

Unintended Consequences of the Market Risk Requirement in Banking Regulation*

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Abstract

We analyze a bank that operates under the Basel credit and market risk requirements, and that maximizes its value through recapitalizations, dividends, and liquid asset investments. According to our model, the market risk requirement may postpone recapitalization and this way increase the bank's default probability. We show that this is indeed the case if the expected return and volatility of the liquid asset portfolio are high, i.e., then the market risk requirement raises the default probability of the bank. In this sense the market risk requirement is inefficient.

Keywords: bank capital, dividends, capital issues, investment, bank regulation

JEL classification: G32, G35

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1. Introduction

The Basel Accord's purpose is to "strengthen the soundness and stability of the international banking system" (Basel Committee Accord (1988)). It tries to achieve this goal by setting different capital requirements. First, there is a credit risk requirement for all banks that fall under its jurisdiction. This is the requirement for the risk of loss due to borrower or counterparty default, and it states that the minimum capital¹ that the banks must hold is at least 8% of their Risk Weighted Assets (RWA). Second, in 1996 an amendment on market risk requirement (MRR) was added to the Basel Accord (Basel Committee on Banking Supervision (1996a, 1996b, and 1996c)). By the amendment, banks must maintain extra capital to account for their market risk and the extra capital is given by the market investment's 10-day Value-at-Risk (VaR)². Hence, the higher the volatility of the market investment the higher the capital requirement. Further, if a bank does not invest in risky traded assets then the VaR is zero and the minimum capital requirement equals the credit risk requirement, i.e., in this case the minimum requirement is 8%.

Basel II has extended the banking regulation to consider also operational risk (see e.g. Hull (2007)). It also tries to set international rules for the supervisory review process and to increase market discipline by requiring banks to disclosure more information about their capital allocations and risks. However, the subprime mortgage crisis has shown that the Basel II is far from perfect and that new regulation rules have to be created (see e.g. discussions in Roubini (2008) and *Financial Times*, May 6 2008, "Seven habits finance regulators must acquire" by Martin Wolf). Basel II is outside the scope of this paper and we focus only on the credit and market risk regulations that are used today and assume naively perfect information between banks, regulators, and investors.³ There are clearly major information problems between these participants (see e.g. the above Financial Times article) and we hope the next regulation addresses these. In our perfect information model we analyze the optimal behavior of a bank that operates under the credit and market regulations. The MRR not only affects the optimal portfolio choice of the bank but also its

¹ Here capital means Tier 1 capital plus Tier 2 capital. Hence, consistent with common bank parlance, our use of the term bank capital refers to banks' book equity. This is the relevant measure of capital in an analysis of bank capital adequacy since minimum capital requirements under the Basel Accord apply to book equity.

² VaR is the maximum loss of a portfolio over a given horizon, at a given confidence level.

³ Bhattacharya et al. (2002) and Marshall and Prescott (2006) study situations where regulators have imperfect information.

recapitalization (equity issuance) and dividend policies. Therefore, the market risk regulation can have unintended consequences and we illustrate an example where the MRR raises the bank's liquidation probability through the bank's changed recapitalization decision. That is, even without the problems in Basel II discussed lately in the press, there is a risk that the current banking regulation has some negative consequences on the banking industry.

Within static settings⁴ risk-based capital requirements lower the probability of default and this way also the expected liability of the deposit insurance. However, static models do not consider the banks' franchise value (expected future income) and, therefore, they give an incomplete explanation of banks' capital. Dynamic models that consider the franchise value and the trade-off between the lower cost of debt finance and the marginal increase in the expected deadweight cost of liquidation are analyzed, e.g., in Milne and Whalley (2001), Milne (2002), and Pelizzon and Schaefer (2003). Peura and Keppo (2006) illustrate that this class of models explain the realized buffer capitals (capital excess the 8% requirement) reasonably well. In the present paper, we continue this dynamic modeling and create a realistic but stylized model of a bank and analyze the effects from MRR. In our model, a bank maximizes its value by taking the Basel minimum capital requirements as constraints. The bank collects gains and losses from two sources: basic banking business and investments in liquid financial assets. The basic banking business corresponds to selling loans and mortgages, while the liquid assets refer to stocks, bonds, and other traded financial instruments. Although during the last few years major US banks have made a significant part of their earnings by securitizing mortgages, about 47% of all their assets are real estate related (see Roubini (2008)). Thus, even after the widespread securitization US banks hold today a significant fraction of the mortgage credit risk on their balance sheets. The liquid asset investment is bounded since most of the bank's RWA are devoted to the basic banking business and since MRR sets constraints on the investment strategy. The bank also has a recapitalization option, but there exist a recapitalization delay and a fixed cost. The recapitalization delay is important since it implies a strictly positive default probability, and thus it makes sense to regulate banks in order to minimize their default probabilities.

We solve the optimal policy for recapitalizations, dividends, and liquid asset investments in a single optimization model. This is important since these decisions affect each other and, hence, cannot be analyzed separately. The bank maximizes the expected discounted

⁴ See Merton (1977), Furlong and Keeley (1989), and Sharpe (1978).

dividends. If the buffer capital violates the market or the credit risk requirement the bank is liquidated. By dynamic programming, the value function satisfies a set of ordinary differential equations that we solve semi-analytically.

The market risk requirement affects the bank behavior and liquidation probability in several ways. First, it lowers the cash flow volatility, which naturally decreases the liquidation probability. Second, the requirement postpones recapitalization which raises the liquidation probability, because then the bank starts the recapitalization process closer to liquidation. Since the MRR lowers the liquid risky asset holdings, it also raises the marginal utility from the capital invested in the market. Therefore, the bank finds it optimal to start the recapitalization at a lower buffer capital level, where the marginal utility from that action is also high. Naturally this decreases the average buffer capital and this way the MRR may raise the liquidation probability. We show that when the liquid risky asset expected return and volatility are high this is indeed the case. Hence, MRR may cause both positive (fall in the cash flow volatility) and negative (fall in the recapitalization level) effects on the liquidation probability. This negative effect is due to the fact that the MRR ignores the recapitalization option. As indicated earlier, when the liquid risky asset expected return and volatility are high then the negative effect is stronger. In order to analyze the total effect from the MRR all the bank's actions have to be considered.

1.1 Prior literature

Several papers have analyzed banks' capitalization decision as a function of the buffer capital. Froot and Stein (1998) demonstrate that a bank investing in illiquid products may adjust its capital structure in order to accommodate the illiquid risks it chooses to bear. They show that equity capital is used as hedge against future losses from illiquid assets. Estrella (2001) uses a variant of the classical inventory or cash management models to study cyclicalities of bank capital. By Heid, Porath, and Stolz (2003), banks with low capital buffers build an appropriate capital buffer by raising capital and lowering risk. In contrast, banks with high capital buffers try to maintain their capital buffer by increasing risk. This is consistent with our model since in the present paper recapitalization is done at a low buffer capital level and the liquid risky asset investment is an increasing function of the buffer capital. Furfine (2001) analyzes banks' portfolio shifts following the introduction of the Basel Accord in 1989. Bhamra et al. (2008) and Meh and Moran (2010) study the impact of time-varying macroeconomic conditions on optimal capital structure.

Jokivuolle and Peura (2004) analyze capital cushions in excess of the minimum requirements on hypothetical bank portfolios that satisfy a Value-at-Risk type constraint. We utilize their framework and add banks' dividend and recapitalization policies into the model. The basic continuous time model of a capital constrained firm is presented in Milne and Robertson (1996). That paper is extended in Milne and Whalley (2001) and Milne (2004) to allow for a recapitalization option. In their model recapitalization is possible without any delays and hence bank liquidation probability is zero. The framework of Milne and Whalley (2001) and Milne (2004) is extended to consider recapitalization delay in Peura and Keppo (2006) while the present paper adds the liquid risky asset investment under the MRR. Peura and Keppo also calibrate the model to accounting data on U.S. commercial banks and show that this class of models explains about 40% of the variation in the buffer capital levels. Højgaard and Taksar (1999, 2001, 2004) consider an insurance company that reinsures and investments in both liquid risky and risk-free assets. We use their results in deriving the portfolio strategy of liquid assets.

Many papers have considered VaR and its effects on bank portfolio decisions. VaR is a special case of the natural risk statistics introduced by Heyde et al. (2006).⁵ It is well known that VaR has some drawbacks (see e.g. Artzner et al. (1999), Danielsson (2003), Rockafellar and Uryasev (2002), Embrechts (2000), and Danielsson et al. (2002)). First, VaR provides only a point estimate of the banks' loss distribution and, hence, it ignores the tail of the distribution. This tail has important information in case of non-normal distributions. Second, VaR does not always capture the diversification effect since in some cases the VaR of a portfolio is higher than the sum of VaRs of the individual assets. Clearly, in these cases the point estimate does not correspond to the portfolio risk and, therefore, banks may want to diversify less. This kind of result is reported with a bank's mean-variance portfolio selection, e.g., in Kahane (1977), Koehn and Santomero (1980), Gennotte and Pyle (1991), and Blum (1999). According to Furlong and Keeley (1989) and Keeley and Furlong (1990) the mean-variance framework is inappropriate to study the capital requirements with deposit insurance and limited liability, because the return distributions are not normally distributed. Pelizzon and Schaefer (2003) show that under the VaR constraint banks may want to increase their risk today since the capital requirement constrains the bank's future ability to take risk. In

⁵ Natural risk statistics is a data based risk measure. The measure is consistent with the prospect theory in psychology and it is characterized by a set of axioms that only require subadditivity for comonotonic random variables.

our model there is a similar effect at low buffer capital levels: Then our bank finds it optimal to delay recapitalization and to invest in liquid assets. In contrast to the above papers, our results are not driven by the used risk measure, instead by the fact that MRR penalizes only the portfolio choice and ignores banks' other actions (recapitalization and dividends).

1.2 Structure of the paper

The rest of the paper is organized as follows. Section 2 presents our model of capital control. Section 3 shows that the non-homogenous problem of capital control can be transformed into a homogenous problem of capital ratio control. Then the solution is characterized in terms of a set of variational inequalities. The optimal policies and the value function are solved in Section 4. Numerical example is presented in Section 5 and Section 6 concludes. Appendices A-C gather some proofs, while Appendix D describes the example parameter values. Appendix E is a list of used notation and abbreviations.

2. The model

In this section we introduce our banking model which is an extension to the credit risk requirement framework of Peura and Keppo (2006). We analyze a single bank that maximizes its expected discounted dividends. Consistent with the 1996 Amendment (Basel Committee on Banking Supervision (1996a, 1996b, and 1996c)) we divide the bank's assets into *banking book* and *trading book*. From these books we get the bank's *risk-weighted assets* by weighting for asset risks according to a formula determined by the regulators. We model directly the risk-weighted asset values.

The banking book consists of loans that are not marked to market for managerial and accounting purposes and, therefore, we call these assets as *illiquid assets*. Because the loans have credit risks, the bank has to keep capital according to the *credit risk requirement*. The trading book is a portfolio of different traded instruments (e.g. stocks, bonds, swaps, forward contracts and other derivative instruments) that are usually marked to market daily. We call these assets as *liquid assets*. Capital charges from trading book accounts between one and forty-five percent of total risk based capital (Basel Committee on Banking Supervision (1999)). By the *market risk requirement* (MRR), the bank has to hold capital to cover their market risks. Some of the assets in the trading book might face also a specific risk charge due to their credit risk (e.g. a corporate bond) and some of the assets might have

counterparty risk if they are traded in OTC-markets. Since we analyze the effect from the MRR, for simplicity, we assume that the trading book's specific risk and counterparty risk charges are zero. This models a situation, where the counterparties are highly rated or the instruments are traded in exchanges (so, there is no counterparty risk) and where we interpret the trading book to consist only of assets that do not have credit risk. Further, regulators sometimes impose capital requirements on their domestic banks that exceed the international Basel requirements (see e.g. Alfon et al. (2004)). In the present paper we ignore these domestic requirements.

[Figure 1 about here]

2.1 Bank portfolio, liabilities, and shareholders' equity

We consider a bank whose portfolio size, measured by its regulatory risk weighted assets (RWA)⁶, grows at a constant positive rate r , so that

$$R_t = R_0 e^{rt} \tag{1}$$

for some initial positive amount R_0 . The growth rate $r > 0$ and it is assumed to equal the risk-free rate. This implies that our bank is in a steady state and its total assets grow at the constant rate due to new loans and investments. The asset growth is funded mainly by debt but also by internally generated equity and external equity. The asset dynamics (1) is also used in Peura and Keppo (2006) and they show that the growth rate of RWA has only a second order effect on the buffer stock. Further justification for the deterministic RWA dynamics comes from the definition of RWA. Under Basel II the risk weights used in the RWA calculation are based on credit ratings or banks' own internal credit ratings (see e.g. Hull (2007)). Both of these methods are stable in the sense that the risk weights do not change according to daily market movements. Then, e.g., for on-balance sheet loans the RWA value equals the risk weight times the underlying principal. Thus, RWA fluctuates significantly less than the corresponding market prices and we model the RWA dynamics by using (1).

⁶ Risk weighted assets, under the current Basel Accord from 1988, are calculated as a weighted sum of a bank's nominal exposures, where the weights depends on product type and counterparty sector. For large banks, risk weighted assets are typically between 65 and 70 percent of total assets.

