configurations ($\Omega_s=30\%$)

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Multi-flexibility Compliant Mechanism (2)

\[\leftarrow\]: Applied force
\[\leftrightarrow\]: Direction of deformation

(1) Deformed shape

(2) Deformed shape

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Flextensional Actuator Design

Piezoceramic + Flexible coupling structure

Coupling Structure → Mechanical Transform

- Amplify output displacement
- Change displacement direction
- Provide stiffness

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OPTISHAPE

Actuator Design

A New Capability to be Implemented
Examples of Flextensional Actuators:

Low-frequency applications are considered (inertia effect is neglected)
Maximize output displacement ($\Delta u$) 

\[
\text{Max } \left\{ \phi_2 \right\}^T \left\{ Q_1 \right\} \\
\text{(mean transduction)}
\]

Maximize blocking force 

\[
\text{Min } \left\{ U_3 \right\}^T \left\{ -F_2 \right\} \\
\text{(mean compliance)}
\]
Example 1

Multilayer actuator (common design)

Design region considered (1/4 symmetry)

Brass
FEM verification:
Example 2

(1/4 symmetry)

FEM verification:

Design Domain (Brass)  B

Δu

PZT

Δw = 0.5

Optimal topology

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Example 3

Design Domain (Brass)

Δu

(1/2 symmetry)

Piezoceramic

Optimal topology ($\Omega_s = 25\%$)

Image interpretation

$w=0.9$

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Structural Optimization in Magnetic Fields

Future OPTISHAPE Capability
Shape of H-magnet

Cross Sectional View  A quarter Model for Analysis
Optimal Shape for Maximizing Total Potential Energy

Design Domain for Optimization (324 elements)

Optimal Shape with 60% Volume Constraint

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Analysis of the Optimal Shape

Increase the value of Flux Densities in Design Domain (25 - 40%)
Stabilize the Flux Densities
Optimal Shape for Prescribed Uniform Fields (432 element model)

Prescribed $B_x = -0.18$

<table>
<thead>
<tr>
<th></th>
<th>Prescribed $B_x = -0.18$</th>
<th>Prescribed $B_x = -0.18$, $B_y = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ave. of x components</td>
<td>-0.18298E+00</td>
<td>-0.16645E+00</td>
</tr>
<tr>
<td>Ave. of y components</td>
<td>0.69363E-01</td>
<td>0.38788E-01</td>
</tr>
<tr>
<td>Stand. Dev. of x components</td>
<td>0.62515E-01</td>
<td>0.57342E-01</td>
</tr>
<tr>
<td>Stand. Dev. of y components</td>
<td>0.68198E-01</td>
<td>0.46641E-01</td>
</tr>
</tbody>
</table>

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Optimal Shape of the Design Domain for Prescribed Uniform Fields
(3-layer, 432 element model)

Prescribed $B_x = -0.20$

| Ave. of x components | Prescribed $B_x = -0.18$ | Prescribed $B_x = -0.18$
<table>
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<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-0.20712E+00$</td>
<td>$-0.20706E+00$</td>
</tr>
<tr>
<td>Ave. of y components</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$0.44251E-01$</td>
<td>$0.44251E-01$</td>
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<tr>
<td>Stand. Dev. of x components</td>
<td>$0.87911E-01$</td>
<td>$0.69931E-01$</td>
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<tr>
<td>Stand. Dev. of y components</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$0.57347E-01$</td>
<td>$0.57347E-01$</td>
</tr>
</tbody>
</table>

Prescribed $B_x = -0.20$, $B_y = 0.05$

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Research Issue in OPTISHAPE

Material Design Optimization
Young’s & Shear Moduli
Poisson’s Ratios
Thermal Expansion Coefficients
Electro-magnetic Properties
Following To
Dr. O. Sigmund
Technical University of Denmark

Jun Ono Fonseca
and
Bing-Chung Chen
Three-Phrase Material Design

- Artificial material mixing rule

$$E = \rho \left[ mE^{(1)} + (1 - m)E^{(2)} \right]$$

$$\alpha = \left[ m\alpha^{(1)} + (1 - m)\alpha^{(2)} \right]$$

- Design layout of two solid phases and void simultaneously

- Possible overlap between two phases when $$m \neq 1$$ or $$m \neq 0$$
Benchmarking
with existing 2-phase bound

