Optimal bank capital with costly recapitalization[*]

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Abstract

We study optimal bank capital choice as a dynamic tradeoff between the opportunity cost of equity, the loss of franchise value following a regulatory minimum capital violation, and the cost of recapitalization. We introduce a recapitalization delay, which results in a strictly positive probability of capital adequacy violation and qualitatively influences optimal capital raising policies. We calibrate the model to bank accounting return data and evaluate the model's ability to explain observed bank capital ratios. Differences in return volatility explain a significant fraction of the cross-sectional variation in bank capital ratios. Differences in the level of capital market imperfections across banks constitute a secondary explanation. Our analysis points to the need for improved forward looking estimates of bank return volatility.

Keywords: bank capital, dividends, capital issues, fixed cost, delay

JEL classification: G32, G35

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1 Introduction

A general risk management lesson from models with frictions is that, in the absence of explicit risk management tools such as financial derivatives, firms will choose to hold buffer stocks of liquid assets and capital as hedges against liquidity and earnings risks. The argument for the buffer stock role of liquid assets has been theoretically presented and empirically verified e.g. by Kim et al. (1998) and Opler et al. (1999), who find that firm liquid asset holdings are positively related to cash flow risks. The buffer stock role of equity capital, on the other hand, is supported by many empirical studies on capital structure (e.g. Harris and Raviv, 1990, Booth et al., 2001, and Titman and Wessels, 1988), who find that firm leverage is negatively related to earnings volatility. In other words, we observe risk management considerations to influence both corporate investment decisions and corporate financing decisions[1].

The capital structure decision in banks is in its very essence a risk management decision. A bank practitioner views bank capital[2] not primarily as a form of financing, but as a buffer against asset risks which needs to be managed so that the bank can satisfy its minimum capital requirement even under relatively adverse future scenarios. It is implicit in this view that the violation of the minimum capital requirement is costly for the bank, and that the bank faces costs and/or constraints associated with portfolio adjustment and recapitalization. Subject to these conditions the role of buffer capital as a hedging mechanism against minimum capital violation is well founded.

Much of academic banking theory has been built on a quite different view on bank capital. A sizable literature has studied conditions or regulatory set-ups which could eliminate bank owners' asset substitution moral hazard problem[3]. This literature concentrates on incentives in asset risk choice while taking bank capital as exogenous and abstracting from dividend and recapitalization choices. Therefore the

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[1] By now there is a large literature on the interactions between financing, investment, and risk management in the presence of capital market frictions (e.g. Acharya et al., 2000, Froot et al., 1993, Froot and Stein, 1998, Leland, 1998, Mello and Parsons, 2000).

[2] Consistent with common bank parlance, our use of the term bank capital refers to book equity. This is the relevant measure of capital in an analysis of bank capital adequacy since minimum capital requirements under the Basel Accord apply to book equity.

[3] This literature includes Kahane (1977), Merton (1977), Koehn and Santomero (1980), Green (1984), Marcus (1984), Crouhy and Galai (1991), Rochet (1992), Fries et al. (1997), and Bhattacharya et al. (2002).

literature is ill suited for explaining banks' actual capital choices[4]. However, the two points of view on bank capital are complementary. The literature which builds on risk shifting incentives provides a rationale for banking regulation in general and for minimum capital regulation in particular. A bank manager thinking on bank capitalization takes the minimum capital constraint (the prevailing form of banking regulation) as well as the consequences of minimum capital violation as additional constraints to her choice. In a minimum capital regime such as the current Basel regime, a bank's capital choice is really a choice of the capital buffer to be held in excess of the minimum requirement. Asset risks, recapitalization constraints and the penalty from regulatory capital violation are the key determinants of this choice.

In this paper we adopt the bank manager's point of view on bank capital and test whether a simple optimizing model built on this view is capable of explaining the observed patterns of bank capital holdings. Our model is one of a value maximizing bank and prescribes an illiquid bank portfolio, imperfections in capital raising transactions, and loss of franchise value associated with the violation of the minimum capital requirement. Our model does not display any risk-shifting opportunities since the bank portfolio is assumed illiquid. The model predicts strictly positive levels of buffer capital. We estimate bank return parameters from bank level accounting data and compare the implied bank capital ratios with actual bank capital ratios. We are not aware of a similar calibration exercise performed in the existing banking literature.

The idea to model banks' capitalization decision based on the buffer stock role of bank capital is not new. Our model builds on the basic continuous time model of a capital constrained firm presented in Milne and Robertson (1996). Hojgaard and Taksar (1999) have extended the basic model into an insurance company setting by assuming that risk reduction at a proportional cost is available, which is interpreted as cheap reinsurance. Milne and Whalley (2001) have extended the model to allow for a recapitalization option. Peura (2002) has analyzed the effects of an equity issuance option which is subject to a proportional cost and an upper bound on the rate at which external capital can be raised. Our main modeling innovation is to impose a

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[4] This point has been emphasized by Milne (2002).

delay on capital issues[5]. We suggest that this is a natural stylized assumption which proxies for the fact that capital raising transactions in reality require heavy preparatory work. It turns out that the delay has a significant effect on the qualitative nature of the resulting capital raising policy. In the presence of a delay, bank liquidation probability is also strictly positive, unlike in a model where recapitalization can be implemented instantaneously.

The determination of bank capital from a trade-off perspective has also been studied in discrete time by Froot and Stein (1998), Estrella (2004) and Furfine (2000). In the context of a model with costly external capital, Froot and Stein (1998) demonstrate that a bank investing in illiquid products may adjust its capital structure in order to accommodate the illiquid risks it chooses to bear. Estrella (2004) uses a variant of the classical inventory or cash management models to study cyclicality of bank capital. In his model, the objective is to minimize the combined costs from over- and undercapitalization, as well as from adjustment of capital. Furfine (2000) presents a model of a value maximizing bank and calibrates the model to panel data in an attempt to explain banks' portfolio shifts following the introduction of the Basel Accord in 1989.

Our calibration exercise is based on a sample of US commercial banks in S&P's Compustat database. Average total capital ratio over the 1993-2002 period in the sample is 13.0%. The minimum requirement imposed by the Basel Accord is 8%, implying that the average capital buffer is 5.0% of risk-weighted assets, or over 50% of the minimum requirement. Our model, implemented with the sample means and volatilities of bank returns, only yields an average capital buffer of 2.4%. The model capital ratio is highly sensitive to the level of return volatility, however, and much of the shortfall between the model and actual capital ratios is attributable to banks with below average return volatilities. There is reason to suspect that these estimates suffer from a peso problem due to insufficient length of the data history. When applied to banks with above average credit loss provisions over the sample period, the model generates an average capital buffer of 3.5% and explains some 40% (R^2) of the variation in capital levels across banks. We find that forward looking volatility

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[5] The effects of delays on irreversible investment decisions have been studied in Bar-Ilan and Strange (1996) and Alvarez and Keppo (2001), and by Subramanian and Jarrow (2001) in connection with optimal liquidation.

estimates could improve model fit, both in terms of average levels and cross-sectionally. Our model can replicate observed capital ratios exactly if volatility is made an implied parameter, analogously to how the Black-Scholes model is used in practice. The average implied volatility is higher with small banks, even after acknowledging the likely differences in capital market imperfections.

The rest of the paper is organized as follows. Section 2 presents our model of capital control and shows that the non-homogenous problem of capital control can be transformed into a homogenous problem of capital *ratio* control. The solution is characterized in terms of a set of variational inequalities. Optimal policies and the value function, also in the limiting cases of the model, are derived in section 3. Section 4 illustrates optimal policies through numerical examples. Model parameters are calibrated and the comparison against actual capital ratios is presented in section 5. Section 6 concludes.