Since the total assets equal the illiquid and liquid assets, in the modeling of the capital structure we assume that

$$(CS1) \quad (1 + \text{positive constant}) \cdot \text{illiquid assets} = \text{bank's debt} + \text{credit risk requirement} \\ = m \cdot \text{RWA},$$

where the credit risk requirement is the minimum equity level (8% of RWA), $m \in (0,1)$ and it is constant, the illiquid assets refer to the basic banking business, i.e., sold loans and mortgages. Thus, the debt plus the minimum equity is greater than the illiquid assets. Further, because all the assets equal RWA, the illiquid assets are a fraction of the risk weighted assets, i.e., by (CS1), $mR_t/(1 + \text{positive constant})$ equals the illiquid assets. According to Peura and Keppo (2006), m is about 0.95 and, hence by (CS1), the debt and the credit risk requirement are about 95% of the risk weighted assets. Note also that by (CS1), if illiquid assets grow at about risk-free rate, then (1) is a natural approximation. Further, according to (1) and (CS1), the bank's debt and the credit risk requirement grow at the steady rate. Thus, the asset growth is mainly funded by debt but also by internally generated equity and external equity. When the bank collects new equity there exist recapitalization delay and fixed cost (subsection 2.3). The equity issuance is the only funding option we consider explicitly since in our model the bank debt growth is exogenous (equations (1) and (CS1)). However, the bank affects internally generated equity through dividends and risky asset investments (see equation (3)).

By definition, *buffer capital* (part of equity) is total shareholders' equity minus the credit risk requirement. Since the total assets are RWA which equals all the liabilities and shareholders' equity, we get from (CS1):

$$(CS2) \quad \text{buffer capital} = \text{RWA} - (1 + \text{positive constant}) \cdot \text{illiquid assets} = (1 - m) \cdot \text{RWA}.$$

Figure 1 illustrates the assets, liabilities, and shareholders' equity. By (CS1) and (CS2), RWA minus the buffer capital corresponds to a constant fraction, m , of RWA. That is, $m = 1 - \bar{X}_t / R_t$, where \bar{X}_t is the buffer equity, i.e., the equity above the credit risk requirement. From now on, we denote the "positive constant" in (CS1) and (CS2) as $1/(1 - \tilde{b}^m) - 1$, i.e., $1 + \text{positive constant} = 1/(1 - \tilde{b}^m)$, where $\tilde{b}^m \in (0,1)$ and it is constant.

In the modeling of the asset portfolio we assume that at time t the bank invests in three sub portfolios. By (1), the RWA is independent of this portfolio choice. In order to write the

buffer capital process as a linear function with respect to RWA, we represent the wealth invested in the sub portfolios as follows (see Figure 1):

(P1) $(1 - \tilde{b}^m)mR_t$ in *illiquid assets*. The term $(1 - \tilde{b}^m)$ is the proportion of mR_t in the illiquid assets.

(P2) $b_t^m mR_t$ in the *liquid risky asset*, where $b_t^m \in [0, \tilde{b}^m]$ and \tilde{b}^m is the *maximum proportion in the liquid risky asset*.

(P3) $(\tilde{b}^m - b_t^m)mR_t + \bar{X}_t$ in the *liquid risk-free asset*, where \bar{X}_t is the buffer equity.

(P1) is the same as (CS1) and it implies that the bank keeps $(1 - \tilde{b}^m)mR_t$ in the illiquid assets in order to run the basic banking business. These are the main assets of the bank. For instance, according to Roubini (2008), about 47% of all US banks' assets are real estate related. Since the bank sells also other loans, we have $(1 - \tilde{b}^m)m > 50\%$. Because the illiquid assets have credit risk, the bank faces the Basel credit risk requirement. The credit risk requirement states that the bank's equity has to be at least 8% of the RWA and, according to (P3), we assume that this equity capital is invested in a risk-free financial asset. (P2) and (P3) imply that the bank invests $\tilde{b}^m mR_t + \bar{X}_t$ in liquid financial assets as follows: $b_t^m mR_t$ in a liquid risky asset and $(\tilde{b}^m - b_t^m)mR_t + \bar{X}_t$ in a risk-free financial asset. These liquid asset investments have to satisfy the MRR that we define in the next subsection.

2.2 Portfolio dynamics and the market risk requirement

The cash flows from the basic banking business, i.e., from the illiquid assets are assumed to follow $dY_t = \bar{\mu}_Y R_t dt + \sigma_Y R_t dW_t^Y$, where W_t^Y is a standard Wiener process, σ_Y is the revenue volatility, $\bar{\mu}_Y$ is the expected proportional net revenue and it is assumed to satisfy $\bar{\mu}_Y > r$. This implies that the illiquid cash flows are modeled with the diffusion process and they are proportional to the bank's portfolio size.

The Y -process can be motivated as follows. Let the cumulative cash flows from the illiquid assets be given by $Y_t = R_t \left(it - \sum_{j=1}^{A(t)} D_j \right)$, where i represents the interest income as a percent of total risk weighted assets, $A(t)$ is the number of customers who have defaulted through time t and it follows a Poisson process with rate λ , $\{D_j\}$ represent the losses due to the defaults as a percentage of the risk weighted assets and they are i.i.d. with mean μ_d and standard deviation σ_d . This implies that the bigger the bank, the bigger loans it can sell

and, therefore, its losses depend on the RWA. If we change time and normalize the state space according to $y_t \mapsto y_{nt} / \sqrt{n}$ then the limiting process follows the Y -process above with $\sigma_Y = \lambda(\mu_d^2 + \sigma_d^2)$ and $\bar{\mu}_Y = i - \lambda\mu_d$. See e.g. Iglehart (1969), Grandell (1990), and Harrison (1977) for the references of diffusion approximations.

By (P2) and (P3), the bank invests $b_t^m m R_t$ in a liquid risky asset that follows a geometric Brownian motion with expected return $\bar{\mu}_Z > r$ and volatility σ_Z , and $(\tilde{b}^m - b_t^m) m R_t + \bar{X}_t$ in a risk-free liquid asset. Therefore, the profits from the liquid financial assets follow

$$dZ_t = \left(\bar{\mu}_Z b_t^m m + r(\tilde{b}^m - b_t^m) m + r \frac{\bar{X}_t}{R_t} \right) R_t dt + \sigma_Z b_t^m m R_t dW_t^Z, \quad (2)$$

where b_t^m is the proportion of $m R_t$ invested in the liquid risky asset, W_t^Z is a standard Wiener process and it is assumed to be independent of W_t^Y . The first term in the drift term in (2) is the expected return from the liquid risky asset, and the two last terms in the drift term are the risk-free return. The uncertainty term is due to the liquid risky asset investment.

As we will see in (5), there are two interpretations for (2): (i) profits and losses from the liquid financial assets under the objective probability measure and (ii) profits and losses under the risk-neutral probability measure. If (2) was under the risk-neutral measure then it would be natural to assume $\bar{\mu}_Z = r$ (see e.g. Björk (2004)). However, this implies that it is optimal for the bank not to trade the liquid risky asset (see Corollary 1) and this is not what we observe in the market. Therefore, if we interpret (2) as the profits and losses process under the risk-neutral probability measure then we assume that the bank is able generate alpha, i.e., also in this case $\bar{\mu}_Z > r$. This excess return could be, for instance, from market making or it just reflects the bank shareholders' expectation for the future returns. Thus, in both the cases ((i) and (ii) above) we assume that the expected return of the risky liquid asset is high enough so that the bank invests in the asset.

The independency of W_t^Y has an important technical merit but it can also be motivated by using the data in Section 5 as follows. Because most of the banks' assets are illiquid (see e.g. Roubini (2008)), let us use here banks' net income as a proxy for the Y -process. For the Z -process we simply use the S&P 500 index. After normalizing the banks' net income changes by the RWA and S&P 500 changes by the S&P 500 index value and then calculating their correlation we get that the correlation estimate between W_t^Z and W_t^Y ranges from -0.25 to 0.40 (over different banks). The average is -0.02 and the standard

deviation is 0.18. Thus, the correlation is not significantly different from zero, i.e., the normalized net income does not correlate significantly with the market index returns. Note that since all the bank assets are not illiquid, this correlation estimate might overstate the dependency between W_t^Z and W_t^Y . On the other hand, S&P 500 might overstate the riskiness of the banks' liquid risky assets.

In addition to the constraint (P2), the portfolio process b_t^m has to satisfy the MRR, which is equal to the larger of the two candidates:

- 1) the average reported two-week VaR at the 99% confidence level in the last 60 trading days times a multiplier
- 2) the last-reported two-week VaR at the 99% confidence level.

The multiplier, k , is between 3 and 4, and it depends on the reporting back testing results (Basel Committee on Banking Supervision (1996c)). For simplicity, in this paper we assume that the MRR is given by the current 10 days VaR at the 99% confidence level (two-week VaR) multiplied by k . That is, the buffer capital has to satisfy $k \cdot VaR(b_t^m) \cdot R_t \leq \bar{X}_t$ where, by the liquid asset portfolio process (2), the value at risk: $VaR(b_t^m) = 2.33 \cdot \sqrt{\frac{10}{250}} \cdot b_t^m \sigma_Z$. Thus, if $k = 0$ then we get the case without the MRR.

From the above VaR constraint, the bank portfolio conditions (P1)-(P3), and the liquid asset portfolio dynamics (2) we get that the liquid asset dynamics can be written as $dZ_t = \left(\bar{\mu}_Z b_t + r \left[(\tilde{b} - b_t) + \frac{\bar{X}_t}{R_t} \right] \right) R_t dt + \sigma_Z b_t R_t dW_t^Z$, where $b_t = m b_t^m$, $\tilde{b} = m \tilde{b}^m$, $b_t R_t \leq v \bar{X}_t$, and $v = m / \left(k \cdot 2.33 \cdot \sqrt{\frac{10}{250}} \cdot \sigma_Z \right)$. If there is no MRR then, obviously, $v = \infty$ since $k = 0$.

2.3 Capital control policy and buffer capital dynamics

Owners control bank capital through liquid financial asset investments, dividend payments, and issues of new capital. There are no frictions in trading the liquid financial assets and the bank can adjust its liquid asset portfolio in continuous time. Dividend payments can be implemented instantaneously, but capital issuance is associated with a delay of length Δ and with a cost which is a fixed proportion K of RWA. Thus, when the bank sells new equity there is a process that takes time and is costly. The recapitalization delay is important since it implies a strictly positive liquidation probability, and thus it makes sense to regulate banks in order to minimize their liquidation probabilities. As mentioned earlier, the equity issuance is the only funding option we consider explicitly. However, the bank affects

internally generated equity through dividends and liquid risky asset investments. Debt financing and internally generated equity are given by (1), (CS1), and (3).

Formally, a capital control policy $\bar{\pi}$ is a collection $(b^{\bar{\pi}}, L^{\bar{\pi}}, \{t_i^{\bar{\pi}}, s_i^{\bar{\pi}}\})$, where $b^{\bar{\pi}}$ is the liquid asset investment under policy $\bar{\pi}$, $L^{\bar{\pi}}$ is a non-decreasing process representing the cumulative amount of dividends, $\{t_i^{\bar{\pi}}\}$ is an increasing sequence of order times of new capital issues, and $\{s_i^{\bar{\pi}}\}$ are the amounts of capital raised at each issue of capital. We denote by Π the class of admissible policies and they satisfy:

- (M1) the liquid risky asset holding is bounded: $R_t b_t^{\bar{\pi}} \in [0, \tilde{b}R_t \wedge v\bar{X}_t]$ for all $t \geq 0$
- (M2) $L_t^{\bar{\pi}}$ is a non-decreasing right-continuous process adapted to F_t and $L_{0-}^{\bar{\pi}} = 0$, where the filtration $\{F_t\}$ is generated by the Wiener processes W_t^Z and W_t^Y
- (M3) each $t_i^{\bar{\pi}}$ is a stopping time of the filtration F_t and each $s_i^{\bar{\pi}}$ is measurable with respect to $F_{(t_i^{\bar{\pi}}+\Delta)-}$
- (M4) $t_{i+1}^{\bar{\pi}} - t_i^{\bar{\pi}} \geq \Delta$ and $dL_t^{\bar{\pi}} = 0$, $b_t^{\bar{\pi}} = 0$ for all $t \in (t_i^{\bar{\pi}}, t_i^{\bar{\pi}} + \Delta]$ and $i \geq 1$.

Condition (M1) follows directly from the MRR above. It implies that the liquid risky asset investment is bounded above by the VaR condition and the maximum investment in the liquid financial assets. Thus, when there is no MRR then $v = \infty$ and the risky asset investment is bounded only by \tilde{b} . Note that when the bank changes the liquid risky asset holding then, by (P2) and (P3), at the same time it also changes the liquid risk-free portfolio. Condition (M1) also prevents the short selling of the liquid risky asset. (M2) states that dividends can be paid in continuous time without any delays and costs. In condition (M3) the measurability of s_i with respect to $F_{(t_i+\Delta)-}$ means that the owners may decide on the exact amount of capital to be raised at time $t_i + \Delta$ based on all then available information. They do not need to precommit to any quantity of capital at time t_i when they order the capital issue. Condition (M4) states first that a new issue may not be ordered while a previously ordered issue is still waiting to be completed. When a capital issue is ordered at time t_i , it takes until $t_i + \Delta$ before new capital can actually be raised. Second, dividends are not paid and the risky liquid asset holding is zero during the periods between the ordering of a capital issue and the actual capital collection. The condition has important technical merit but also an economic justification: Ruling out simultaneous capital orders and dividend payments as well as the bank portfolio's high risk levels during the

recapitalization process is likely to reduce conflicts of incentives between existing and new equity holders. Therefore, (M4) can be viewed as a constraint set by the capital markets.⁷

Bank *buffer capital* as a function of policy $\bar{\pi}$ is denoted $\bar{X}_t^{\bar{\pi}}$ and it satisfies:

$$\begin{aligned}\bar{X}_t^{\bar{\pi}} &= \bar{X}_0 + \int_0^t dY_u + \int_0^t dZ_u - L_t^{\bar{\pi}} + \sum_i s_i^{\bar{\pi}} \mathbf{I}_{\{t_i^{\bar{\pi}} + \Delta \leq t\}} \\ &= \bar{X}_0 + \int_0^t \left(\bar{\mu}_Y + \bar{\mu}_Z b_u + r \left[(\tilde{b} - b_u) + \frac{\bar{X}_u^{\bar{\pi}}}{R_u} \right] \right) R_u du + \int_0^t \sigma_Y R_u dW_u^Y \\ &\quad + \int_0^t \sigma_Z b_u R_u dW_u^Z - L_t^{\bar{\pi}} + \sum_i s_i^{\bar{\pi}} \mathbf{I}_{\{t_i^{\bar{\pi}} + \Delta \leq t\}}\end{aligned}\tag{3}$$

where $\mathbf{I}_{\{\cdot\}}$ is the indicator function of the event defined in the braces. Thus, cumulative profits and new issues of capital feed to the buffer capital, while dividend payments and cumulative losses represent a leakage from the capital. The bank invests the buffer capital in the risk-free asset and, hence, it earns the risk-free rate. The first integral term in the first line of (3) is from the illiquid bank portfolio and the second term is from the liquid assets. The two last terms are the cumulative dividend and recapitalization processes, respectively.