- \( E^{(1)} = 10, \nu^{(1)} = 0.3, \alpha^{(1)} = 1.0, V = 50\% \)
- \( E^{(2)} = 1.0, \nu^{(2)} = 0.3, \alpha^{(2)} = 10.0, V = 50\% \)

- “Good” expansion material surrounded by “Bad” expansion material results in the “Worst” expansion composite

\[
\alpha^H = \begin{bmatrix}
6.5 \\
0 \\
6.48
\end{bmatrix}
\]
Negative Expansion
in the vertical direction

$E^{(1)} = 10, \nu^{(1)} = 0.3, \alpha^{(1)} = 1.0, V = 25\%$

$E^{(2)} = 1.0, \nu^{(2)} = 0.3, \alpha^{(2)} = 10.0, V = 10\%$

.Void

$\alpha^H = \begin{bmatrix} 2.0 & 0 \\ 0 & -1.1 \end{bmatrix}$

Re-entrant structure

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Near Zero Expansion in the Horizontal Direction

- Almost disconnected in the y
- Again, very complicated structure in terms of manufacturing

\[ E^H = \begin{bmatrix} 1.0 & 0.0008 & 0 \\ 0.6 & 0.0008 & 0.0011 \\ 0 & 0 & 0.0005 \end{bmatrix} \]

\[ \alpha^H = \begin{bmatrix} 0.08 & 0 \\ 0 & 1.6 \end{bmatrix} \]

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Construction of three-phase material by two stage design

- Given distribution of phase 1, design phase 2 distribution, excluding the domain occupied by phase 1
- Mark phase 2 as exclusion, design phase 1
- The final micro-structure should be non-complex and easy to manufacture.
Example: Reinforcement Design

Given a material with properties to be improved

\[
E^H = \begin{pmatrix}
0.9 & 0.32 & 0 \\
0.32 & 0.6 & 0 \\
0 & 0 & 0.324
\end{pmatrix}
\]

\[E_1 = 10, \nu = 0.3\]
Find the Optimal Distribution of Reinforcement. Phase 2

- Add the reinforcement in a particular pattern to achieve the design goal.

E₂ = 5, ν = 0.3
The Optimal Distribution of Reinforcement

- The reinforcement phase is non-overlapping with the original phase.
Superimpose the two non-overlapping phases

- Super-impose the two phases to achieve the design goal (increased rigidity and negative Poisson’s ratio)

\[
E^H = \begin{pmatrix}
10 & -0.27 & 0 \\
-0.27 & 1.0 & 0 \\
0 & 0 & 0.32
\end{pmatrix}
\]

- \( E_1 = 10, \nu = 0.3 \)
- \( E_2 = 5, \nu = 0.3 \)

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Negative expansion in the horizontal direction

\[ E^{(1)} = 10, \nu^{(1)} = 0.3, \alpha^{(1)} = 1.0, V = 30\% \]

\[ E^{(2)} = 10, \nu^{(2)} = 0.3, \alpha^{(2)} = 10.0, V = 25\% \]

- Stretch in the y due to temperature rise
- Shrink in the x due to Poisson’s effect
- Unusual CTE material must encompass structure-like mechanism
Negative expansion in the vertical direction

Initial phase 1 distribution (inverted honeycomb)

\[
\alpha^H = \begin{pmatrix}
0.8 & 0 \\
-0.5 & 0
\end{pmatrix}
\]

- \( E^{(1)} = 5, \nu^{(1)} = 0.3, \alpha^{(1)} = 10, V = 35\% \)
- \( E^{(2)} = 10, \nu^{(2)} = 0.3, \alpha^{(2)} = 5, V = 25\% \)

Void
Near zero Thermal Expansion

Phase 1

Phase 2

\[ \alpha^H = \begin{bmatrix} 0.22 & 0 \\ 0 & 0.21 \end{bmatrix} \]

- \( E^{(1)} = 5, \nu^{(1)} = 0.3, \alpha^{(1)} = 10, V = 25\% \)
- \( E^{(2)} = 10, \nu^{(2)} = 0.3, \alpha^{(2)} = 5, V = 30\% \)

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Topology Optimization Algorithm Examinations / Research