2 The model

2.1 A model of bank capital

We imagine a bank whose portfolio size, measured by its regulatory risk weighted assets[6] R, grows at a constant positive rate r, so that

$$R_t = R_0 e^{rt} \tag{1}$$

for some initial positive R_0. The growth rate r is assumed to coincide with the risk free rate. The bank's relative profitability remains unchanged over time, so that the scale of the bank's profits also grows at the rate r. Cumulative bank profit Y_t therefore satisfies

$$dY_t = \mu R_t dt + \sigma R_t dW_t\,,$$

where W_t is a standard Wiener process[7]. The parameter μ is the expected return on (risk weighted) assets and σ is the asset return volatility. These are both assumed to

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[6] Risk weighted assets, under the current Basel Accord from 1988, are calculated as a weighted sum of a bank's nominal exposures, where the weights depends on product type and counterparty sector. For large banks, risk weighted assets are typically between 65 and 70 percent of total assets.

[7] The wiener process is defined on an underlying probability space (Ω, F, P), where we take F to be the standard filtration generated by the Wiener process.

be positive constants. This implies that expected instantaneous bank profit is proportional to bank portfolio size (measured by risk weighted assets), and that instantaneous profit is stochastic and non-predictable.

Owners control bank capital through dividend payments and issues of new capital. Dividends payments can be implemented instantaneously, but capital issuance is associated with a delay of length Δ and with a cost which is a fixed proportion K of the size of the bank (as measured by R). Formally, a capital control policy $\hat{\pi}$ is a collection $\left(L^{\hat{\pi}}, \left\{t_i^{\hat{\pi}}, s_i^{\hat{\pi}}\right\}\right)$, where $L^{\hat{\pi}}$ is a non-decreasing process representing the cumulative amount of dividends paid under policy $\hat{\pi}$, $\left\{t_i^{\hat{\pi}}\right\}$ is an increasing sequence of order times of new capital issues, and $\left\{s_i^{\hat{\pi}}\right\}$ are the amounts of capital raised at each issue of capital. We denote by Π the class of admissible policies which satisfy: $L_t^{\hat{\pi}}$ is a non-decreasing right-continuous process adapted to F_t such that $L_{0-}^{\hat{\pi}} = 0$; each $t_i^{\hat{\pi}}$ is a stopping time of the filtration F_t; each $s_i^{\hat{\pi}}$ is measurable with respect to $F_{\left(t_i^{\hat{\pi}}+\Delta\right)-}$. The measurability of s_i with respect to $F_{(t_i+\Delta)-}$ means that owners may decide on the exact amount of capital to be raised at time t_i + Δ based on all then available information. They do not need to precommit to any quantity of capital at time t_i when they order the capital issue. Additionally, admissible controls satisfy

$$t_{i+1}^{\hat{\pi}} - t_i^{\hat{\pi}} \geq \Delta \quad \text{for all } i \geq 1, \tag{2i}$$

$$dL_t^{\hat{\pi}} = 0 \text{ for all } t \in \left(t_i^{\hat{\pi}}, t_i^{\hat{\pi}} + \Delta\right], \quad i \geq 1. \tag{2ii}$$

Condition (2i) states that a new issue may not be ordered while a previously ordered issue is waiting to be completed. Condition (2ii) states that dividends may not be paid between the ordering of a capital issue and the actual capital collection. The condition has important technical merit but also an economic justification, in that ruling out simultaneous capital issues and dividend payments is likely to reduce conflicts of incentives between existing and new equity holders. The potential incentive conflicts are not explicitly present in our model and we do not analyze the division of bank value between existing and new equity holders. We simply think of constraint (2ii) as a restriction set by the capital markets.

Bank capital stock as a function of policy $\hat{\pi}$ is denoted $\hat{X}_t^{\hat{\pi}}$, and satisfies the integral dynamics

$$\begin{aligned}\hat{X}_t^{\hat{\pi}} &= \hat{X}_0 + \int_0^t r\hat{X}_{u-}^{\hat{\pi}} du + \int_0^t dY_u - L_t^{\hat{\pi}} + \sum_i s_i^{\hat{\pi}} \mathrm{I}_{\left\{t_i^{\hat{\pi}}+\Delta \le t\right\}} \\ &= \hat{X}_0 + \int_0^t r\hat{X}_{u-}^{\hat{\pi}} du + \int_0^t \mu R_u du + \int_0^t \sigma R_u dW_u - L_t^{\hat{\pi}} + \sum_i s_i^{\hat{\pi}} \mathrm{I}_{\left\{t_i^{\hat{\pi}}+\Delta \le t\right\}}\end{aligned} \tag{3}$$

where $\mathrm{I}_{\{\cdot\}}$ is the indicator function of the event defined in the parenthesis. This implies that cumulative profits and new issues of capital feed to the capital stock, while dividend payments represent a leakage from the capital stock. Bank capital also earns the risk free rate[8].

The minimum capital requirement under the current Basel Accord states that bank capital must at all times exceed 8% of the bank's risk weighted assets. We assume that the corrective action from violation of the minimum capital requirement will be liquidation[9]. The model bank therefore only operates up to the liquidation time

$$\hat{\tau}_{\hat{\pi}} = \inf\left\{t : \hat{X}_t^{\hat{\pi}} \le 8\% \cdot R_t\right\}. \tag{4}$$

The value of bank under policy $\hat{\pi}$ to its owners, given initial level of capital $\hat{X}_0$, is the expected discounted present value of dividends less capital issues until liquidation

$$\hat{V}_{\hat{\pi}}\left(\hat{X}_0\right) = E_{\hat{X}_0}\left[\int_0^{\hat{\tau}_{\hat{\pi}}} e^{-(r+\rho)t} dL_t^{\hat{\pi}} - \sum_i e^{-(r+\rho)\left(t_i^{\hat{\pi}}+\Delta\right)}\left(s_i^{\hat{\pi}} + KR_{t_i^{\hat{\pi}}+\Delta}\right)\mathrm{I}_{\left\{t_i^{\hat{\pi}}+\Delta<\hat{\tau}_{\hat{\pi}}\right\}}\right], \tag{5}$$

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[8] This assumption can be justified in several ways. We could assume that bank capital is explicitly invested in a risk free asset. Alternatively, we could postulate that any capital the bank has replaces an equivalent amount of borrowing/deposit funding, and that the effective cost of borrowing/deposits to the bank equals the risk free rate. The latter assumption in turn could be justified by the presence of deposit insurance.

[9] In practice, a violation of the minimum capital requirement will not result in immediate liquidation, but will generate additional costs and constraints to the bank due to increased regulatory surveillance (Peek and Rosengren, 1997, list the provisions for Prompt Corrective Action specified in the FDICIA). Also the bank's competitive position is likely to be affected. Therefore the bank's owners are likely to lose a significant amount of the bank's economic rent.

where ρ is a positive constant representing the wedge between debt and equity finance, due to capital market frictions such as taxation and agency costs of equity[10], and K is a non-negative constant representing the cost of capital issuance. (5) implies that the cost from capital issuance is proportional to bank size, as measured by risk weighted assets. The capital control problem is to identify the value of an optimally managed bank

$$\hat{V}(x) = \sup_{\hat{\pi}\in\Pi} \hat{V}_{\hat{\pi}}(x) \tag{6}$$

and an admissible policy which achieves this value. The model has six parameters in total. μ and σ characterize bank returns. r is the risk free rate (and also the implicit cost of debt, see footnote 8) and ρ is the wedge between debt and equity finance. Δ and K are the capital raising delay and cost, respectively.

2.2 A normalized model of bank capital ratio

The capital dynamics defined in (3) is not time-homogenous, which makes direct solution of the problem (6) difficult. The problem of capital control can, however, be transformed into a time-homogenous problem of capital ratio control, through a simple normalization. The normalized state variable, the bank capital ratio X, is defined as

$$X_t = \frac{\hat{X}_t}{R_t} \,. \tag{7}$$

The following proposition presents the capital ratio control problem and shows its connection to the capital control problem (6).