2.4 Value of the bank

The minimum capital requirement under the Basel Accord states that bank capital must at all times exceed 8% of the bank's risk weighted assets, i.e., the buffer capital has to be positive. We assume that the corrective action from violation of the minimum capital requirement will be liquidation. The model bank therefore only operates up to the *liquidation time*:

$$\bar{\tau}_{\bar{\pi}} = \inf \{t : \bar{X}_t^{\bar{\pi}} \leq 0\}.\tag{4}$$

This is a stylized assumption. In practice, a violation of the minimum capital requirement will not result in immediate liquidation, but will generate additional costs and constraints to the bank due to increased regulatory surveillance (see e.g. Peek and Rosengren (1997)).

By (4), our recapitalization model (M3) and (M4) can be viewed as a recapitalization under liquidation risk. When the recapitalization is started there is a small probability that

⁷ We have also derived and tested a model where the risky asset holding is not zero during the delay. This does not change our qualitative results, but complicates the model. For instance, with that model the MRR raises the liquidation probability when the risky liquid asset volatility is close to 30% (see Figure 5).

the bank is not able to raise new capital and is liquidated. The liquidation probability falls in the buffer capital and rises in the bank's cash flow volatility. Our delay Δ captures these effects. Thus, we can view (M3) and (M4) as a model for the liquidation risk that corresponds to the tail probability $P\left(\inf_{0 \leq t \leq \Delta} \bar{X}_t \leq 0\right)$. That is, any recapitalization model (with or without delay) that has the same liquidation probability and the same recapitalization cost (see (5) below) gives the same results as our model.

Similarly as e.g. in Milne and Whalley (2001), the value of the bank under policy $\bar{\pi}$ to its owners, given initial buffer capital \bar{X}_0 , is the expected discounted present value of dividends less capital issues until liquidation:

$$\bar{V}_{\bar{\pi}}(\bar{X}_0) = E_{\bar{X}_0} \left[\int_0^{\bar{\tau}_{\bar{\pi}}} e^{-(r+\rho)t} dL_t^{\bar{\pi}} - \sum_i e^{-(r+\rho)(t_i^{\bar{\pi}} + \Delta)} \left(s_i^{\bar{\pi}} + KR_{t_i^{\bar{\pi}} + \Delta} \right) I_{\{t_i^{\bar{\pi}} + \Delta < \bar{\tau}_{\bar{\pi}}\}} \right], \quad (5)$$

where ρ is a positive constant representing the wedge between debt and equity finance, due to capital market frictions such as taxation and agency costs of equity⁸, and it satisfies $\rho > \bar{\mu}_Z - r$, which gives $\bar{V}_{\bar{\pi}}(\bar{X}_0) < \infty$. K is a non-negative constant representing the cost of capital issuance and the cost is proportional to the risk weighted assets. If the expectation in (5) is taken with respect to the risk-neutral probability measure then (3) is the buffer capital process under the risk-neutral measure and (5) is the usual risk-neutral pricing formula (see e.g. Björk (2004) and Bhamra et al. (2009)) and, therefore, it gives the bank value under the given policy. Alternatively, as in Milne (2002), the share holders are risk-neutral and this gives directly (5) with the buffer capital process (3) under the objective probability measure.

The capital control problem is to identify the value of an optimally managed bank:

$$\bar{V}(\bar{X}) = \sup_{\bar{\pi} \in \Pi} \bar{V}_{\bar{\pi}}(\bar{X}) \quad (6)$$

and an admissible policy which achieves this value. Note that the model has ten parameters in total: μ_Y and σ_Y characterize the net revenue process of the illiquid portfolio, $\hat{\mu}_Z$ and σ_Z correspond the bank's liquid risky asset returns, \tilde{b} is the maximum proportional investment in the liquid risky asset, k is the regulatory constant, r is the risk-free rate, ρ is the wedge

⁸ The parameter ρ should not be interpreted as equity risk premium since it is assumed constant and does not depend on bank leverage. This suggests that our modeling framework is risk-neutral (risk-neutral share holders or risk-neutral probability measure). If the risk-neutral probability measure is used then the drift term of buffer capital process need not coincide with its observed value. In particular, since uncertainty in our model is driven by a Brownian Motion, a change of measure would influence the drift in (3), but not the volatility.

between debt and equity finance, Δ and K determine the magnitude of the capital market imperfections.

3. Characterization of optimum

The capital dynamics defined in (3) are not time-homogenous, which makes direct solution of the problem (6) difficult. The problem of capital control can, however, be transformed into a time-homogenous problem of buffer capital ratio control through a simple normalization. The normalized state variable, the bank *capital ratio* is given by

$$X_t = \bar{X}_t / R_t. \quad (7)$$

From Peura and Keppo (2006) we get the following lemma that presents the capital ratio control problem and shows its connection to the capital control problem (6).

Lemma 1 (Capital control problem) *Given an admissible policy $\pi \in \Pi$, the buffer capital ratio satisfies*

$$X_t^\pi = X_0 + \int_0^t (\mu_Y + \mu_Z b_u) du + \int_0^t \sigma_Y dW_u^Y + \int_0^t b_u \sigma_Z dW_u^Z - L_t^\pi + \sum_i s_i^\pi \mathbf{I}_{\{t_i^\pi + \Delta \leq t\}}, \quad (8i)$$

where $\mu_Y = \bar{\mu}_Y + \tilde{b}r$ and $\mu_Z = \bar{\mu}_Z - r$. Define the first time of credit risk violation by

$$\tau_\pi = \inf \{t : X_t^\pi \leq 0\} \quad (8ii)$$

and the bank equity value under policy π by

$$V_\pi(X_0) = E_{X_0} \left[\int_0^{\tau_\pi} e^{-\rho t} dL_t^\pi - \sum_i e^{-\rho(t_i^\pi + \Delta)} (s_i^\pi + K) \mathbf{I}_{\{t_i^\pi + \Delta < \tau_\pi\}} \right], \quad (8iii)$$

where the expectation is conditional on the capital ratio dynamics (8i). The value function:

$$V(x) = \sup_{\pi \in \Pi} V_\pi(x). \quad (8iv)$$

Then (6) can be expressed in terms of (8iv) as

$$\bar{V}(\bar{X}_0) = R_0 V(X_0). \quad (9)$$

Moreover, let π^* be the policy which achieves the optimum in (8iv). Then the policy which achieves the optimum in (6), $\bar{\pi}^*$, can be expressed in terms of π^* by

$$b_t^{\bar{\pi}^*} = b_t^{\pi^*}, \quad L_t^{\bar{\pi}^*} = \int_0^t R_u dL_u^{\pi^*}, \quad t_i^{\bar{\pi}^*} = t_i^{\pi^*}, \quad \text{and} \quad s_i^{\bar{\pi}^*} = R_{t_i^{\pi^*} + \Delta} s_i^{\pi^*} \quad \text{for all } i \geq 1. \quad (10)$$

The key to this result is to understand that when π and $\bar{\pi}$ are related through (10) then the buffer capital ratio process (8i) is the process $\bar{X}_t^{\bar{\pi}}/R_t$. The proof is based on Ito's lemma and equations (1) and (3). Equation (9) implies that the objective function of the capital ratio control problem, (8iv), can be interpreted as the value of bank equity as a percentage of risk weighted assets.

We characterize the value function (8iv) through a set of variational inequalities. For this purpose we define two operators. Let D be the set of real-valued functions on \mathbf{R}_+ . Let the operator $M:D \rightarrow D$ be given by

$$Mf(x) = E_x \left[e^{-\rho\Delta} \sup_s [f(X_\Delta + s) - s - K] \mathbf{I}_{\{\tau_0 > \Delta\}} \right], \quad (11)$$

where $X_\Delta = x + \mu_Y \Delta + \sigma_Y W_\Delta^Y$ and it is the buffer capital ratio after the recapitalization delay under zero risky liquid asset holding. τ_0 is the first time X hits zero and the expectation is conditional on $X_0 = x$. Operator M can be interpreted as the expected value of the decision to order new capital immediately given the continuing value f . Second, we define the infinitesimal generator A_b for all sufficiently regular f as follows

$$A_b f(x) = \frac{1}{2} (\sigma_Y^2 + b^2 \sigma_Z^2) f''(x) + (\mu_Y + b\mu_Z) f'(x). \quad (12)$$

The proposition below characterizes the optimum. The proof follows from standard arguments (see e.g. Højgaard and Taksar (1999) or Fleming and Soner (1993)).

Proposition 1 (Value function)

(a) *Necessary conditions: Assume that the value function (8iv) satisfies Ito's formula. Then it satisfies the following set of inequalities for all $x \geq 0$:*

$$\text{LIQUIDATION:} \quad V(0) = 0 \quad (13i)$$

$$\text{RECAPITALIZATION:} \quad V(x) \geq MV(x) \quad (13ii)$$

$$\text{LIQUID ASSET INVESTMENT:} \quad \max_{b \in [0, b^{\max}]} (A_b - \rho)V(x) \leq 0 \quad (13iii)$$

$$\text{DIVIDENDS:} \quad V'(x) \geq 1 \quad (13iv)$$

For each buffer capital ratio x , one of the inequalities (13ii) – (13iv) is tight.

(b) *Sufficient conditions: If f is twice differentiable and concave satisfying (13iv) with equality for all $x > u_D$ and V replaced by f , (13iii) with equality for all $x \in (u_C, u_D)$, (13ii) with equality for all $x \in (0, u_C)$, and $f(0) = 0$, where $0 < u_C < u_D < \infty$, then f equals the value function V . If*

$b(x)$ maximizes (13iii) with V replaced by f then π^* in Appendix A is optimal, i.e., $f(x) = V(x) = V_{\pi^*}(x)$.

(13) is a system of first order conditions to our problem which follow from standard dynamic programming arguments applied to the Bellman equation. (13i) follows from (4) and (5) since when the capital hits the credit requirement, i.e., when the buffer capital ratio hits zero the bank is liquidated. (13ii) holds since the value of immediate order of new capital can never exceed the value function by the definition of the value function. (13iii) and (13iv) hold since investing in the liquid assets and paying dividends are always admissible. Note that if there is no MRR then $v = \infty$ and, thus, (13iii) is $\max_{b \in [0, \bar{b}]} (A_b - \rho)V(x) \leq 0$.

The sufficient conditions in Appendix A can be understood as follows: (a) Every time Mf and f coincide outside the order periods, a new capital issue is ordered. During the order periods, capital stock cannot be controlled. At the end of capital order periods, just enough capital is raised to shift the buffer capital ratio to u_D . (b) Dividends are paid so as to never let the buffer capital ratio rise above u_D outside the order periods. (c) Between the recapitalizations and dividends the bank invests in the liquid financial assets, i.e., it solves

$$\max_{b \in [0, \bar{b} \wedge vx]} (A_b - \rho)f(x) = 0.$$

4. Optimal policy

The optimal structure of the bank policy depends on when the VaR constraint and the maximum investment constraint in (M1) are active. For simplicity, in this section we mostly consider the most general structure, where the VaR constraint is active with high and low buffer capital levels and the maximum investment is active only with high buffer capital levels. This structure is not always optimal and, therefore, when our model is implemented all possible structure candidates have to be considered (see Section 5). The optimal structure is the one that produces smooth value function and this has to be tested numerically.

Related to the policy structure, let $0 < u_C < x_0 < x_1 < x_2 < u_D < \infty$. Motivated by Proposition 1, our policy structure assumption can be stated as follows:

(i) For $x \in (0, u_C]$ it is optimal to immediately order new capital.

(ii) For $x \in (u_C, u_D)$ it is optimal to invest in liquid financial assets and neither to order new capital nor to pay dividends. The interval (u_C, u_D) is divided as follows: For $x \in (u_C, x_0]$ and $x \in [x_1, x_2]$ the MRR is active, for $x \in (x_0, x_1)$ none of the liquid risky asset constraints is active, and for $x \in [x_2, u_D)$ the maximum liquid risky asset investment constraint is active.

(iii) For $x \in [u_D, \infty)$ it is optimal to pay dividends.

Figure 2 illustrates the situation. As can be seen, the liquid risky asset investment policy is an increasing function of the buffer capital ratio: The higher the ratio the lower the liquidation probability and, therefore, the bank takes more risk in order to raise its revenues. By (P2) and (P3), this implies that the risk-free asset holding falls in the buffer capital ratio. Note that because the unconstrained investment policy (between x_0 and x_1) is convex (Appendix C) and because the VaR constraint is a straight line, the MRR is active at most twice.