Various Filtering Schemes Proposed using SLP

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Example 1

Design Domain

P 24

10

Va: 10%
Va: 20%
Va: 30%

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Example 2

Design domain

$E = 100 \text{Gpa}$

$P = 1$

$V_a: 37.5\% \quad CE: 0.9454$

$V_a: 25\% \quad CE: 1.422$

$V_a: 50\% \quad CE: 0.7157$

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Example 3

Design domain

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Maximization of Attractive Force

\[ g = \sum_{i=1}^{N} \sum_{k=1}^{4} \frac{\rho_i \rho_k}{r_{ik}^2} \rightarrow \text{max} \]

\[ \rho_i = 1 - \alpha_i \beta_i \]

\[ r_{ik} : \text{distance} \]

\[ N : \text{number of element} \]

\[ w_g : \text{weight} \quad \bar{g} = g / g_{\text{ini}} \]

Objective

\[ f = C \sqrt{1 - (w_g \bar{g})^2} \]
Attractive Forces

\[ r_{ik} = 1 \]

\[ \rho_i = 1 \]

\[ g_i = 4 \]
\[ g_i = 3 \]
\[ g_i = 2 \]
\[ g_i = 1 \]
\[ g_i = 0 \]

\[ \rho_i = 0.5 \]

\[ g_i = 1 \]
\[ g_i = 0.75 \]
\[ g_i = 0.5 \]
\[ g_i = 0.25 \]
\[ g_i = 0 \]

\[ \rho_i = 0 \]

\[ g_i = 0 \]
\[ g_i = 0 \]
\[ g_i = 0 \]
\[ g_i = 0 \]
\[ g_i = 0 \]

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Example 1A

$w_g = 0.1$

Design Domain

$P$

10

24

Va: 10%

Va: 20%

Va: 30%

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Example 2A

Design domain

\[ E = 100 \text{Gpa} \]
\[ P = 1 \]

\[ w_g = 0.1 \]

\[ P = 10 \]

Va: 37.5%  CE: 0.9576

Va: 25%  CE: 1.499

Va: 50%  CE: 0.7275

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Example 3A

\[ w_g = 0.1 \]
Gray Scale Penalty

\[
\begin{bmatrix}
D^H_{1,1}
\end{bmatrix}
\begin{bmatrix}
D_{1,1}
\end{bmatrix}
\]

\[\rho = 1 - \alpha \beta \quad (\alpha = \beta)\]
Example 1B

$w_g = 0.1, \ p = 0.8$

Design Domain

$P$ 24

10

Va: 10%  Va: 20%  Va: 30%

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Example 2B

Design domain

\[ E = 100 \text{Gpa} \]
\[ P = 1 \]

\[ w_g = 0.1, \quad p = 0.8 \]

\[ V_a : 37.5\% \quad \text{CE: 1.011} \]

\[ V_a : 25\% \quad \text{CE: 1.642} \]

\[ V_a : 50\% \quad \text{CE: 0.7530} \]

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Example 3B

\[ w_g = 0.1, \quad p = 0.8 \]
Perimeter Control
(Muriel BECKERS, 1997)

\[ p_r = \sum_{i=1}^{N} \sum_{k=1}^{4} l_{ik} |\rho_i - \rho_k| \rightarrow \text{min} \]

<table>
<thead>
<tr>
<th>( \rho_2 )</th>
<th>( \rho_i )</th>
<th>( \rho_3 )</th>
<th>( \rho_4 )</th>
</tr>
</thead>
</table>

\( \rho_i = 1 - \alpha_i \beta_i \)

\( l_{ik} \) : Length of Common Boundary

\( N \) : number of element

\( w_g \): weight\((g)\), \( w_p \): weight\((p_r)\), \( \bar{g} = g / g_{ini} \), \( \bar{p}_r = p_r / L \)

Objective

\[ f = C \sqrt{1 - (w_g \bar{g})^2 + (w_p \bar{p}_r)^2} \]

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Perimeter Length

\[ \rho_i = 1 \]

\[ p_{ri} = 0 \quad  p_{ri} = 1 \quad  p_{ri} = 2 \quad  p_{ri} = 3 \quad  p_{ri} = 4 \]

\[ \rho_i = 0.5 \]

\[ p_{ri} = 0 \quad  p_{ri} = 0.5 \quad  p_{ri} = 1 \quad  p_{ri} = 1.5 \quad  p_{ri} = 2 \]

\[ \rho_i = 0 \]

\[ p_{ri} = 4 \quad  p_{ri} = 3 \quad  p_{ri} = 2 \quad  p_{ri} = 1 \quad  p_{ri} = 0 \]
Example 1C

\[ w_g = 0.1, \ p = 0.8, \ w_p = 0.01 \]