Proposition 1. *Given a policy $\pi \in \Pi$, let bank capital ratio satisfy*

$$X_t^{\pi} = X_0 + \mu t + \sigma W_t - L_t^{\pi} + \sum_i s_i^{\pi} \mathrm{I}_{\{t_i^{\pi}+\Delta\le t\}} \,, \tag{8i}$$

and define the first time of capital ratio violation by

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[10] As pointed out by a referee, ρ should not be interpreted as equity risk premium since it is assumed constant and does not depend on bank leverage. This suggests that our modeling framework is risk-neutral and the parameter μ under a risk-neutral measure need not coincide with its observed value. In particular, since uncertainty in our model is driven by a Brownian Motion, a change of measure would influence the drift, but not the volatility, of the earnings process. We ignore this potential source of error in estimating μ. This should not be crucial since, as we will demonstrate in Section 5, the drift term only has a secondary effect on model capital ratios.

$$\tau_{\pi} = \inf\left\{t : X_t^{\pi} \leq 8\%\right\} \quad . \tag{8ii}$$

Define bank equity value, given policy π, by

$$V_{\pi}\left(X_0\right) = E_{X_0}\left[\int_0^{\tau_{\pi}} e^{-\rho t} dL_t^{\pi} - \sum_i e^{-\rho\left(t_i^{\pi}+\Delta\right)}\left(s_i^{\pi} + K\right)\mathrm{I}_{\left\{t_i^{\pi}+\Delta<\tau_{\pi}\right\}}\right], \tag{8iii}$$

where the expectation is conditional on the capital ratio dynamics (8i), and let the value function be

$$V(x) = \sup_{\pi\in\Pi} V_{\pi}(x)\,. \tag{8iv}$$

Then (6) can be expressed in terms of (8iv) as

$$\hat{V}\left(\hat{X}_0\right) = R_0 V\left(\frac{\hat{X}_0}{R_0}\right). \tag{9}$$

Moreover, let π^* *be the policy which achieves the optimum in (8iv). Then the policy which achieves the optimum in (6),* $\hat{\pi}^*$*, can be expressed in terms of* π^* *by*

$$L_t^{\hat{\pi}^*} = \int_0^t R_u dL_u^{\pi^*} \tag{10i}$$

$$t_i^{\hat{\pi}^*} = t_i^{\pi^*}, \quad i \geq 1 \tag{10ii}$$

$$s_i^{\hat{\pi}^*} = R_{t_i^{\pi^*}+\Delta} s_i^{\pi^*}, \quad i \geq 1. \tag{10iii}$$

The key to this result is to understand that when π and $\hat{\pi}$ are related through (10), then the capital ratio process X^{π} given by (8i) is exactly the process $\hat{X}^{\hat{\pi}}\big/R$, derived from (1) and (3) with the help of Ito's lemma, given that the initial values satisfy $X_0 = \hat{X}_0\big/R_0$. The complete proof of the proposition is available from the authors upon request.

(9) implies that the objective function of the capital ratio control problem, (8iii), can be interpreted as the value of bank equity *as a percentage of risk weighted assets.* Moreover, since the capital ratio dynamics in (8i) does not depend on the level of the capital ratio, we may without loss of generality normalize the default point in (8ii) to 0, and interpret X as the excess capital ratio, i.e. capital ratio in excess of 8%. The relation (9) between the solution to the original capital control problem and the solution to the capital ratio control problem is preserved, once we interpret $\hat{X}$

correspondingly as excess capital, i.e. capital stock in excess of 8% of risk weighted assets. We will use this reinterpretation in the following.

2.3 Characterization of optimum

We characterize the value function (8iv) through a set of variational inequalities. For this purpose we define two operators. Let D be the set of real-valued functions on $\mathbf{R}_+$. We define the operator $M{:}D \to D$ by

$$Mf(x) = E_x\left[e^{-\rho\Delta}\sup_s\left[f\left(X_\Delta + s\right) - s - K\right]\mathrm{I}_{\{\tau_0 > \Delta\}}\right], \tag{11}$$

where X_Δ is the value at time Δ of X defined by $dX_t = \mu dt + \sigma dW_t$, τ_0 is the first hitting time of 0 of X, and the expectation is conditioned on $X_0 = x$. Operator M can be interpreted as the expected value of the decision to order new capital immediately, given that the 'continuing value' of the problem is f. Also we define the infinitesimal generator A by

$$Af(x) = \frac{1}{2}\sigma^2 f''(x) + \mu f'(x), \tag{12}$$

for all sufficiently regular f. This may be interpreted as the 'expected instantaneous change in the value of the function f, given no immediate controls are undertaken'.

Now the following characterization of optimum can be established using standard arguments (see e.g. Hojgaard and Taksar, 1999, or Fleming and Soner, 1993).

Proposition 2. *Assume that the value function* (8iv) *satisfies Ito's formula. Then it satisfies the following set of inequalities for all $x > 0$:*

$$V(0) = 0 \tag{13i}$$

$$V(x) \geq MV(x) \tag{13ii}$$

$$(A - \rho)V(x) \leq 0 \tag{13iii}$$

$$V'(x) \geq 1 \tag{13iv}$$

$$\left(V(x) - MV(x)\right)(A - \rho)V(x)\left(V'(x) - 1\right) = 0. \tag{13v}$$

(13) is a system of first order conditions to our problem which follow from standard dynamic programming arguments applied to the Bellman equation. With the

exception of (13i), they must be understood as functional (in)equalities, i.e. to hold for all positive x (the domain of the state variable in our problem). (13i) follows from 0 capital buffer (remember our reinterpretation of the state variable as excess capital) being an absorbing state in our model. (13ii) holds since the value of immediate order of new capital can never exceed the value function by definition of the value function. (13iii) holds since applying no control to the capital stock is always an admissible policy. (13iv) must hold since paying dividends is an admissible policy. (13v) states that in an optimum, one of the inequalities must be tight. That is, for all x either taking no action or taking some of the admissible actions must represent the optimal policy.

We note that Proposition 2 does not assume that the value function is twice continuously differentiable everywhere. It is well known that the second derivative of a value function in general exhibits a discontinuity at the boundary of the region where impulse control actions are optimal (see Dumas, 1991). This does not prevent Ito's formula from applying, but the differential generator in inequality (13iii) is to be interpreted in terms of left or right derivatives[11].

3 Solutions

Constructing a solution to (13) requires a guess on the form of the solution, i.e. on the order of the 'optimality regions' for each of the policies. Our assumption on the form of the solution in the general case (i.e. under parameter combinations such that capital issues are optimally undertaken) is the following: i) for $x \in (0, u_1]$, it is optimal to immediately order new capital, ii) for $x \in (u_1, u_2)$, it is optimal neither to order new capital nor to pay dividends, and iii) for $x \in [u_2, \infty)$ it is optimal to pay dividends. Furthermore, we expect to have $u_1 < u_2$ and u_2 finite. u_1 may be 0 when capital market imperfections are prohibitively high. Figure 1 illustrates the model with this form of solution.

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[11] We have presented no proof of sufficiency of the first-order conditions (13). Subject to general restrictions on the form of the solution, such proof of sufficiency can be formulated and can be obtained from the authors upon request.

According to our initial guess, we look for a function that solves (13ii) with equality for $x \le u_1$, solves (13iii) with equality for $x \in [u_1, u_2]$, and solves (13iv) with equality for $x \ge u_2$. Such function also solves (13i) and (13v). The function is to be continuously differentiable at the impulse control barrier u_1 and twice continuously differentiable at the singular control barrier u_2.