If μ_Z / σ_Z^2 is small (high) enough then $x_0 = u_C$ and $x_1 = x_2 = u_D$ (if the fraction is high then $x_0 = x_1$), i.e., the liquid risky asset's investment constraints are never (always) active. If there is no MRR then $x_0 = u_C$ and $x_1 = x_2$. Further, u_C may be 0 when capital market imperfections are prohibitively high. In this section we mostly consider the most general model structure and the other cases can be solved in the same way. That is, here we assume that all the model barriers exist and they satisfy $0 < u_C < x_0 < x_1 < x_2 < u_D < \infty$. We call this model structure as the *general model structure*.

[Figure 2 about here]

Next we introduce different value function and optimal strategy parts that will be used in Proposition 2. These parts correspond to different cases in (13) and they are derived in Appendix C. The value function in the beginning of the refinancing period, i.e., corresponding to MV in (13ii), is given by⁹

⁹ From (11) and (13) we get the equation below and then Peura and Keppo (2006) gives the result.

$$\begin{aligned} \Psi(x; b_D, u_D, K, \Delta) &= e^{-\rho\Delta} E_x \left[\left(X_\Delta + \frac{\mu(b_D)}{\rho} - K - u_D \right) \mathbf{I}_{\{X_\Delta < u_D\}} \mathbf{I}_{\{\tau_0 > \Delta\}} + \left(X_\Delta + \frac{\mu(b_D)}{\rho} - K - u_D \right) \mathbf{I}_{\{X_\Delta \geq u_D\}} \mathbf{I}_{\{\tau_0 > \Delta\}} \right] \\ &= e^{-\rho\Delta} E_x \left[\left(X_\Delta + \frac{\mu(b_D)}{\rho} - K - u_D \right) \mathbf{I}_{\{\tau_0 > \Delta\}} \right] \end{aligned}$$

$$\begin{aligned} \Psi(x; b_D, u_D, K, \Delta) = & e^{-\rho\Delta} \left\{ \left(x + \mu_Y \Delta + \mu(b_D) / \rho - K - u_D \right) \Phi \left(\frac{x + \mu_Y \Delta}{\sigma_Y \sqrt{\Delta}} \right) + \sigma_Y \sqrt{\Delta} \varphi \left(\frac{x + \mu_Y \Delta}{\sigma_Y \sqrt{\Delta}} \right) \right. \\ & \left. - e^{-\frac{2\mu_Y x}{\sigma_Y^2}} \left[\left(-x + \mu_Y \Delta + \mu(b_D) / \rho - K - u_D \right) \Phi \left(\frac{-x + \mu_Y \Delta}{\sigma_Y \sqrt{\Delta}} \right) + \sigma_Y \sqrt{\Delta} \varphi \left(\frac{-x + \mu_Y \Delta}{\sigma_Y \sqrt{\Delta}} \right) \right] \right\}, \end{aligned} \quad (14i)$$

where $\varphi(y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right)$ denotes the standard normal density, $\Phi(y) = \int_{-\infty}^y \varphi(x) dx$ is the cumulative standard normal distribution, b_D is the optimal liquid risky asset holding at the dividend barrier¹⁰, $\mu(b) = \mu_Y + b\mu_Z$, and $\sigma(b) = \sqrt{\sigma_Y^2 + b^2\sigma_Z^2}$. By Peura and Keppo (2006), if $\mu(b_D) / \rho - K - u_D \geq 0$ then $\frac{\partial \Psi(x; b_D, u_D, K, \Delta)}{\partial x} > 0$ and $\frac{\partial^2 \Psi(x; b_D, u_D, K, \Delta)}{\partial x^2} < 0$ for all x . Further, if this condition does not hold then the recapitalization is not optimal.

The value function corresponding to (13iii) with $b(x) = vx$, i.e., the case when the VaR constraint is active, is given as follows

$$f_v(x; k_1, k_2) = k_1 D(x; \gamma + 1) + k_2 E(x; \gamma + 1), \quad (14ii)$$

where k_1 and k_2 are constants, $D(x; \omega) = \int_x^\infty (t - x)^\omega K(t) dt$, $E(x; \omega) = \int_{-\infty}^x (x - t)^\omega K(t) dt$,

$$\begin{aligned} K(t) &= (\sigma_v^2 t^2 + \sigma_Y^2)^{-(\gamma+1+\mu_v/\sigma_v^2)} \exp\left(-2 \arctan(\sigma_v t / \sigma_Y) \mu_Y / \sigma_Y \sigma_v\right), \\ \gamma &= \frac{1}{2} \left(\sqrt{(2\mu_v / \sigma_v^2 - 1)^2 + 8\rho / \sigma_v^2} - (1 + 2\mu_v / \sigma_v^2) \right), \end{aligned}$$

ω satisfies $-1 < \omega < 1 + 2\gamma + 2\mu_v / \sigma_v^2$, $\sigma_v = \sigma_Z v$, and $\mu_v = \mu_Z v$.

The value function when none of the liquid risky asset's investment constraints is active (unconstrained value function: (13iii) with $0 < b(x) < \tilde{b} \wedge vx$) is given by

$$f_u(x; x_0, b_0, k_3, k_4) = k_3 \int_{x_0}^x \exp\left(-\int_{x_0}^z \frac{\mu_Z}{\sigma_Z^2 b(y)} dy\right) dz + k_4, \quad (14iii)$$

¹⁰ b_D depends on the model structure: If MRR is active just before u_D then $b_D = vu_D$, if none of the constraints is active then b_D equals the unconstrained asset holding at u_D , and if the maximum investment is active just before u_D then $b_D = \tilde{b}$. Note that under the general model structure we have $b_D = \tilde{b}$.

where x_0 , b_0 , k_3 , and k_4 are constants, $b(x) = b_0 + \int_{x_0}^x g(y)dy$,

$$g(y) = \frac{\left(\frac{\mu_Z^2}{\sigma_Z^2} + 2\rho\right)b^2(y) + \left(2b(y) - \frac{\sigma_Y^2}{\sigma_Z^2} \frac{\mu_Z}{\mu_Y}\right) \frac{\mu_Z \mu_Y}{\sigma_Z^2}}{\mu_Z b(y) + \sigma_Y^2 \mu_Z / \sigma_Z^2},$$

and the liquid risky asset holding is solved numerically by $b(x^n) = b_0 + \sum_{i=0}^{n-1} g(x_0 + i\Delta x)\Delta x$, where $\Delta x = x^i - x^{i-1}$ for all $i \in \{1, 2, \dots\}$ and $x^i = x_0 + i\Delta x$ is the i 'th discrete buffer capital ratio value.

The value function under the maximum liquid risky asset investment ((13iii) with $b(x) = \tilde{b}$):

$$f_m(x; u_D) = a_+ e^{-d_+(\tilde{b})(u_D - x)} + a_- e^{-d_-(\tilde{b})(u_D - x)}, \quad (14iv)$$

where $a_+ = d_-(\tilde{b}) / (d_+(\tilde{b})d_-(\tilde{b}) - d_+^2(\tilde{b})) > 0$, $a_- = d_+(\tilde{b}) / (d_+(\tilde{b})d_-(\tilde{b}) - d_-^2(\tilde{b})) < 0$, and $d_{\pm}(\tilde{b}) = \frac{1}{\sigma^2(\tilde{b})} \left[-\mu(\tilde{b}) \pm \sqrt{\mu^2(\tilde{b}) + 2\rho\sigma^2(\tilde{b})} \right]$.

Finally, the value under the dividend payment, i.e., the case for (13iv):

$$f_d(x; u_D) = \frac{\mu(b_D)}{\rho} + (x - u_D), \quad (14v)$$

where b_D is the optimal liquid risky asset investment at the dividend barrier and in Proposition 2 it equals \tilde{b} , because here we assume that the general model structure is optimal. Constants $k_1 - k_4$ in (14ii) and (14iii) and the barriers u_C , u_D , and $x_0 - x_2$ are solved from the *smooth pasting and value matching conditions* at the barriers¹¹ as illustrated in Appendix B. If there is a solution to the equations in the appendix then the general model structure creates a smooth concave value function candidate in terms of (14) and then, by Proposition 1, this candidate equals the value function. The following proposition gives this result (the proof is in Appendix C).

Proposition 2 (Optimal policy) *Let the general structure be optimal. Then the value function can be represented as follows*

¹¹ We have value and 1st derivative matching at the impulse control u_C and value, 1st and 2nd derivative matching at all the other barriers.

$$V(x) = \begin{cases} f_d(x; u_D) & u_D \leq x \\ f_m(x; u_D) & x_2 \leq x \leq u_D \\ f_v(x; k_5(x_0, x_1, u_C, u_D; vx_0), k_6(x_0, x_1, u_C, u_D; vx_0)) & x_1 \leq x \leq x_2 \\ f_u(x; x_0, vx_0, k_3(x_0, u_C, u_D), k_4(x_0, u_C, u_D)) & x_0 \leq x \leq x_1 \\ f_v(x; k_1(u_C, u_D; 0), k_2(u_C, u_D; 0)) & u_C \leq x \leq x_0 \\ \Psi(x; b_D, \tilde{b}, u_D, K, \Delta) & 0 \leq x \leq u_C \end{cases} \quad (15i)$$

where the functions are given by (14) and Appendix B. Outside the recapitalization process the optimal liquid risky asset holding satisfies:

$$b(x) = \begin{cases} \tilde{b} & x_2 \leq x \leq u_D \\ xv & x_1 \leq x \leq x_2 \\ x_0 v + \int_{x_0}^x g(y) dy & x_0 \leq x \leq x_1 \\ xv & u_C \leq x \leq x_0 \end{cases} \quad (15ii)$$

Outside the recapitalization process it is optimal to order new capital if $x \leq u_C$ and to pay dividends if $u_D \leq x$.

The interpretation of the solution (15i) and (15ii) is as follows. The value function coincides with the function Ψ in the region where immediate ordering of capital issues is optimal. By condition (M4), below u_C the bank does not invest in the liquid risky asset. Above the region for Ψ we have different control strategies for the liquid risky asset investment. Under the general structure the market risk constraint is active immediately when $x > u_C$ and, therefore, the risky asset holding equals vx . Note that the liquid risky asset holding $b(\cdot)$ is discontinuous at the recapitalization barrier u_C , because at that point the holding jumps from 0 to vu_C (see Figure 2). x_0 is the point where MRR is no longer active and x_1 is the point, where MRR is active again. Hence, between x_0 and x_1 the bank uses the optimal unconstrained strategy. Then between x_1 and x_2 the MRR is active which gives $b(x) = vx$. Finally, before the dividend barrier the bank is maximally invested in the liquid risky asset, i.e., $b(x) = \tilde{b}$.

Comparative statics with respect to the recapitalization frictions are considered in Peura and Keppo (2006):

- The bank value falls with respect to recapitalization frictions K and Δ .
- Dividend barrier rises with respect to the recapitalization frictions.

- Recapitalization barrier falls in K . With low values of Δ the recapitalization barrier rises in Δ and with high values of Δ it falls due to the dividend constraint during the delay.

Further, as explained in Peura and Keppo (2006), our model is a volatility model: Cash flow volatility drives the optimal recapitalization and dividend barriers.

The following proposition gives the comparative statics with respect to model parameters that are not in Peura and Keppo (2006).

Proposition 3 (Comparative statics) *We have the following effects:*

- (a) *VOLATILITY: The value function falls in liquid risky asset volatility σ_Z .*
- (b) *MRR: VaR parameter k decreases the value function and the liquid risky asset holding.*

By the concavity of the value function, the volatility decreases the value of the bank. Since the MRR set constraints on the bank's portfolio choice it also decreases the value of the bank. Thus, the regulation penalizes the banking industry and decreases their expected discounted revenues. The definition of v implies that v falls in k . By MRR condition (M1) and Proposition 1, VaR is more active the higher the parameter k , and the value function falls in k . Thus, the barriers x_0 and x_2 rise in k and x_1 falls in k . Therefore, the bank uses less the unconstrained strategy.

By using our results we get the following limiting cases. First note that the unconstrained optimal strategy naturally rises in μ_Z / σ_Z^2 . Therefore, when μ_Z / σ_Z^2 falls the model with MRR approaches the model without MRR. Further, when μ_Z / σ_Z^2 approaches zero then our model approaches Peura and Keppo (2006), because then the liquid risky asset holding approaches zero. On the other hand, when μ_Z / σ_Z^2 is high enough then $x_0 = x_1$ and, thus, the unconstrained optimal strategy is no longer active. By Proposition 1, when the maximum risky asset investment \tilde{b} is zero our model equals Peura and Keppo (2006). When Δ or K get large then capital issue is no longer optimal. The critical values for Δ or K depends on the other model parameters, but the recapitalization frictions only affects the delay operator (14i) which approaches zero when the frictions approaches to infinity (see Peura and Keppo (2006)), i.e., leads to no option to recapitalize. In the absence of capital raising delay, new capital can be issued instantaneously and, therefore, there is a perfect control on the minimum level of capital. Clearly, in this case it is optimal to wait until the capital stock falls arbitrarily close to zero before issuing new capital and, hence, only ε

optimal policies can be constructed which set the capital issue barrier arbitrarily close to zero. If the regulatory constant k approaches zero then $x_0 \rightarrow u_C$ and $x_1 \rightarrow x_2$. This implies that the market risk requirement is never active and the model with MRR equals the model without it. If k approaches infinity then the market risk requirement is violated with all strictly positive liquid risky asset investments. Therefore, the risky asset holding must be zero and our model equals again Peura and Keppo (2006).