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Example 2C

$w_g = 0.1, \ p = 0.8, \ w_p = 0.01$

Design domain

$E = 100Gpa$

$P = 1$

P

10

Va: 37.5\% \ CE: 1.009

Va: 25\% \ CE: 1.632

Va: 50\% \ CE: 0.7545
Example 3C

Design domain

\[ w_g = 0.1, \ p = 0.8, \ w_p = 0.005 \]

\[ w_g = 0.1, \ p = 0.8, \ w_p = 0.01 \]
Post Processing of OPTISHAPE

Smooth Surface Extrusion
Example: Caliper
Mesh From CT Scan
150,000 3-D Elements

9% Weight Reduction
Comparison by Sections

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Interpolation Functions

Meshless Approach

\[ f(x) = \sum_{j=1}^{n} c_j \Phi_j(x) \quad \text{where} \quad c_j = f(x_j) \]

\[ \Phi_j(x) \quad \text{is defined with non-polynomial function:} \]

\[ \Phi_j(x) = a_o(x)w_j(x) \]

where \[ w_j(x) = w(x - x_j) \quad \text{and} \quad w(x) = \exp(-\alpha x^2) \]
Approximation Functions (2)

\[ f(x) = \sum_{j=1}^{n} c_j \Phi_j(x) \]

To Determine \( \Phi_j(x) \), which yield \( k \)-th degree polynomial, let’s assume:

\[ \Phi_j(x) = \left\{ a_o(x) + x_j a_1(x) + \ldots + x_j^k a_k(x) \right\} w_j(x) \]

\[ = \begin{bmatrix} a_o(x) \\ \vdots \\ a_k(x) \end{bmatrix} \begin{bmatrix} 1 \\ \ldots \\ x_j^k \end{bmatrix} w_j(x) \]

\[ f(x) = f_o + f_1 x + \ldots + f_k x^k \]

Solve for \( \{a_o(x) \ldots a_k(x)\} \)
Approximation Functions (3)

\[ \begin{align*}
\{a_o(x) & \} = \left[ \sum_{j=1}^{n} w_j(x) \quad \cdots \quad \sum_{j=1}^{n} x_j^k w_j(x) \right]^{-1} \\
\{a_k(x) & \} = \left[ \sum_{j=1}^{n} x_j^k w_j(x) \quad \cdots \quad \sum_{j=1}^{n} x_j^{2k} w_j(x) \right]^{-1} \\
\{ & \} = \left[ \sum_{j=1}^{n} x_j^k w_j(x) \quad \cdots \quad \sum_{j=1}^{n} x_j^{2k} w_j(x) \right]^{-1} \\
\end{align*} \]

Recall: \( \Phi_j(x) = a_o(x)w_j(x) \)

\[ \begin{align*}
\Phi_j(x) = \left\{ \begin{array}{c}
1 \\
\vdots \\
x_j^k \\
\end{array} \right\} = \left[ \sum_{j=1}^{n} w_j(x) \quad \cdots \quad \sum_{j=1}^{n} x_j^k w_j(x) \right]^{-1}
\left\{ \begin{array}{c}
1 \\
\vdots \\
x^k \\
\end{array} \right\} w_j(x)
\end{align*} \]
Reconstruction of a 3-D Model

\[ \chi^h_{\Omega}(x, y, z) = \sum_{k=1}^{k_{\text{max}}} \chi^h_{\Omega,k}(x, y)\Phi_k(z) \]

\( \chi^h_{\Omega,k} \) : Characteristic function of each image
Greyscale values (0-255)

\( \Phi_k(z) \) : Approximation functions
Brake Caliper
Analysis Result

low stress

high stress
Possible image-based design software

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Optimization
Prototypes
Summary

Concept of OPTISHAPE: Topology Optimization is continuously extended not only to structures but also materials, mechanisms, electro-magnetic fields, and others.
VOXELCON for I-DEAS
OPTISHAPE for I-DEAS
NASTRAN-OPTISHAPE

Toward
Image Based CAE