3.1 General case

We define the following functions:

$$\begin{aligned} M(x;u_2) = e^{-\rho\Delta} \Bigg\{ & \left(x + \mu\Delta + \mu/\rho - K - u_2\right)\Phi\left(\frac{x+\mu\Delta}{\sigma\sqrt{\Delta}}\right) + \sigma\sqrt{\Delta}\varphi\left(\frac{x+\mu\Delta}{\sigma\sqrt{\Delta}}\right) \\ & - e^{-\frac{2\mu x}{\sigma^2}}\left[\left(-x + \mu\Delta + \mu/\rho - K - u_2\right)\Phi\left(\frac{-x+\mu\Delta}{\sigma\sqrt{\Delta}}\right) + \sigma\sqrt{\Delta}\varphi\left(\frac{-x+\mu\Delta}{\sigma\sqrt{\Delta}}\right)\right]\Bigg\}; \end{aligned} \tag{14}$$

$$f_1(x;u_2) = a_1 e^{-d_{1+}(u_2 - x)} + a_2 e^{-d_{1-}(u_2 - x)}, \tag{15}$$

where $a_1 = \dfrac{d_{1-}}{d_{1+}d_{1-} - d_{1+}^2} > 0$, $\qquad a_2 = \dfrac{d_{1+}}{d_{1+}d_{1-} - d_{1-}^2} < 0$, and

$$d_{1\pm} = \frac{1}{\sigma^2}\left[-\mu \pm \sqrt{\mu^2 + 2\rho\sigma^2}\right]; \text{ and}$$

$$f_2(x;u_2) = \frac{\mu}{\rho} + (x - u_2). \tag{16}$$

In (14), Φ is the cumulative standard normal distribution and φ is the density of the standard normal distribution. The following result gives the value function (8iv) in terms of these functions, as well as a sufficient condition on the problem parameters for the existence of the general solution.

Proposition 3. *Let* $\dfrac{\partial M(x;u_0)}{\partial x}\Big|_{x=0} > \dfrac{\partial f_1(x;u_0)}{\partial x}\Big|_{x=0}$, *where* M *and* f_1 *are given by (14) and (15), and where*

$$u_0 = \frac{2}{d_{1+} - d_{1-}}\log\left(-\frac{d_{1-}}{d_{1+}}\right). \tag{17}$$

Then there exists a solution (u_1,u_2) *to the set of algebraic equations*

$$M(u_1;u_2) = f_1(u_1;u_2) \tag{18i}$$

$$\frac{\partial M(x;u_2)}{\partial x}\Big|_{x=u_1} = \frac{\partial f_1(x;u_2)}{\partial x}\Big|_{x=u_1} \tag{18ii}$$

satisfying $0 < u_1 < u_2 < u_0$ *and such that* $M(x;u_2) \le f_1(x;u_2)$ *for all* $0 \le x \le u_2$*. In terms of the solution* (u_1,u_2) *to (18), the value function (8iv) is*

$$V(x) = \begin{cases} M(x;u_2) & 0 \le x \le u_1 \\ f_1(x;u_2) & u_1 < x < u_2 \\ f_2(x;u_2) & u_2 \le x \end{cases} \tag{19}$$

where M *is given by (14),* f_1 *is given by (15) and* f_2 *is given by (16).*

The algebraic system (18) is non-linear, but standard numerical optimization procedures (secant method, Newton's method) converge well, given reasonable initial values[12].

The interpretation of the solution (19) is simple. The value function coincides with the function M in the region where immediate ordering of capital issues is optimal, with the function f_1 in the 'wait and see' region, and with the linear function f_2 in the dividend payment region. Figure 2 illustrates the solution in this general case. Smooth pasting (up to second derivatives) of f_1 and f_2 takes place at u_2, the dividend payment barrier. The solution to (18) in turn imposes smooth pasting (up to first derivatives) of M and f_1 at u_1, the capital issue order barrier. M is more concave at u_1 than is f_1, and the second derivative of the value function at this point experiences a discontinuity.

3.2 Limiting cases

The limiting cases of the model emerge as either the capital issue cost or the capital issue delay approaches zero, or as either of these exceeds a critical value so that capital issues are no longer optimal. We find the limiting cases interesting since they help to understand the comparative statics of the general model, and show exactly which of the capital market imperfections drive the qualitative results.

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[12] We have done the optimization with the Solver™ add-in in Microsoft Excel.

Case I: K or Δ above a critical value

When Δ or K are above their critical values (which depend on other problem parameters) such that capital issuance is no longer optimal, the optimal policy and the value function reduce to those of the Milne and Robertson (1996) model without the capital issue option. This value function is a special case of (19), obtained by setting $u_1 = 0$ and $u_2 = u_0$ as given in (17). The value function takes the form

$$V(x) = \begin{cases} a_1 e^{-d_{1+}(u_0 - x)} + a_2 e^{-d_{1-}(u_0 - x)} & x < u_0 \\ \dfrac{\mu}{\rho} + x - u_0 & x \geq u_0 \end{cases} \tag{20}$$

where a_1, a_2, d_{1+} and $d_{1\text{-}}$ are as in (15).

Case II: $\Delta = 0$

In the absence of any capital raising delay, new capital can be issued instantaneously and there is perfect control on the minimum level of capital. Given the opportunity cost of capital, it is clearly optimal to wait until the buffer capital falls arbitrarily close to zero before issuing new capital. As zero is an absorbing boundary, however, new issues would have to be implemented before buffer capital actually hits zero. Non-surprisingly, an optimal policy in the model without delays does not exist. ε-optimal policies can be constructed which set the capital issue barrier arbitrarily close to 0. One may think of the value function in the $\Delta = 0$ case as the limit of the values associated with such ε-optimal policies.

Taking the limit as Δ approaches 0, the function M in (14) simplifies to

$$M\left(x; u_2\right) = x + \mu/\rho - K - u_2,$$

for all $x > 0$. Taking the limit of this as we let the capital issue point x approach zero, we obtain the boundary condition satisfied by the limiting value function in the $\Delta = 0$ case

$$V(0) = \max\left(\lim_{x \to 0+} M\left(x; u_2\right)\Big|_{\Delta=0}, 0\right) = \max\left(\mu/\rho - K - u_2, 0\right). \tag{21}$$

This condition now replaces (13i). By the previous reasoning, the capital issue barrier in the limiting case is located at zero, so that the solution only has one free barrier, the dividend barrier. As in the general case, the dividend barrier is also the level up to which the capital ratio is replenished each time a new capital issue is implemented. The solution is given in the following proposition, proven in the Appendix.

Proposition 4. *If* $K < \mu/\rho$ - u_0, *where* u_0 *is given by* (17), *the value function is*

$$V(x) = \begin{cases} a_1 e^{-d_{1+}(\hat{u}-x)} + a_2 e^{-d_{1-}(\hat{u}-x)} & x < \hat{u} \\ \dfrac{\mu}{\rho} + x - \hat{u} & x \geq \hat{u} \end{cases} \tag{22}$$

where $\hat{u} < u_0$ *is the unique positive solution for* u_2 *in the equation*

$$a_1 e^{-d_{1+}u_2} + a_2 e^{-d_{1-}u_2} = \frac{\mu}{\rho} - u_2 - K\,. \tag{23}$$

Else, the value function and the barrier $\hat{u}$ *are identical to* (20) *and* (17).

The parametric form of (22) is the same as that of (20), the difference being the location of the barriers u_0 and $\hat{u}$. When the condition of Proposition 4 holds, $\hat{u} < u_0$, and in this case (22) is a left-shifted version of (20).

If both Δ and K are equal to 0, then (23) is solved by $\hat{u} = 0$. This limiting case represents perfect market conditions. In perfect markets, no buffer stocks of capital are held and all profits are immediately paid out as dividends. When losses are realized, the capital to cover the losses is instantaneously raised from capital markets. The controlled capital ratio would be a constant equal to the 8% minimum.