Based on the discussion above and (A5) in Appendix C, we get the following result.

Corollary 1 (Zero excess return) *If $\bar{\mu}_Z = r$ then the bank never invests in the liquid risky asset.*

Since in this case the liquid risky asset is never used, the VaR constraint and the maximum risky asset investment cannot be active and our model equals Peura and Keppo (2006). As discussed earlier, if (2) is under the risk-neutral probability measure then one could assume $\bar{\mu}_Z = r$ (see e.g. Björk (2004)) and Corollary 1 would apply. This would also be consistent with a typical corporate finance lesson: Companies should not do anything that the investors can do by themselves with the same costs, e.g., now invest in different liquid assets and funds. However, in practice banks do invest in liquid risky assets such as equities, derivative securities, and corporate bonds. At least, part of this (e.g. market making) is something that investors cannot cheaply replicate by themselves and we have assumed in Propositions 2 and 3 that the risky liquid investment is expected to add value and, thus, $\bar{\mu}_Z > r$. That is, since banks invest in risky liquid assets and the shareholders and the regulators allow this, we have simply assumed that there is a good reason for that.

5. Numerical example

In this section we illustrate our model with a numerical example. We use the following parameter values: For the recapitalization friction parameters we have $K = 0.25\%$ and $\Delta = 0.5$ years; the wedge between debt and equity finance $\rho = 4\%$; the illiquid portfolio parameters are $\mu_Y = 1\%$ and $\sigma_Y = 0.35\%$; for the liquid risky asset parameters we have $\mu_Z = 1\%$ and $\sigma_Z = 16\%$; the maximum proportion of RWA invested in the liquid risky asset $\tilde{b} = 11.5\%$. These are modified parameter values from Peura and Keppo (2006) and they are estimated from a sample of 29 U.S. commercial banks. However, since Peura and Keppo do not have liquid risky asset we have decreased their illiquid portfolio volatility and introduced the liquid risky asset process and the maximum proportion in the liquid risky asset. These

example parameter values are loosely motivated in Appendix D. As explained in the appendix, for simplicity we set $\mu_Y = \mu_Z = 1\%$. If our model is based on risk-neutral share holders (see equation (5)) then $\mu_Z = 1\%$ implies that under the objective probability measure the expected return of the liquid risky asset is the risk-free rate plus 1%. If (5) is under the risk-neutral probability measure then $\mu_Z = 1\%$ means that the bank's liquid asset investments generate 1% alpha, e.g., due to market making or this is just the bank shareholders' forward looking estimate.

5.1 Optimal strategy of the example bank

Next we calculate the optimal policy for the example bank with and without MRR, i.e., the cases where $k = 4$ and $k = 0$. In solving the optimal strategy, we test all the possible model structure candidates and, hence, not only the general structure in Proposition 2. The optimal structure is the one which gives a concave and smooth value function. Figure 3 illustrates the optimal liquid risky asset holdings and the value functions. As can be seen, the general model structure is not optimal, since the MRR is active only at low buffer capital levels. Note first that the liquid risky asset holding rises in the buffer capital and, therefore, also the bank's earnings volatility rises. This is consistent with the empirical findings in Heid, Porath, and Stolz (2003).

Naturally, the liquid risky asset holding and the value function with MRR are lower than the corresponding functions without MRR (Proposition 3 (b)). The average difference between the risky asset holdings is 0.18% of the risk weighted assets, and the average difference between the value functions is 0.37% of the bank value without MRR. Thus, the MRR decreases the value of the banking industry. The average buffer capital ratio levels are 3.18% with the MRR and 3.29% without it. This difference is from the lower cash flow volatility under MRR. That is, the buffer capital can be viewed as a hedge against possible losses and, therefore, the lower the cash flow volatility the less is hedged (i.e. the lower the buffer capital). The average liquid risky asset investments: 6.97% of the RWA with the MRR and 7.20% without it (see Figure 3).

The MRR has both negative and positive effects on the bank's liquidation probability. The positive effect is due to the fact that the MRR decreases the bank's cash flow volatility. However, since the MRR considers only the liquid asset holdings and ignores its effect on the other actions, the regulation has also a negative effect. We should first note that, as can be

seen from Figure 3, an optimally behaving bank without the MRR mostly operates in a way the bank regulators want them to behave: The bank decreases its liquid risky asset holding when the buffer capital decreases. The risk in using the MRR is that it may change the optimal behavior. The MRR is active when the buffer capital is low and this raises the marginal value from the liquid investments. Therefore, the other possible action, recapitalization, is found optimal only at a lower buffer capital level, i.e., when the bank is closer to liquidation, because then the marginal utility from that action is also high.

By Monte Carlo simulation with 10,000 runs and reflection principle we solve the liquidation probability with and without the MRR. The liquidation probability for a hundred-year time period with MRR is 0.78% and without it 0.79%. Thus, the improvement from the MRR seems to be slim, about 0.01%. Recall that MRR decreases the bank value on average by 0.37%. Hence, it is not obvious that the benefits from the MRR are greater than its costs.

[Figure 3 about here]

5.2 High liquid asset volatility

In order to illustrate that the MRR can also have a negative total effect on the bank's liquidation probability, we now consider cases with different liquid risky asset volatility.

Let us first set $\sigma_z = 30\%$. Figure 4 illustrates the optimal risky asset holding and the value function. Note that the optimal structure is the same as in Figure 3. The main difference is that the optimal barriers have increased in the buffer capital ratio, shifting to the right. This can be viewed as a hedge against the higher cash flow volatility. Further, by the increased volatility, the bank invests less in the liquid risky asset. The average liquid risky asset holding with the MRR is 5.99% of the RWA and without it 6.67%. Now the average buffer capital ratios are close: with the MRR it is 3.81% and without it 3.82%. Therefore, also the liquidation probabilities over the hundred-year time period are almost equal: 0.907% (with the MRR) and 0.914% (without the MRR).

[Figure 4 about here]

Let us analyze the liquidation probability and the average buffer capital ratio as a function of the liquid risky asset volatility. Figure 5 illustrates the situation. The higher the

volatility the smaller the benefit from the MRR, and finally at 40% it has a negative effect on the liquidation probability. This effect is driven by the recapitalization level. That is, when the volatility is high then the bank under the MRR allows the buffer capital to fall too close to zero (relative to the risks in its cash flows) before it starts the recapitalization process. Clearly, this raises the liquidation probability during the recapitalization process. As discussed in Section 2, our recapitalization process (M3) and (M4) can be viewed as a simple model for the recapitalization with liquidation risk. Our results are driven by the recapitalization cost and the fact that the liquidation probability falls in the buffer capital and rises in the cash flow volatility. Thus, any model with these realistic characteristics would give the same qualitative results.

[Figure 5 about here]

6 Conclusions

We have derived a new model for bank behavior. The bank maximizes its value by selecting optimal recapitalization, dividends, and liquid asset strategies. These decisions affect each other and, therefore, we have considered them in the same optimization model.

The optimal bank behavior is as follows. The bank triggers recapitalization at a low buffer capital ratio level and at a high buffer capital ratio level it pays dividends. Between these levels the bank's liquid risky asset investment is an increasing function of the buffer capital ratio level and, therefore, the liquid risk-free asset investment falls in the buffer capital ratio. Thus, the closer to liquidation the lower the cash flow volatility.

Based on our numerical example the market risk requirement can be questioned. First, in the numerical example the requirement decreases the bank's liquidation probability only by 0.01% over a hundred-year time period, while the bank value falls by 0.37%. Second, if a bank invests in highly risky liquid assets then the market risk requirement raises the liquidation probability. That is, it is not guaranteed that the requirement has positive effects on the banking industry.

We have shortly discussed some extensions of the model. First, we have already solved numerically an extension where the liquid risky asset holding is nonzero during the recapitalization delay. This does not change our qualitative results but complicates the model (see footnote 6). Second, our recapitalization model with delay can be viewed as a

proxy for models (with or without delay) that have the same liquidation probability and the same recapitalization cost. In addition to these, there are also several other ways the model can be extended. For instance, frictions such as taxes and transaction costs, partially observed bank assets as well as extreme events by jump processes could improve the model fit (e.g. Cadenillas et al. (2007), Framstad et al. (2001)). These will complicate the model setup and, therefore, not considered in the present paper.

Our model could also be extended to consider different risk measures, such as coherent risk measures.¹² However, note that our results were not primarily driven by the used risk measure, instead by the fact that the market risk requirement focuses only on the banks' portfolio choice. We believe that as long as the regulation ignores banks' other actions (recapitalization and dividend payment) it is likely to face similar problems as reported in the present paper. Thus, if new market risk requirements are developed they should be robust: decrease banks' liquidation probability under different stochastic processes for the liquid and illiquid assets. Further, the costs of the regulation should be analyzed carefully.

¹² Different risk measures are analyzed e.g. in Heyde et al. (2006).

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Appendix A: An admissible policy π^* in Proposition 1.

(i) $\{t_i^{\pi^*}, s_i^{\pi^*}\}$ solve:

$$t_1^{\pi^*} = \inf \{t \geq 0 : f(X_t^{\pi^*}) = Mf(X_t^{\pi^*})\}, \quad t_{i+1}^{\pi^*} = \inf \{t \geq t_i^{\pi^*} + \Delta : f(X_t^{\pi^*}) = Mf(X_t^{\pi^*})\},$$

$$s_i^{\pi^*} = u_D - X_{(t_i^{\pi^*} + \Delta)^-}^{\pi^*} \quad \text{for all } i \geq 1.$$

(ii) Outside the recapitalization processes, i.e., on $\bigcup_i [t_i^{\pi^*} + \Delta, t_{i+1}^{\pi^*}]$, $\{X_t^{\pi^*}, L_t^{\pi^*}\}$ solve:

$$\int_0^\infty \mathbf{I}_{\{X_t^{\pi^*} < u_D\}} dL_t^{\pi^*} = 0, \quad X_t^{\pi^*} \leq u_D,$$

$$dX_t^{\pi^*} = (\mu_Y + \mu_Z b(X_t^{\pi^*})) dt + \sigma_Y dW_t^Y + b(X_t^{\pi^*}) \sigma_Z dW_t^Z + d \left(\sum_i s_i^{\pi^*} \mathbf{I}_{\{t_i^{\pi^*} + \Delta \leq t\}} \right) - dL_t^{\pi^*},$$

where $b(x)$ solves $\max_{b \in [0, \bar{b} \wedge vx]} (A_b - \rho) f(x) = 0$ for all $x \in [u_C, u_D]$.

During the recapitalization processes, i.e., on $\bigcup_i (t_i^{\pi^*}, t_i^{\pi^*} + \Delta)$, $\{X_t^{\pi^*}, L_t^{\pi^*}\}$ solve:

$$dL_t^{\pi^*} = 0, \quad dX_t^{\pi^*} = \mu_Y dt + \sigma_Y dW_t^Y.$$

The interpretation of (i) is that every time Mf and f coincide outside the capital order periods, a new capital issue is ordered. $s_i^{\pi^*}$ states that at every capital collection, just enough capital is raised to shift the capital stock to u_D . Note that $s_i^{\pi^*}$ is solved from (11). If $X_{(t_i^{\pi^*} + \Delta)^-}^{\pi^*} > u_D$ then $s_i^{\pi^*} < 0$ and it is understood as a dividend payment at $t_i^{\pi^*} + \Delta$. According to (ii), dividends are paid so as to never let the capital stock rise above u_D outside the order periods. During the order periods, capital stock cannot be controlled.

Appendix B: Value and smooth pasting conditions.

The constants in (14ii) and (14iii) are solved from the smooth pasting and value matching conditions:

$$\begin{aligned} \text{at } u_C: & \begin{cases} k_1(u_C, u_D; b_D) = \frac{1}{J(u_C; \gamma)} \left[\Psi(u_C; b_D, u_D, K, \Delta) (\gamma + 1) E(u_C; \gamma) - \frac{\partial \Psi(x; b_D, u_D, K, \Delta)}{\partial x} \Big|_{x=u_C} E(u_C; \gamma + 1) \right] \\ k_2(u_C, u_D; b_D) = \frac{1}{J(u_C; \gamma)} \left[\frac{\partial \Psi(x; b_D, u_D, K, \Delta)}{\partial x} \Big|_{x=u_C} D(u_C; \gamma + 1) - \Psi(u_C; b_D, u_D, K, \Delta) (\gamma + 1) D(u_C; \gamma) \right] \end{cases} \\ \text{at } x_0: & \begin{cases} k_3(x_0, u_C, u_D) = -k_1(u_C, u_D) (\gamma + 1) D(x_0; \gamma) + k_2(u_C, u_D) (\gamma + 1) E(x_0; \gamma) \\ k_4(x_0, u_C, u_D) = k_1(u_C, u_D) D(x_0; \gamma + 1) + k_2(u_C, u_D) E(x_0; \gamma + 1) \end{cases} \quad (\text{A1}) \\ \text{at } x_1: & \begin{cases} k_5(x_0, x_1, u_C, u_D; b) = \frac{1}{J(x_1; \gamma)} H_1(x_0, x_1, u_C, u_D; b) \\ k_6(x_0, x_1, u_C, u_D; b) = \frac{1}{J(x_1; \gamma)} H_2(x_0, x_1, u_C, u_D; b) \end{cases} \end{aligned}$$

where $J(x; \gamma) = (\gamma + 1)[D(x; \gamma + 1)E(x; \gamma) + E(x; \gamma + 1)D(x; \gamma)]$,

$$H_1(x_0, x_1, u_C, u_D; b) = (\gamma + 1)E(x_1; \gamma)p(x_0, u_C, u_D; b) - E(x_1; \gamma + 1)k_3(x_0, u_C, u_D)e(x_1; x_0, b),$$

$$H_2(x_0, x_1, u_C, u_D; b) = (\gamma + 1)D(x_1; \gamma)p(x_0, u_C, u_D; b) + D(x_1; \gamma + 1)k_3(x_0, u_C, u_D)e(x_1; x_0, b),$$

$$p(x_0, u_C, u_D; b) = k_3(x_0, u_C, u_D) \int_{x_0}^{x_1} e(z; x_0, b) dz + k_4(x_0, u_C, u_D),$$

$$\text{and } e(z; x_0, b) = \exp \left(-\frac{\mu z}{\sigma^2} \int_{x_0}^z \left(b + \int_{x_0}^y g(z) dz \right)^{-1} dy \right).$$

The barrier values are solved from the following value matching and smooth pasting conditions at the barriers:

$$\begin{aligned} \text{at } x_0: & \frac{\partial^2 f_v(x; k_1(u_C, u_D; 0), k_2(u_C, u_D; 0))}{\partial x^2} \Big|_{x=x_0} = \frac{\partial^2 f_u(x; x_0, vx_0, k_3(x_0, u_C, u_D), k_4(x_0, u_C, u_D))}{\partial x^2} \Big|_{x=x_0} \\ \text{at } x_1: & \frac{\partial^2 f_v(x; k_5(x_0, x_1, u_C, u_D; vx_0), k_6(x_0, x_1, u_C, u_D; vx_0))}{\partial x^2} \Big|_{x=x_1} = \frac{\partial^2 f_u(x; x_0, vx_0, k_3(x_0, u_C, u_D), k_4(x_0, u_C, u_D))}{\partial x^2} \Big|_{x=x_1} \\ \text{at } x_2: & \begin{cases} \frac{\partial^2 f_v(x; k_5(x_0, x_1, u_C, u_D; vx_0), k_6(x_0, x_1, u_C, u_D; vx_0))}{\partial x^2} \Big|_{x=x_2} = \frac{\partial^2 f_m(x; u_D)}{\partial x^2} \Big|_{x=x_2} \\ \frac{\partial f_v(x; k_5(x_0, x_1, u_C, u_D; vx_0), k_6(x_0, x_1, u_C, u_D; vx_0))}{\partial x} \Big|_{x=x_2} = \frac{\partial f_m(x; u_D)}{\partial x} \Big|_{x=x_2} \\ f_v(x_2; k_5(x_0, x_1, u_C, u_D; vx_0), k_6(x_0, x_1, u_C, u_D; vx_0)) = f_m(x_2; u_D) \end{cases} \end{aligned} \quad (\text{A2})$$

such that $0 < u_C < x_0 < x_1 < x_2 < u_D < \infty$.

Appendix C: Proof of Proposition 2.

The value function has the following parts:

$$V(x) = \begin{cases} f_d(x) & u_D \leq x \\ f_b(x) & u_C < x < u_D \\ \Psi(x) & 0 \leq x \leq u_C \end{cases} \quad (\text{A3})$$

where $f_b(\cdot)$ solves $\max_{b \in [0, b \wedge vx]} (A_b - \rho) f_b(x) = 0$ and it is the value function that is derived when (13iii) is at equality. We construct a function that solves (13ii) with equality for $x \leq u_C$, (13iii) with equality for $x \in [u_C, u_D]$, and (13iv) with equality for $x \geq u_D$. The function will be continuously differentiable at the impulse control barrier u_C and twice continuously differentiable at the singular control barrier u_D . Peura and Keppo (2006) show that $\Psi(\cdot)$ and

$f_d(\cdot)$ equal the value function when $x \leq u_C$ and $x \geq u_D$, respectively. Therefore, we consider only the f_b part in this proof.

C.1 Parts of f_b function

f_b function can have several parts: the part where the constraints for $b(x)$ are not active, the part where $b(x) = \tilde{b}$, and the parts where $b(x) = vx$.

Next we solve the different parts of f_b . From (13iii) we get the following ODE

$$\max_{b \in [0, \tilde{b} \wedge vx]} \left[\frac{1}{2} (\sigma_Y^2 + b^2(x) \sigma_Z^2) V''(x) + (\mu_Y + \mu_Z b(x)) V'(x) - \rho V(x) \right] = 0. \quad (\text{A4})$$

UNCONSTRAINED SOLUTION, f_u

If there are no constraints then by the first order condition get from (A4):

$$b(x) = -\frac{\mu_Z V'(x)}{\sigma_Z^2 V''(x)}, \quad (\text{A5})$$

which with (A4) gives $V(x) = k_3 \int_{x_0}^x \exp \left(-\int_{x_0}^z \frac{\mu_Z}{\sigma_Z^2 b(y)} dy \right) dz + k_4$, i.e., (14iii). By Højgaard and Taksar (2004), this value function part is increasing and concave. Therefore, (A5) implies $b(x) > 0$. From Højgaard and Taksar (2004) we get

$$b'(x) = \frac{\left(\frac{\mu_Z^2}{\sigma_Z^2} + 2\rho \right) b^2(x) + \left(2b(x) - \frac{\sigma_Y^2}{\sigma_Z^2} \frac{\mu_Z}{\mu_Y} \right) \frac{\mu_Z \mu_Y}{\sigma_Z^2}}{\mu_Z b(x) + \sigma_Y^2 \mu_Z / \sigma_Z^2}. \quad (\text{A6})$$

Instead of applying the Inverse Function Theorem as in Højgaard and Taksar (2004) we

directly use $b(x) = b_0 + \int_{x_0}^x b'(y) dy$, because the theorem can be applied only if

$x_0 > \frac{\mu_Z \mu_Y}{\mu_Z^2 + 2\rho \sigma_Z^2} \left(\sqrt{1 + \left(\frac{\mu_Z^2}{\sigma_Z^2} + 2\rho \right) \frac{\sigma_Y^2}{\mu_Y^2}} - 1 \right)$ and there is no guarantee that this is the case.

Note that when the MRR is not active, the optimal investment at the recapitalization barrier is solved from (A4) and (A5) and by using the smooth pasting and value matching conditions at u_C as follows. First, from (A4) and (A5) we get

$$\frac{1}{2} \sigma_Y^2 (V''(x))^2 + (\mu_Y V'(x) - \rho V(x)) V''(x) - \frac{1}{2} \frac{\mu_Z^2}{\sigma_Z^2} (V'(x))^2 = 0,$$

which is a parabolic function of $V''(x)$. Next taking $x \downarrow u_C$ and using the value and first derivative matching at the impulse control u_C we get

$$\frac{1}{2} \sigma_Y^2 S^2 + \left(\frac{\partial \Psi(x; b_D, u_D, K, \Delta)}{\partial x} \Big|_{x=u_C} \mu_Y - \rho \Psi(u_C; b_D, u_D, K, \Delta) \right) S - \frac{1}{2} \left(\frac{\mu_Z}{\sigma_Z} \frac{\partial \Psi(x; b_D, u_D, K, \Delta)}{\partial x} \Big|_{x=u_C} \right)^2 = 0,$$

where $S = V''(u_C)$. Since V is concave we take the negative solution to this equation:

$$S(b_D, u_D, K, \Delta) = -\frac{1}{\sigma_Y} \widehat{\Psi}(u_C; b_D, u_D, K, \Delta) - \frac{1}{\sigma_Y} \sqrt{\widehat{\Psi}^2(u_C; b_D, u_D, K, \Delta) + \sigma_Y^2 \left(\frac{\mu_Z}{\sigma_Z} \frac{\partial \Psi(x; b_D, u_D, K, \Delta)}{\partial x} \Big|_{x=u_C} \right)^2},$$

where $\widehat{\Psi}(u_C; b_D, u_D, K, \Delta) = \frac{\partial \Psi(x; b_D, u_D, K, \Delta)}{\partial x} \Big|_{x=u_C} \mu_Y - \rho \Psi(u_C; b_D, u_D, K, \Delta)$. This gives the optimal investment at the recapitalization barrier when VaR is not active:

$$b_C(b_D, u_D) = -\frac{\mu_Z}{\sigma_Z^2} \frac{\partial \Psi(x; b_D, u_D, K, \Delta)}{\partial x} \Big|_{x=u_C} / S(b_D, u_D, K, \Delta),$$

where b_D is the holding at u_D .

MARKET RISK REQUIREMENT ACTIVE, f_v

Next we consider the case where the VaR constraint is active. In this case $b(x) = vx$ and, by (A4), we have the following ODE:

$$\frac{1}{2} V''(x) (\sigma_Y^2 + v^2 x^2 \sigma_Z^2) + V'(x) (\mu_Y + vx \mu_Z) - \rho V(x) = 0 \quad (\text{A7})$$

and, by Højgaard and Taksar (2001), its solution is (14ii). The concavity is given by the following lemma.

Lemma A1. *Let $\frac{\partial f_v(x; k_1, k_2)}{\partial x} \Big|_{x=u} > 0$ and $\frac{\partial^2 f_v(x; k_1, k_2)}{\partial x^2} \Big|_{x=u} < 0$, where $u \geq 0$, $vu = b(u)$, and b is given by (A5). Then $f_v(x; k_1, k_2)$ is concave for all $x \geq 0$.*

Proof. Differentiating (A7) with respect to x gives

$$\frac{1}{2} V'''(x) (\sigma_Y^2 + v^2 x^2 \sigma_Z^2) + V''(x) \left((\mu_Z + v \sigma_Z^2) vx + \mu_Y \right) - V'(x) (\rho - v \mu_Z) = 0 \quad (\text{A8})$$

which implies that $V'''(x) \geq 0$ iff

$$V'(x) (\rho - v \mu_Z) \geq V''(x) \left((\mu_Z + v \sigma_Z^2) vx + \mu_Y \right). \quad (\text{A9})$$

This holds if $\rho - v \mu_Z \geq 0$ (since μ_Z and μ_Y are assumed to be positive).

Next let us consider the case $\rho - v \mu_Z < 0$. When $x = u$ we have $vu = b(u)$ and, by (A5) and (A9), $-vu \left(\frac{\sigma_Z^2}{\mu_Z} \rho + \mu_Z \right) \leq \mu_Y$ which holds for all $u \geq 0$. Thus, $V'''(u) > 0$ and, by assumption, $V''(u) < 0$. Therefore, $V''(x)$ is an increasing function for all x close to u . If there is a point $x \geq u$, where $V''(x) = 0$, then by (A9), $V'''(x) < 0$ and, hence, $V''(x) \leq 0$ for $x \geq u$.

Because $V''(u) < 0$ and $V'''(u) > 0$, $V(x)$ is concave for all $x \in (u - \varepsilon, u]$ and for small $\varepsilon > 0$. Differentiating (A8) gives

$$\frac{1}{2}V''''(x)(\sigma_Y^2 + v^2x^2\sigma_Z^2) + V'''(x)\left((\mu_Z + 2v\sigma_Z^2)vx + \mu_Y\right) + V''(x)\left[(\mu_Z + v\sigma_Z^2)v - (\rho - v\mu_Z)\right] = 0,$$

where the last term is negative. If there is a point where $V'''(x) = 0$ then $V''''(x) > 0$. This implies that $V'''(x) \geq 0$ and, because $V''(u) < 0$, we have $V''(x) \leq 0$ for $x \leq u$. \square

MAXIMUM RISKY ASSET CONSTRAINT ACTIVE, f_m

Finally, we consider the case where the maximum risky asset constraint is active. Then $b(x) = \tilde{b}$ and we have the following ODE

$$\frac{1}{2}V''(x)(\sigma_Y^2 + \tilde{b}^2\sigma_Z^2) + V'(x)(\mu_Y + \tilde{b}\mu_Z) - \rho V(x) = 0$$

and the value function for this domain of x is given by (14iv) (see e.g. Peura and Keppo (2006)).

C.2 Optimality

First we present the following lemma.

Lemma A2. *If there are no constraints and if the value function is concave then the optimal investment policy is strictly convex.*

Proof. Differentiating (A6) and simplifying gives

$$b''(x) = \left(\frac{\mu_Z(\mu_Z + 2\mu_Y)\sigma_Y^2 + 2(\mu_Z^2 + 2\rho\sigma_Z^2)\sigma_Y^2 b(x) + \sigma_Z^2(\mu_Z^2 + 2\rho\sigma_Z^2)b^2(x)}{\mu_Z(\sigma_Y^2 + b(x)\sigma_Z^2)^2} \right) b'(x),$$

where the term in parentheses is positive since $b(x) > 0$ and all the variables are positive. Thus, $b''(x)$ and $b'(x)$ have the same sign. If they were negative then the investment policy would be concave and decreasing. Therefore, in this case there would exist x with $b(x) < 0$. This contradicts (A5) and, thus, we get the result. \square

Lemma A2 implies that the linear value-at-risk constraint will be active at most twice. As in Proposition 2, we assume here that there are two distinct intervals for x , where $b(x) = vx$. All the other optimal structure cases follow from this case. More specifically, under the most general model structure the value-at-risk constraint is active when x is low or high: $b(x) = vx$ for all $x \in [u_C, x_0]$ and $x \in [x_1, x_2]$, where $u_C < x_0 < x_1 < x_2$. Between x_0 and x_1 the bank invests based on the optimal unconstrained policy and the smoothness of $b(\cdot)$ at x_0 gives

$b_0 = vx_0$. Finally, $b(x) = \tilde{b}$ for $x \in [x_2, u_D]$. Thus, the general structure implies the following value function candidate:

$$f_C(x) = \begin{cases} f_m(x) & x_2 \leq x \leq u_D \\ f_v(x; k_5, k_6) & x_1 \leq x \leq x_2 \\ f_u(x; x_0, k_3, k_4) & x_0 \leq x \leq x_1 \\ f_v(x; k_1, k_2) & u_C \leq x \leq x_0. \end{cases}$$

Now we state the following lemma.