Case III: $K = 0$

We can say little more about this limiting case than about the general case. The limit $K = 0$ does not involve a degeneracy as the limit $\Delta = 0$ does. This is evident from the formula (14) for the function M, where setting K to zero does not influence the qualitative properties of M.

This observation implies that the presence of delay drives the qualitative nature of the solution, in particular the existence of the non-zero capital issue barrier. The sole presence of the fixed cost does not generate a positive capital issue barrier, and will not result in a positive probability of liquidation. We find these observations to support the presence of a capital issue delay, which is precisely the additional ingredient in our modeling strategy, relative to earlier contributions in the banking literature.

4 Comparative statics

In this section we illustrate the effects of the capital issue cost and delay on optimal capital issue and dividend policies, as well as on the value of the capital issue option.

4.1 Optimal dividend and capital raising policies

Figure 3 shows the response of the barriers u_2 and u_1 to the parameters Δ and K determining the degree of capital market imperfections. We have drawn the figure with μ, σ and ρ fixed at representative values. We will return to the estimation of these parameters in the following section.

The optimal dividend barrier in the upper picture of Figure 3 is non-decreasing with respect to K and Δ. The optimal choice of the dividend barrier balances the expected cost of new capital issues as well as the expected loss of continuing value from liquidation, against the time value of delayed dividends. The dividend barrier is non-decreasing in the capital issue cost since the latter increases expected capital raising costs. The dividend barrier is non-decreasing in the length of the delay since a longer delay, ceteris paribus, implies a higher probability of liquidation. Also, the dividend barrier is quite sensitive to the introduction of small costs of and delays in capital issuance, given that these are initially at a low level. When the cost or the delay is already sizable, the dividend barrier is relatively insensitive to small increases in them.

The optimal capital issue barrier in the lower picture in Figure 3 is non-increasing in the capital issue cost K. The higher the capital issue cost, the higher risk of liquidation the owners of the bank are willing to take in order to avoid the capital issue cost. However, the capital issue barrier does not behave monotonically with respect to the delay Δ. When the delay is relatively short, the optimal response to an increase in the delay is an increase in the capital issue barrier. In this case a longer delay induces earlier (in time) ordering of new capital. When the delay is relatively long, on the other hand, the optimal response to an increase in the delay is a decrease in the capital issue barrier. This happens because the value of ordering a new capital issue is affected two ways by changes in the delay. First, an increase in the delay increases the probability of liquidation during the delay, ceteris paribus, inducing an increase in the capital issue barrier. Second, the model forbids dividend payments during delay, so that an increase in the delay defers potential dividend payments further into the future, should a capital issue be ordered now, suggesting a decrease in the capital issue barrier. It turns out that for short delays, the former effect dominates, while for sufficiently long delays, the latter effect dominates. Figure 3 also suggests that the point where the positive response of the capital issue barrier with respect to the delay turns negative is the lower, the higher is the cost of capital issuance. Finally, consistent with Proposition 4, the capital issue barrier converges to zero as the delay approaches zero.

Comparing the dividend and capital issue barriers in Figure 3, we observe that the expected size of a new issue (which is well approximated by the difference between the dividend barrier and the capital issue barrier) increases with the capital raising cost K. The owners optimally issue new equity in larger quantities but less frequently, when the cost of issuance is high, relative to the case where the cost of issuance is low. We will use this observation in the next section to identify plausible values for the parameters Δ and K describing capital market imperfections.

4.2 Value of the capital issue option

The opportunity to issue new equity in our model, being an option, cannot reduce bank value[13]. The value of the capital issue option in the model is the difference between the value functions (19) and (20), for a given initial capital ratio.

Figure 4 shows the value of the capital issue option as a function of the capital buffer. We observe that the value of the option is monotonically declining in the capital issue cost K (and also in the delay Δ, although this is not shown in Figure 4). This is unsurprising since both parameters are business constraints. As a function of the capital buffer, the value of the capital issue option displays a humped shaped behavior. The option value is at its highest when the capital buffer is somewhat below the optimal capital issue barrier (marked by a rectangle in Figure 4). The value of the capital issue option decreases as the capital buffer falls sufficiently below the optimal capital issue barrier. Here it becomes increasingly unlikely that the bank will survive until the end of the delay. As the capital buffer approaches zero, the

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[13] This is not generally true in models with asymmetric information, such as Myers and Majluf (1984).

value of the capital issue option approaches zero as well. The value of the capital issue option also goes down when the capital buffer increases above the capital issue barrier and above the dividend barrier (marked with a cross). Here the probability of capital shortages in the near future is low. The value of the option ultimately levels off to a constant. This happens somewhat above the dividend barrier (this is easily grasped from (19) and (20)). To conclude, the option to issue new capital is most valuable to banks that are in the neighborhood of the optimal capital issue barrier, but are still a reasonable distance above their minimum capital requirement.

It turns out that the value of the capital issue option, as a percentage of the total (cum-option) value of the bank, may be substantial for troubled banks. In terms of the example in Figure 4, when the capital issue cost equals 0.25% of risk-weighted assets and the capital issue delay is 0.5 years, the value of the capital issue option is over 15% of the value of a troubled bank, i.e. one whose capital buffer approaches zero. However, the option value is less than 2% of bank value for an otherwise identical bank which is optimally capitalized, i.e. one whose capital buffer equals the dividend barrier.

The recapitalization option may also have value despite substantial capital market imperfections. Figure 4 is based on an annual expected bank income (μ) of 1.0% of the bank's risk weighted assets. The option to issue capital still has value when the cost of a capital issue is 2% of risk weighted assets, i.e. worth of two year's expected profit. This suggests that we should observe bank owners optimally recapitalizing even when this means paying out several years' worth of expected earnings in capital issue related expenses.

5 Calibration and empirical tests

Our purpose is to provide a crude assessment of the potential of the model to explain observed bank capitalizations, both the average level of bank capital and the variation in capital levels across banks. In our tentative calibration, only μ and σ are estimated at bank level from accounting data. Other parameters are treated as common to all banks. Δ and K are fixed based on their implied capital issuance frequency and cost, while ρ is fixed based on its implied bank valuation multiple. We will later evaluate the cross-sectional variation in bank capital ratios that can be generated by variations in Δ and K.

5.1 Calibration of model parameters

The accounting identity that governs the evolution of bank equity is of the form

$$C_t = C_{t-1} + NI_t - D_t + S_t\,, \tag{24}$$

where C_t is bank equity at time t, NI_t is net income over period (t-1,t), D_t is dividends over period (t-1,t), and S_t is equity issuance over period (t-1,t). Consistent with our model, we decompose net income as

$$NI_t = ROA_t \cdot R_{t-1} + r_t X_{t-1} R_{t-1}\,, \tag{25}$$

where ROA_t stands for return on (risk weighted) assets over period (t-1,t), and r_t is the riskfree rate over period (t-1,t). R is risk weighted assets and X is bank capital ratio as before. The first summand in (25) is the stochastic return on bank asset portfolio. The second summand is the return on bank equity, where equity is assumed to be invested at the riskfree rate. Also consistent with our modeling assumptions, we impose the condition that bank risk weighted assets grow at the riskfree rate

$$R_t = \left(1 + r_t\right) R_{t-1}\,. \tag{26}$$

Combining (24)-(26), an approximate expression for the discrete dynamics of the bank capital ratio X_t in terms of the accounting variables is

$$\Delta X_t = \frac{C_t}{R_t} - \frac{C_{t-1}}{R_{t-1}} \approx ROA_t - \frac{D_t}{R_{t-1}} + \frac{S_t}{R_{t-1}}\,, \tag{27}$$

where, according to (25), ROA_t has the expression

$$ROA_t = \frac{NI_t}{R_{t-1}} - r_t X_{t-1}\,. \tag{28}$$

A comparison of the model capital ratio dynamics (8i) and the discrete dynamics (27) then suggests that we should interpret the model parameters μ and σ in terms of accounting data as

$$\mu = E\left[\frac{NI_t}{R_{t-1}} - r_t X_{t-1}\right],$$

$$\sigma^2 = VAR\left[\frac{NI_t}{R_{t-1}} - r_t X_{t-1}\right].$$

We use annual Compustat data on actual capital and accounting returns for a sample of US national commercial banks over the period 1983-2002. We qualify all banks with at least 15 years of data. We drop one additional bank from the sample because its μ-estimate is negative (in the model such bank would be liquidated at once), ending up with a sample of 61 banks. For each bank, we estimate μ and σ from the time series of the bank's ROA, calculated according to (28)[14]. The riskfree rate in this calculation is taken to be the prevailing Fed funds rate. Table 1 summarizes the parameter estimates.