Lemma A3. *If there is a solution to (A1) and (A2) then u_C, x_0, x_1, x_2, u_D , and k_1 - k_6 satisfy the value matching and smooth pasting conditions at u_C, x_0, x_1, x_2 , and u_D .*

Proof. Value matching and smooth pasting conditions at u_C :

$$\begin{aligned} \Psi(u_C; b_D, u_D, K, \Delta) &= k_1 D(u_C; \gamma + 1) + k_2 E(u_C; \gamma + 1) \\ \frac{\partial \Psi(x; b_D, u_D, K, \Delta)}{\partial x} \Big|_{x=u_C} &= -k_1(\gamma + 1)D(u_C; \gamma) + k_2(\gamma + 1)E(u_C; \gamma) \end{aligned}$$

which gives k_1 and k_2 in (A1). Note that at the impulse barrier u_C we have the smooth pasting only up to the first derivative. At every other barrier we have also the second derivative matching and these conditions give (A1) and (A2). \square

By Proposition 1 (b), to prove that $f_C(x) = f_b(x)$ we have to show:

(C1) each part of f_C is twice differentiable and concave

(C2) each part of f_C solves $\max_{b \in [0, \tilde{b} \wedge vx]} (A_b - \rho) f_C(x) = 0$.

Given Lemma A3 and (C1) and (C2), f_C is concave and we can use Ito's lemma in Proposition 1. As discussed earlier, Peura and Keppo (2006) show that the value function satisfies (C1) and (C2) above u_D and below u_C . Højgaard and Taksar (2004) shows that (C1) and (C2) hold with f_u . For f_v we first note that (14ii) is smooth enough and Lemma A1 gives the concavity. By Lemma A2, (C2) holds with $f_v(x)$ for all $x \in [u_C, x_0]$, where $x_0 = \inf \left\{ x > u_C \mid xv > \frac{-\mu_Z f_C'(x)}{\sigma_Z^2 f_C''(x)} \right\}$. That is, by the concavity of the unconstrained strategy $xv < \frac{-\mu_Z f_v'(x)}{\sigma_Z^2 f_v''(x)}$ for all $x \in (u_C, x_0)$ and, hence, $b(x) = vx$ is optimal and condition (C2) equals (A7) for all $x \in [u_C, x_0]$. For the same reasons condition (C2) holds for all $x \in [x_1, x_2]$. Therefore, $f_v(\cdot)$ satisfies conditions (C1) and (C2).

Peura and Keppo (2006) show that $f_m(\cdot)$ satisfies (C1) and (C2). Thus, each part of f_C satisfies the conditions (C1) and (C2) which gives that $f_C(x) = f_b(x)$. However, even if there exists a solution to (15i) this has to be solved numerically. This means solving numerically the value matching and smooth pasting conditions with all the model structure candidates, i.e., not only with the general model structure. The one that gives numerically the best fit is selected as the optimal structure and the corresponding strategy gives the optimal strategy.

Appendix D: Example parameters.

In order to select the model parameters of the example bank, we use the data in Peura and Keppo (2006) and assume that the risky asset holding is a linear function of the buffer capital: $b(x) = \beta x$. This implies $\sigma_Y^2 + b^2(x)\sigma_Z^2 = \sigma_Y^2 + \beta^2\sigma_Z^2x^2$ and $\mu_Y + b(x)\mu_Z = \mu_Y + \beta\mu_Zx$. Hence, we estimate the following regressions from Table 1 (see below):

$$\begin{aligned} Y_1 &= (0.35\%)^2 + (20.28\%)^2 X^2 \\ (R^2: 42\%) \quad (t\text{-stat: } 0.47) \quad (t\text{-stat: } 4.39) \\ Y_2 &= 1.00\% - 4.22\% X, \\ (R^2: 4\%) \quad (t\text{-stat: } 5.49) \quad (t\text{-stat: } -1.15) \end{aligned}$$

where Y_1 is the variance of the return on risk weighted assets, X is the buffer capital ratio, and Y_2 is the average return on risk weighted assets. The regression for the variance has a high R^2 (42%) and the average return has a low R^2 (4%). The high R^2 is important since, as illustrated in Peura and Keppo (2006), the variance is the driving factor in our model. The above regressions imply the following parameter estimates: $\sigma_Y = 0.35\%$, $\beta\sigma_Z = 20.28\%$, $\mu_Y = 1.00\%$, and $\beta\mu_Z = -4.22\%$. Based on the t-statistics the most significant parameters are the liquid risky asset volatility and the average net revenue from the illiquid portfolio. The negative average excess return from the liquid risky asset suggests that the risky asset holding also includes some investments in the basic banking business because then the risky asset volatility raises credit losses which decreases the average return. However, we do not use the negative value since it is not convenient forward looking estimate in our model.

Based on Table 1, the average buffer capital ratio is 4.61% and therefore we set $m = 95.39\%$. We assume that the regulatory multiplier k equals 4 which is its maximum value. Now, by the definition of VaR, $\beta \leq v = m / (k \cdot 2.33 \cdot \sqrt{\frac{10}{250}} \cdot \sigma_Z)$, i.e., $\beta\sigma_Z \leq m / (k \cdot 2.33 \cdot \sqrt{\frac{10}{250}})$. Thus, we get $\beta\sigma_Z \leq 51.17\%$. On the other hand, based on the regression model, the 99% confidence interval for $\beta_0\sigma_Z$ is [12.33%, 25.90%], which suggests that the example bank does

not violate the VaR constraint at most of the buffer capital ratio levels. However, as we will see, there are still buffer levels, where VaR is locally active since, by Figure 3, the investment slope β is not constant.

[Table 1 about here]

We estimate σ_Z by using the model without the market risk requirement. Motivated by (14iii), here we simply assume that $b(x) = \int_0^x g(y)dy \approx \frac{(\mu_Z^2/\sigma_Z^2 + 2\rho)\bar{x}^2 + (2\bar{x}\mu_Y - \sigma_Y^2/\sigma_Z^2)\mu_Z/\sigma_Z^2}{(\sigma_Y^2/\sigma_Z^2 + \bar{x}^2)\mu_Z}x$ for all $x \geq 0$, i.e., the recapitalization buffer is zero and \bar{x} is the buffer level where $b(\bar{x}) = \bar{x}$. Next we set $\rho = 4\%$ and $\bar{x} = 2\%$ that are the wedge between debt and equity finance in Peura and Keppo (2006) and about the half of the dividend barrier in that paper. Further, we set the expected risky asset's excess return $\mu_Z = 1\%$ (i.e. equal to μ_Y) since we cannot use the negative estimate from the regression (it is also insignificant). Now fitting the $b(x)$ -approximation to $\beta_b\sigma_Z = 20.28\%$ gives $\sigma_Z = 16\%$. Further, assuming that the bank with the highest cash flow volatility is maximally invested in the liquid risky asset ($b(x) = \tilde{b}$) gives $\tilde{b} = 11.5\%$.

For the recapitalization friction parameters K and Δ we use the estimates from Peura and Keppo (2006), i.e., $K = 0.25\%$ and $\Delta = 0.5$ years. In summary, we have the following model parameters for the example bank: $\sigma_Y = 0.35\%$, $\mu_Y = 1\%$, $\sigma_Z = 16\%$, $\mu_Z = 1\%$, $\rho = 4\%$, $K = 0.25\%$, and $\Delta = 0.5$ years.

Appendix E: Used notation and abbreviations.

MRR = Market risk requirement

VaR = Value-at-risk

RWA = Risk weighted assets

R_t = Risk weighted asset at time t

r = risk-free rate

\bar{X}_t = buffer equity

$m = 1 - \bar{X}_t / R_t$

\tilde{b}^m = maximum investment in the liquid risky asset

b_t^m = liquid risky asset investment

Y_t = cumulative cash flows from the illiquid portfolio

i = interest income as a percent of total risk weighted assets

$A(t)$ = the number of customers who have defaulted through time t

λ = the rate of a Poisson process

$\{D_j\}$ = i.i.d. default losses (proportional to RWA) with mean μ_d and standard deviation σ_d

W_t^Y = a standard Wiener process

σ_Y = net revenue volatility of the illiquid portfolio

$\bar{\mu}_Y$ = expected proportional net revenue from the illiquid portfolio

$\bar{\mu}_Z$ = expected return of the liquid risky asset

σ_Z = volatility of the liquid risky asset

Z_t = profits from the liquid financial assets

W_t^Z = a standard Wiener process

k = VaR multiplier

$$v = m / \left(k \cdot 2.33 \cdot \sqrt{\frac{10}{250}} \cdot \sigma_Z \right)$$

Δ = recapitalization delay

K = recapitalization cost

$\bar{\pi}$ = admissible capital buffer control policy

$L^{\bar{\pi}}$ = cumulative amount of dividends

$\{t_i^{\bar{\pi}}\}$ = increasing sequence of order times of new capital issues

$\{s_i^{\bar{\pi}}\}$ = the amounts of capital raised at each issue of capital

Π = class of admissible policies

$\{F_t\}$ = information filtration generated by the Wiener processes W_t^Z and W_t^Y

$\bar{\tau}_{\bar{\pi}}$ = liquidation time

$\bar{V}_{\bar{\pi}}(\bar{X}_0)$ = value of the bank under policy $\bar{\pi}$

ρ = wedge between debt and equity finance

$\bar{V}(\bar{X})$ = value of an optimally managed bank

$$X_t = \bar{X}_t / R_t$$

π = admissible buffer capital ratio control policy

$$\mu_Y = \bar{\mu}_Y + \tilde{b}r$$

$$\mu_Z = \bar{\mu}_Z - r$$

τ_{π} = first credit risk violation time

$$V(\bar{X}_0 / R_0) = \bar{V}(\bar{X}_0) / R_0$$

$\bar{\pi}^*$ = optimal capital buffer control policy

π^* = optimal capital buffer ratio control policy

$$L_t^{\pi^*} = \int_0^t R_u dL_u^{\pi^*}$$

$$t_i^{\pi^*} = t_i^{\pi^*}$$

$$s_i^{\pi^*} = R_{t_i^{\pi^*} + \Delta} s_i^{\pi^*}$$

M = recapitalization operator

$$A_b f(x) = \frac{1}{2}(\sigma_Y^2 + b^2 \sigma_Z^2) f''(x) + (\mu_Y + b\mu_Z) f'(x)$$

u_C = recapitalization barrier

x_0 = barrier for MRR

x_1 = barrier for MRR

x_2 = barrier for maximum liquid risky asset investment

u_D = dividend barrier

k_1, k_2, k_3, k_4 = value function parameters

b_0 = liquid risky asset holding when the buffer capital ratio is x_0

b_D = liquid risky asset holding at the dividend barrier

$\Psi(x; b_D, u_D, K, \Delta)$ = recapitalization function

$$\mu(b) = \mu_Y + b\mu_Z$$

$$\sigma(b) = \sqrt{\sigma_Y^2 + b^2 \sigma_Z^2}$$

$f_v(x; k_1, k_2)$ = value function when MRR is active

$f_u(x; x_0, b_0, k_3, k_4)$ = value function when the constraints are not active

$f_m(x; u_D)$ = value function under the maximum liquid risky asset investment

$f_d(x; u_D)$ = value under the dividend payment

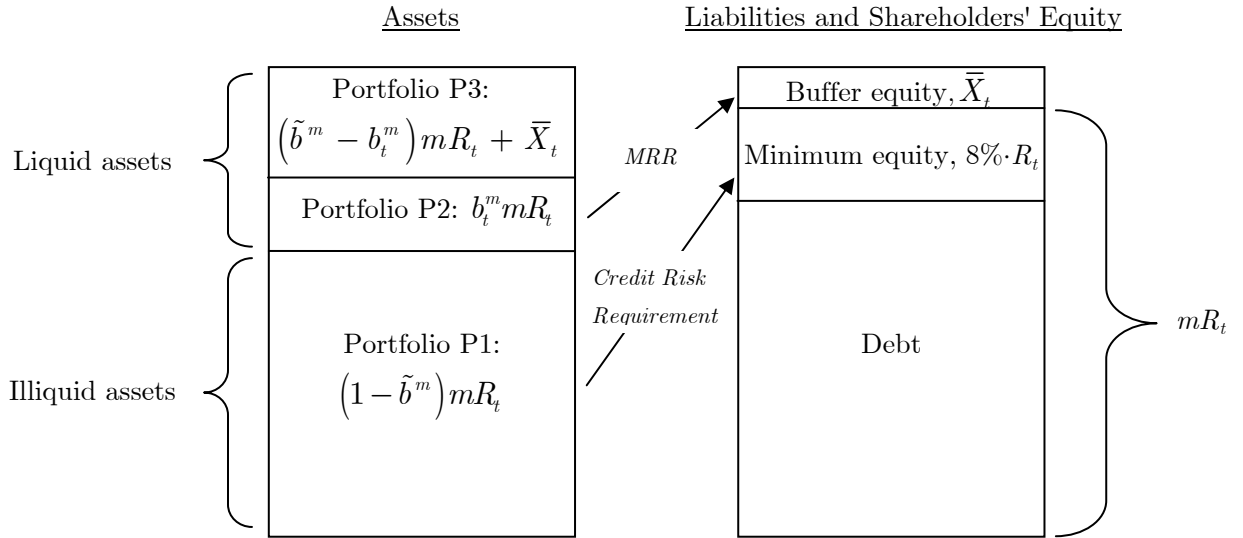


Figure 1. Assets, liabilities and shareholders' equity. Portfolio P1 is the illiquid assets (due to P1, there is the credit risk requirement), portfolio P2 is the liquid risky assets (P2 satisfies the MRR), and portfolio P3 is the liquid risk-free portfolio. MRR is the market risk requirement, $m = 1 - \bar{X}_t / R_t$, R_t is the risk weighted assets at time t , \bar{X}_t is the buffer equity, $\tilde{b}^m \in (0,1)$, and $b_t^m \in [0, \tilde{b}^m]$.

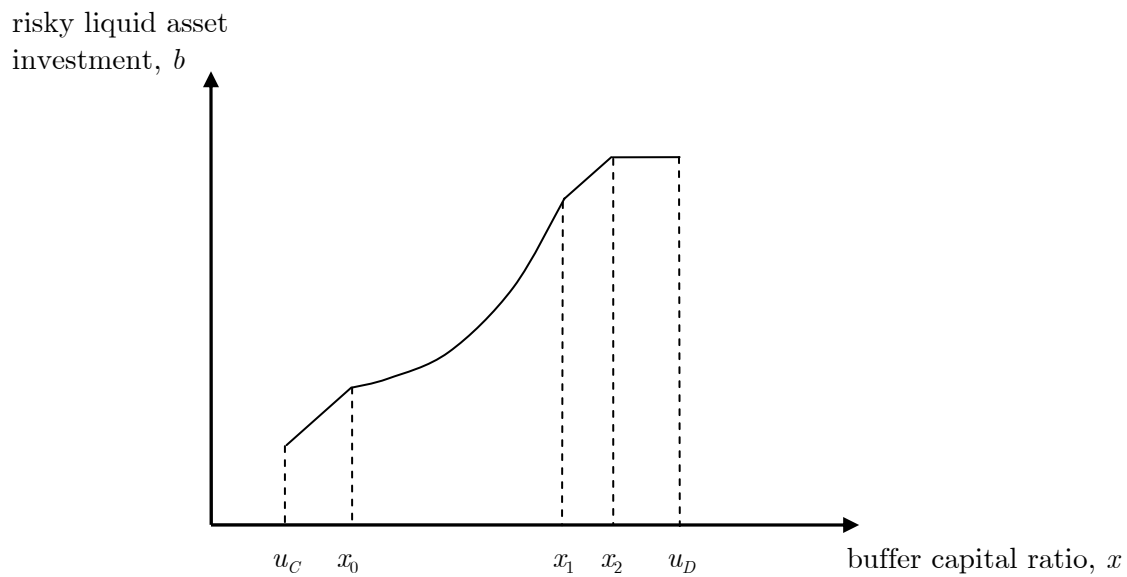


Figure 2. Risky liquid asset investment. The MRR is active on $[u_C, x_0]$ and on $[x_1, x_2]$, the maximum investment \tilde{b} is active on $[x_2, u_D]$, and none of the constraints is active on (x_0, x_1) .

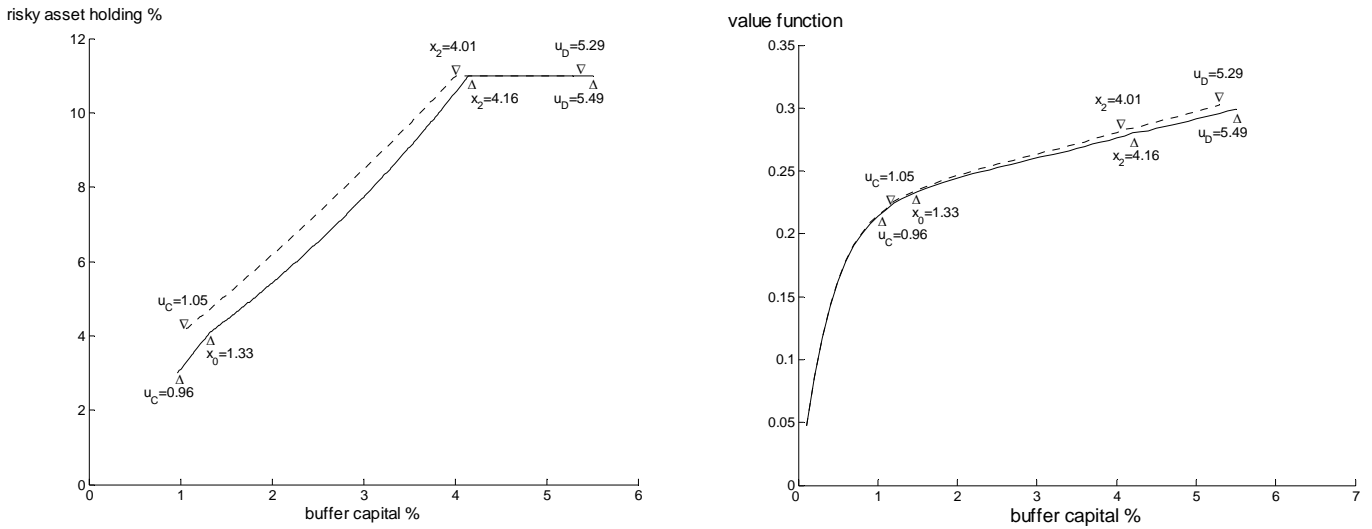


Figure 3. Example bank's optimal liquid risky asset investment and value function. The dotted (solid) lines are without (with) MRR. The points in the graphs indicate the barrier levels. Parameter values: $\sigma_Y = 0.35\%$, $\mu_Y = 1\%$, $\sigma_Z = 16\%$, $\mu_Z = 1\%$, $\rho = 4\%$, $K = 0.25\%$, and $\Delta = 0.5$ years.

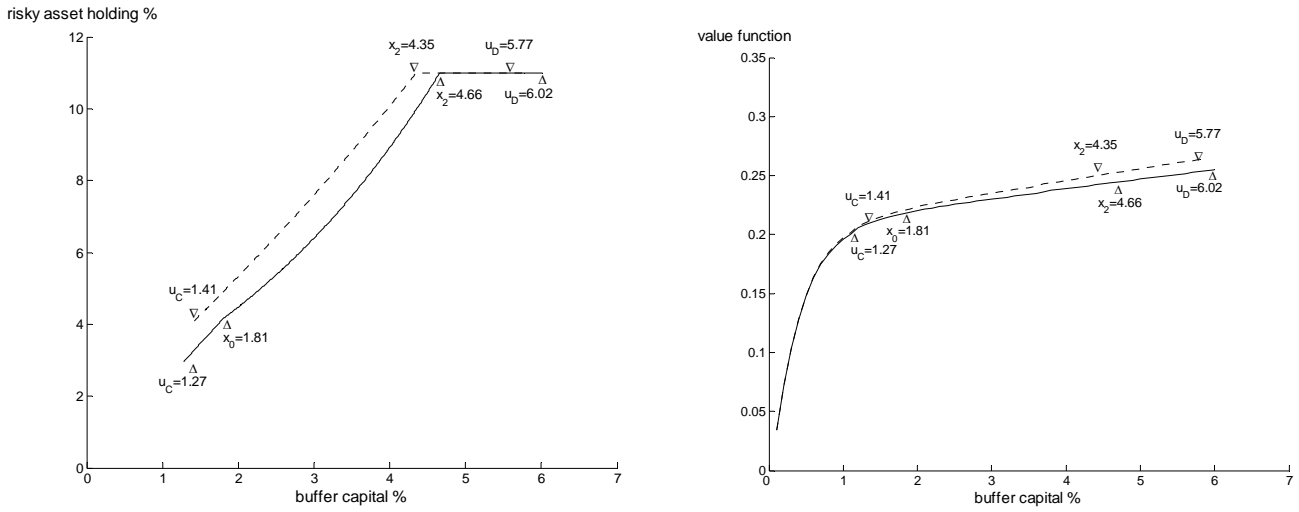


Figure 4. Example bank's optimal risky asset investment and value function with 30% risky asset volatility. The dotted (solid) lines are without (with) MRR. The points in the graphs indicate the barrier levels. Parameter values: $\sigma_Y = 0.35\%$, $\mu_Y = 1\%$, $\sigma_Z = 30\%$, $\mu_Z = 1\%$, $\rho = 4\%$, $K = 0.25\%$, and $\Delta = 0.5$ years.

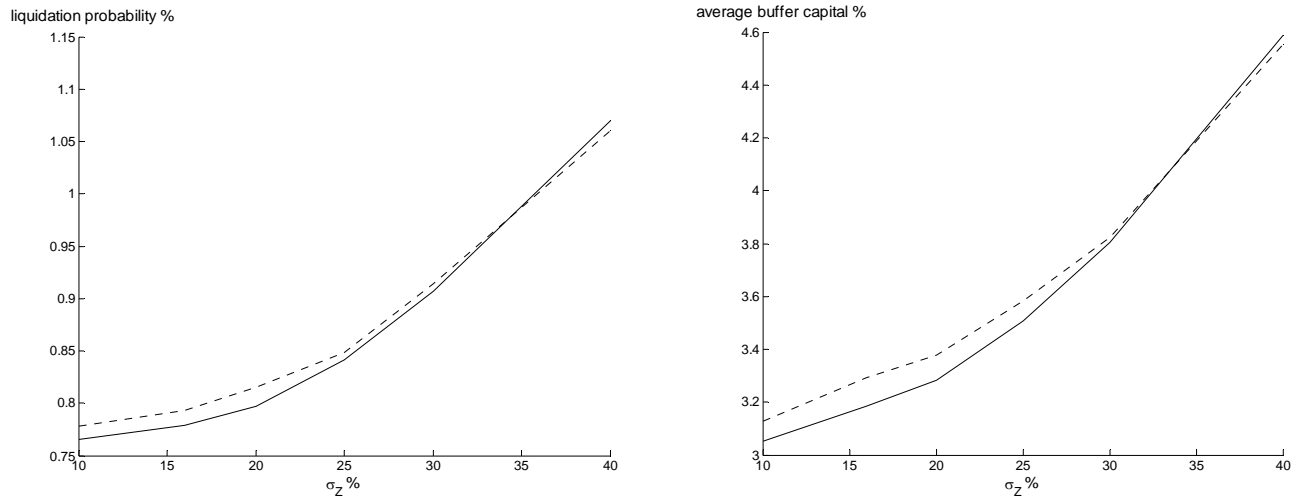


Figure 5. Example bank's liquidation probability and average buffer capital ratio with different liquid risky asset volatilities. The dotted (solid) lines are without (with) MRR. Parameters: $\sigma_Y = 0.35\%$, $\mu_Y = 1\%$, $\mu_Z = 1\%$, $\rho = 4\%$, $K = 0.25\%$, and $\Delta = 0.5$ years.

Table 1. Bank return parameters. The sample is 29 commercial banks of annual Compustat data over the period 1983-2002 (from Peura and Keppo (2006)). Each bank's estimate of $\mu_Y + b\mu_Z \left(\sqrt{\sigma_Y^2 + b^2\sigma_Z^2} \right)$ is the time-series average (standard deviation) of the bank's return on risk weighted assets, see (8i)¹³. Average buffer capital is the bank level average over 1993-2002.

Bank	Average capital	$\mu_Y + b\mu_Z$	$\sqrt{\sigma_Y^2 + b^2\sigma_Z^2}$	Bank	Average capital	$\mu_Y + b\mu_Z$	$\sqrt{\sigma_Y^2 + b^2\sigma_Z^2}$
WESTPAC BANKING -SPON ADR	2.91%	0.59%	1.06%	CITICORP	4.19%	0.60%	1.05%
BARCLAYS PLC/ENGLAND -ADR	2.98%	0.95%	0.82%	FLEETBOSTON FINANCIAL CORP	4.19%	0.74%	0.78%
WELLS FARGO & CO	3.00%	0.71%	0.87%	BANK OF NEW YORK CO INC	4.49%	1.01%	1.13%
COMERICA INC.	3.22%	0.84%	0.65%	BANK ONE CORP	4.67%	1.11%	0.57%
M & T BANK CORP	3.27%	0.73%	0.58%	SYNOVUS FINANCIAL CP	5.08%	1.28%	0.53%
U S BANCORP	3.61%	0.97%	0.57%	HIBERNIA CORP -CL A	5.20%	0.32%	1.53%
BANK OF AMERICA CORP	3.63%	0.81%	0.64%	POPULAR INC	5.27%	0.92%	0.41%
HUNTINGTON BANCSHARES	3.64%	0.85%	0.65%	ZIONS BANCORPORATION	5.42%	0.76%	1.14%
KEYCORP	3.69%	0.90%	0.66%	CULLEN/FROST BANKERS INC	5.86%	0.37%	1.40%
PNC FINANCIAL SVCS GROUP INC	3.70%	0.89%	0.70%	CITY NATIONAL CORP	6.12%	0.64%	1.27%
FIRST TENNESSEE NATL CORP	3.81%	1.14%	0.76%	ALLFIRST FINANCIAL INC	6.44%	0.33%	0.65%
BANCO SANTANDER CENT -ADR	4.01%	1.26%	0.50%	NORTH FORK BANCORPORATION	7.47%	1.35%	1.89%
J P MORGAN CHASE & CO	4.02%	0.43%	1.19%	UNION PLANTERS CORP	7.54%	0.38%	1.25%
NATIONAL CITY CORP	4.08%	1.12%	0.66%	WHITNEY HOLDING CORP	8.11%	0.63%	1.87%
MELLON FINANCIAL CORP	4.09%	0.77%	1.74%				

¹³ Risk weighted assets, and therefore also capital ratios, do not exist in the data prior to 1993. In calculating the average returns and the standard deviations prior to 1993, each bank's risk weighted assets are based on the average post 1993 risk weighted assets-to-total assets ratio of the bank.