Strikingly, the correlation between μ and σ estimates is significantly negative, -0.37 (bottom part of Table 1). Ex ante a positive correlation between risk and return would be expected. The negative correlation suggests that the sample period is of insufficient length to properly capture the risk–return tradeoff in banking, and that some banks' estimates may suffer from a peso-problem, i.e. non-existence in a finite sample of a crisis which takes place with small probability. The worst loss years in the data are 1987, 1990, and 1991. Banks which experienced losses (as measured by net income) or high credit loss provisions during these years have systematically below average mean returns and above average volatilities of returns over the sample. Many banks which have not experienced a such loss episodes, on the other hand, display both a high mean return and a low volatility of return.

In order to separate the banks which have undergone a loss episode from those which have not, we split the sample into two groups based on the average level of loan loss provisions over the sample period. Table 1 shows the parameter estimates for both groups. We observe that banks (29 in total) with higher than average loan loss provisions have (1) on average lower μ-estimates; (2) on average higher σ-estimates; (3) a positive and highly significant correlation between capital levels and σ-estimates. The last observation is important since the σ-estimate should be the key parameter driving capital levels in the model. Moreover, the correlation between capital levels and μ-estimates is of the correct sign (negative although not significant) in the group of banks with high loan loss provisions. None of the correlations are

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[14] Our estimate for μ is time-series average, and of σ is time-series standard deviation. Risk weighted assets, and therefore capital ratios, do not exist in the data prior to 1993. In calculating (28) prior to 1993, we estimate each bank's risk weighted assets based on the average post 1993 risk weighted assets-to-total assets ratio of the bank.

significant in the other group of banks, which supports our claim that the μ-and σ-estimates in that group are likely to suffer from peso problem.

As for the other parameters, ρ has an alternative interpretation within our model as the earnings-to-price ratio of an optimally capitalized bank with stationary growth. This is due to the result $V\left(u_2\right) = \mu/\rho$ obtained from formulas (16) and (19)[15]. We use a common estimate for ρ equal to 4% across all banks, implying a maximal price-to-earnings ratio of 25.

For Δ and K we identify common estimates across banks such that their implied capital raising cost and frequency are of plausible magnitude. In this exercise we set μ and σ close to their average values in the bank level data. Table 2 shows proportional capital raising cost and expected capital raising frequency subject to different combinations of Δ and K. The table implies that Δ has only a second-order effect on the proportional cost of capital issues. This is because the delay has only a modest impact on the difference $u_2 - u_1$, which in turn determines the average size of a capital issue. The difference $u_2 - u_1$ is driven by K and hence we may choose K rather independently from Δ, so as to yield a desired proportional cost of capital issuance. The empirical evidence on capital issuance costs (see e.g. Lee et al., 1996, or Bajaj et al., 2002) suggests that direct costs of equity issuance can be as high as 10% of issue size. There are likely to be some indirect costs as well, such as the cost of the bank's own effort, so that direct and indirect costs together could be more than 10% of the proceeds of the issue. Table 2 implies that a K of 0.25% (of risk weighted assets) achieves proportional capital issuance costs in the 10% - 12% range. We treat this as our best estimate for K.

Given K fixed at 0.25%, Table 2 shows that Δ equal to 0.08/0.5/1.0 years implies an expected capital issuance frequency of 18/28/44 years. We do not possess comprehensive data on bank capital issues to evaluate these values empirically. However, 18 years would imply an emergency recapitalization in every second or third recession, which to our intuition is more frequent than most banks are likely to,

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[15] Remember that μ denotes expected earnings, while $V(u_2)$ is bank value when capital ratio is at the optimal dividend barrier, both evaluated as a percentage of risk weighted assets. Hence the ratio of μ to $V(u_2)$ should proxy for the average earnings-to-price ratio of a well capitalized bank.

or expect to, experience. We find 0.5 years a plausible estimate for Δ, yielding an average recapitalization frequency of 28 years.

5.2 Cross-section and level of capital ratios

We begin by illustrating the performance of our model in explaining the cross-sectional variation in capital ratios. Figure 5 plots actual capital buffer against the model predicted capital level, the latter taken to be the dividend barrier (u_2). The left picture uses the average over 1993-2002 as the measure of actual capital for each bank, while the right picture uses the maximum capital over the same period. These are both shown since it is not immediately obvious which are the relevant quantities to be compared against each other[16].

Figure 5 shows that the model has predictive power only within the group of banks with higher than average loan loss provisions. Among this group of banks, the model dividend barrier explains over 30% of the variation in both average capital and maximum capital (correlations 0.57 and 0.60 against the respective variables). The slope coefficient in the linear least squares fit of actual capital on model dividend barrier is positive, yet significantly less than one, and the intercept in the same fit is significantly positive. This suggests that there could be a component in actual capital buffers that is unrelated to the volatility of bank returns.

As for the group of banks with lower than average loan loss provisions, the R^2's are virtually zero and the slope coefficients of actual capital on model capital, if nonzero, are negative. However, this is consistent with the hypothesis that mean returns are overestimated, and volatilities of returns are underestimated, within this group of banks. Both these biases lower the model dividend barrier, which systematically (with only one exception!) underestimates actual capital buffer.

Figure 6 illustrates the behavior of the model dividend barrier as a function of the μ-and σ-estimates. The left picture shows that the dividend barrier is effectively driven by the σ-estimate, the correlation between the two being 0.99. This picture

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[16] In the continuous-time model, the dividend barrier is the highest capital ratio that would ever be observed, so that a time-average of capital ratios *within the model* would invariably be below u_2. Given annual dividend payments, we would expect banks to deviate upwards from the u_2 level between annual dividend payments, even considerable so in years with unexpectedly high profits. Yet banks which have experienced large unexpected losses should have average capital ratios well below u_2.

also sorts out the two groups of banks, showing that a significant number of the banks with lower than average loan loss provisions have σ-estimates that are no higher than 0.5%. These banks systematically have model dividend barriers less than 2%, and they constitute most of the observations in the utmost left region in Figure 5. The right picture in Figure 6 in turn verifies that μ is negatively correlated (-0.48) with the dividend barrier. The effect of μ on the dividend barrier is also non-linear and depends on the level of volatility[17]. This is highlighted in the right picture by separating the observations into four groups according to the level of the σ-estimate.

The model falls somewhat short of explaining the *level* of capital held by banks. In the aggregate sample, the average of model dividend barriers (u_2) is 2.4%, while the average of (time series averages of) actual capital is 5.0%. The difference is even wider if the comparison is made against maximum capital (7.2%); however we do not endorse this comparison mainly because discreteness of dividend payments and temporary investment considerations are likely to distort the maximum measure. Among the group of banks with higher than average loan loss provisions, the difference between average actual capital (4.6%) and average model dividend barrier (3.2%) is narrower but still large in proportional terms. The averages also cannot be reconciled through an adjustment of the capital market imperfections. Even in the absence of the capital issue option, the average of model dividend barriers (u_0) is only 2.7% in the aggregate sample, and 3.8% among banks with higher than average loan loss provisions. Lowering the remaining model parameter ρ down to 2% would generate 0.35% higher average dividend barrier (0.6% higher among banks with higher than average loan loss provisions), not sufficiently to match the average capital buffer in the sample. Also, the implied price-earnings ratio would no longer be realistic (ρ of 2% implies a price-earnings ratio of 50 for a well capitalized bank).

It is possible that our model underestimates actual capital because we have misspesified the minimum capital requirement. This is suggested by the significantly positive intercepts in the regressions of actual capital on model capital in Figure 5. The model restricts the capital buffer of a zero volatility bank to zero. A positive volatility intercept could mean that the effective minimum capital requirement faced by banks is higher than 8%, perhaps closer to the 10% ‘well capitalized’ requirement

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[17] Milne and Whalley (2001) have observed the non-linearity of the relationship between μ and u_2.

imposed by the FDICIA (we will discuss alternative regulatory requirements in Section 6). Interestingly, no bank in the sample has a time series average capital buffer less than 2%, i.e. total capital ratio less than 10%.

Table 3 shows correlations between model capital and the μ-and σ-estimates as well as some possible explanatory factors which are not present in our model. The latter include bank asset size, asset growth rate, and the level of loan loss provisions (this has been used as a grouping variable). Also shown are correlations between the model residual (the difference between actual capital buffer and the model proxy, either u_2 or u_0) and the explanatory variables. These can provide signals concerning possible model misspecification. Concentrating on the restricted sample in the lower panel of Table 3, we observe that the volatility estimate (σ) is significantly negatively correlated with the model residual. This is not surprising, given the significantly positive intercept of actual capital against model capital (Figure 5). There is also a negative correlation between average loan loss provisions and the model residual, suggesting that the time series average may underestimate actual capital with those banks that have experienced large credit losses. On the positive side, we observe that asset growth rate is not significantly correlated with the model residual, suggesting that our constant growth rate (across banks) assumption does not affect the cross-sectional performance of the model.

Finally, there is in Table 3 a material negative correlation between bank asset size and actual capital (-0.43 in the restricted sample). The correlation between bank asset size and the model residual is lower in absolute value but still negative, suggesting that larger banks hold less capital than small banks, even after controlling for possible differences in their μ and σ estimates. So far we have not varied Δ and K across banks but it is quite plausible that these are inversely related to bank size. A simple way to test this is to apply our estimated Δ and K to large banks only (those with higher than average assets), and assume that capital issues are prohibitively costly for small banks. Therefore u_2 (u_0) is treated as the model capital buffer for large (small) banks. The rightmost column in Table 3 confirms that this reduces the correlation between bank size and model residual close to zero. Moreover, the R^2 in a regression of average (maximum) actual capital on model dividend barrier is now 39.7% (42.9%), suggesting that differences in capital market imperfections across banks can explain an additional 10% of the variation in capital levels across banks.

Continuing with the idea that only large banks posses the recapitalization option, Figure 7 shows the implied volatilities that replicate the average level of the capital buffer among large and small banks, within the restricted sample. For large banks (average assets over 40 bnUSD), the average capital buffer of 3.8% implies a 1.06% volatility. For small banks the average capital buffer of 5.3% implies a volatility of 1.19%. The average σ-estimate is 0.82% for large banks and 1.06% for small banks, implying that the required volatility adjustment is in fact higher for large banks, after controlling for likely differences in capital market imperfections.

6. Discussion and conclusions

The banking literature so far lacks good attempts to explain observed bank capital holding behavior with optimizing models of capital choice. In this paper we have expanded existing trade-off models of bank capital choice by introducing a recapitalization delay. This has intuitive appeal since in the presence of a delay, banks face liquidation risk despite their option to recapitalize and are forced to initiate recapitalization at positive buffer capital levels. We have also illustrated the option value accruing from the recapitalization option and the nature of the optimal capital raising policies and dividend policies. Finally, we have calibrated our model to bank level accounting return data and tested its ability to replicate actual bank capital ratios.

In an 'unconditional' sample, the model's predictive power is low, mainly because the correlation between bank level volatility estimates and actual capital is low. Once we restrict to banks with above average levels of loan loss provisions over the sample, however, the model explains some 40% of the variation in capital ratios across banks, and 70% of the average level of the capital buffer. The variation in model capital ratios is mainly driven by the variation in the volatility estimates. Our restricting to the more ex post risky banks is grounded by the belief that these banks' ex post measure of volatility is close to the ex ante uncertainty faced by the banks. That there is a peso problem in our bank return sample is supported by the fact that in the unconditional sample, the correlation between average bank return and volatility of bank return is significantly negative.

Our calibration exercise has been somewhat arbitrary. We spesified a model, estimated some of its parameters from bank level accounting returns, imposed common values for other parameters mainly guided by high level economic intuition, and evaluated the correlation between model output and actual capital buffers. It is the task of future research to test alternative model spesifications. A possible source of model misspecification is the nature of the minimum capital requirement. In addition to the Basel Committee's minimum solvency standard, the Federal Deposit Insurance Corporation Improvement Act (FDICIA) imposes minimum leverage requirements. These should not be binding over risk-based capital requirements with banks that have relatively risky portfolios, but this need not be the case with banks that have a significant portion of their portfolios invested e.g. in bank assets. There are also soft capital ratio targets set by the regulators, such as the FDICIA 'well capitalized' category which necessitates a 10% total capital ratio. This is soft regulation since violating the 10% limit does not imply prompt corrective action or significant restrictions on the bank's activities. The FDICIA first stipulates corrective regulatory intervention once the 8% limit is breached[18]. In light of the FDICIA stipulations, our assumption concerning the penalty from regulatory capital violation is extreme. The FDICIA contains 5 categories of capitalization, a lower category always implying more restrictions on bank behavior. A model where the penalty gradually increases as the capital ratio deteriorates would hence be more realistic.

Besides regulators, market discipline could dictate an implicit minimum capital requirement. This could be pressure coming from rating agencies, competitors and peers (swap market participants), or customers. Our model abstracts from market interactions. Whether these omissions matter reduces to the issue of which constituency provides the binding constraint on bank capitalization. Banking theory traditionally holds that customers, to the extent that they enjoy the security of deposit insurance, are not motivated in monitoring banks. Rating agencies' view is that of debt holders. Jokivuolle and Peura (2004) suggest that regulatory capital buffer (that is what we measure) dominates the economic capital constraint which measures bank riskiness from debtors' perspective. However, there is some evidence

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[18] Peek and Rosengren (1997) contains a description of the FDICIA from 1991 containing the well capitalized category.

that competitor reactions, in particularly the access to swap markets, could constitute a binding constraint on bank capital (Jackson et al, 2002).

Our model assumes normally distributed bank returns, while bank portfolio returns, and credit returns in particular, are known to be negatively skewed and positively serially correlated. This is evident from bottom-up portfolio models such as CreditMetrics^{TM} (J.P.Morgan, 1997), where a positive correlation between counterparty credit standings invariably generates a skewed bank portfolio return distribution. The Compustat bank return data over 1983-2002 indicates that there is negative skewness as well as positive serial correlation in the sample banks' accounting returns. It is not clear how our volatility estimate captures these effects. Incorporating non-normality or serial correlation into the model is likely to destroy the analytic tractability of the current model.

Accounting conventions could bias the volatility estimates as well. Banks in most jurisdictions have some options for income smoothing in the form of discretionary loan loss provisions (see e.g. Pain, 2003, on banks' provisioning behavior). Discretionary provisioning allows banks to distribute credit losses, which are typically realized only during a fraction of quarters over each 'credit cycle', more evenly over time[19]. This will cause banks' accounting returns to be less volatile than their actual portfolio (cash) returns. Of course, bank capital requirements apply to an accounting measure of capital, hence the dynamics of accounting measures could be all that matters.

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[19] Statistical tests which affirm banks' income smoothing behavior have been provided e.g. by Bhat (1996) and Lobo and Yang (2001).

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Appendix: Proofs

Proof of Proposition 3. We attempt to construct a function that solves (13ii) with equality for $x \le u_1$, solves (13iii) with equality for $x \in [u_1, u_2]$, and solves (13iv) with equality for $x \ge u_2$. The function should be continuously differentiable at the impulse control barrier u_1 and twice continuously differentiable at the singular control barrier u_2.

We first construct the solutions to (13ii)–(13iv), when these hold as equalities. Denoting our candidate solution by f_1, (13iii) becomes

$$\frac{1}{2}\sigma^2 f_1''(x) + \mu f_1'(x) - \rho f_1(x) = 0. \tag{A1}$$

The general solution to this is the exponential function

$$f_1(x) = c_1 e^{d_{1+}x} + c_2 e^{d_{1-}x}, \tag{A2}$$

$$d_{1\pm} = \frac{1}{\sigma^2}\left[-\mu \pm \sqrt{\mu^2 + 2\rho\sigma^2}\right].$$

The general solution to (13iv), denoted f_2, is

$$f_2(x) = x + c \tag{A3}$$

Twice continuous differentiability at u_2 therefore requires that $f_1'(u_2) = 1$ and that $f_1''(u_2) = 0$. Imposing these on (A2) yields

$$f_1(x; u_2) = a_1 e^{-d_{1+}(u_2 - x)} + a_2 e^{-d_{1-}(u_2 - x)}, \tag{A4}$$

$$a_1 = \frac{d_{1-}}{d_{1+}d_{1-} - d_{1+}^2} > 0, \qquad a_2 = \frac{d_{1+}}{d_{1+}d_{1-} - d_{1-}^2} < 0.$$

Also substituting $f_1'(u_2) = 1$ and $f_1''(u_2) = 0$ into (A1) gives

$$f_1(u_2) = \frac{\mu}{\rho}. \tag{A5}$$

Subject to this boundary condition, (A3) becomes

$$f_2(x; u_2) = \frac{\mu}{\rho} + (x - u_2). \tag{A6}$$

Now assume that (13ii) holds with equality, when applied to some concave function f that satisfies $f'(u_2)=1$. The supremum in (11) is then achieved by $s^* = u_2 - X_\Delta$. Also using (A5), (11) simplifies to

$$M(x;u_2) = e^{-\rho\Delta} E_x\left[\left(X_\Delta + \mu/\rho - K - u_2\right) \mathrm{I}_{\{\tau_0 > \Delta\}}\right]. \tag{A7}$$

Let $\beta = \mu/\rho - K - u_2$, which measures the net benefits from new issues of equity. (A7) further simplifies to

$$\begin{aligned} M(x;u_2) &= e^{-\rho\Delta} E_{x+\beta}\left[X_\Delta \mathrm{I}_{\{\tau_\beta > \Delta\}}\right] \\ &= e^{-\rho\Delta} E_{x+\beta}\left[X_\Delta(\Delta \wedge \tau_\beta) \mathrm{I}_{\{\tau_\beta > \Delta\}}\right] \\ &= e^{-\rho\Delta} \int_\beta^\infty y g(y;\Delta, x+\beta)\, dy, \end{aligned} \tag{A8}$$

where $\tau_\beta = \inf\{t : X_t = \beta\}$ and $g(y;\Delta, x+\beta)$ is the density of the absorbed process $X_\Delta(\Delta \wedge \tau_\beta)$ that starts at $x + \beta$. The first equality in (A8) is due to the spatial homogeneity of arithmetic Brownian motion, the second equality follows since the values of X outside the event defined in the indicator function do not affect the expectation, and the third equality is due to the fact that for the absorbed process $X_\Delta(\Delta \wedge \tau_\beta)$, the event in the indicator function is exactly the event that the process has not been absorbed by time Δ. Using the Reflection Principle (see e.g. Borodin and Salminen, 1997), the density can be written as

$$g(y;\Delta, x+\beta) = \varphi\left(y;\mu\Delta + x + \beta, \sigma\sqrt{\Delta}\right) - \exp\left(-\tfrac{2\mu x}{\sigma^2}\right) \varphi\left(y;\mu\Delta - x + \beta, \sigma\sqrt{\Delta}\right)$$

where $\varphi(y;\mu,\sigma)$ denotes the density of a normal distribution with mean μ and variance σ^2, i.e.

$$\varphi(y;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\tfrac{(y-\mu)^2}{2\sigma^2}\right).$$

Substituting this into (A8) and integrating, we get

$$\begin{aligned} M(x;u_2) = e^{-\rho\Delta} &\left\{\left(x + \mu\Delta + \mu/\rho - K - u_2\right) \Phi\left(\frac{x+\mu\Delta}{\sigma\sqrt{\Delta}}\right) + \sigma\sqrt{\Delta}\varphi\left(\frac{x+\mu\Delta}{\sigma\sqrt{\Delta}}\right)\right. \\ &\left. -e^{-\frac{2\mu x}{\sigma^2}} \left[\left(-x + \mu\Delta + \mu/\rho - K - u_2\right) \Phi\left(\frac{-x+\mu\Delta}{\sigma\sqrt{\Delta}}\right) + \sigma\sqrt{\Delta}\varphi\left(\frac{-x+\mu\Delta}{\sigma\sqrt{\Delta}}\right)\right]\right\}, \end{aligned} \tag{A9}$$

where $\Phi(y)$ and $\varphi(y)$ are the cumulative standard normal distribution and its density. Direct substitution shows that $Mf(0) = 0$.

Value matching and smooth pasting conditions at u_1 can now be formulated as

$$M(u_1, u_2) = f_1(u_1, u_2) \tag{A10i}$$

$$\frac{\partial M(x, u_2)}{\partial x}\Big|_{x=u_1} = \frac{\partial f_1(x, u_2)}{\partial x}\Big|_{x=u_1} \tag{A10ii}$$

This non-linear system of equations is quite complicated algebraically, and closed form solutions for u_1 and u_2 do not exist. A sufficient condition for the existence of the solution is provided by Lemma 4. The condition is expressed in terms of a positive barrier u_0 defined by $f_1(0, u_0) = 0$, which can be solved for

$$u_0 = \frac{2}{d_{1+} - d_{1-}} \ln\left(-\frac{d_{1-}}{d_{1+}}\right). \tag{A11}$$

Lemma 5 then shows that the function (19) constructed in terms of the solution to (A10) is concave and satisfies (13), and hence by a sufficiency theorem coincides with the value function (8iv). This completes the proof of Proposition 3.

Lemma 1 (this will be needed in the proofs of lemmas 2 and 3).

$$E_x\left[X_\Delta \mathrm{I}_{\{\tau_0 > \Delta\}}\right] = x + \mu\Delta - \mu\int_0^\Delta p(x,t)\,dt\,,$$

where $p(x,t) = P\left[\tau_0 \le t \middle| X_0 = x\right]$.

Proof:

$$\begin{aligned}
E_x\left[X_\Delta \mathrm{I}_{\{\tau_0 > \Delta\}}\right] &= E_x\left[X_\Delta \mathrm{I}_{\{\Omega\}}\right] - E_x\left[X_\Delta \mathrm{I}_{\{\tau_0 \le \Delta\}}\right] \\
&= E_x\left[X_\Delta\right] - \int_0^\Delta E\left[X_\Delta \middle| X_t = 0\right] \frac{\partial}{\partial t} p(x,t)\,dt \\
&= x + \mu\Delta - \int_0^\Delta \mu(\Delta - t) \frac{\partial}{\partial t} p(x,t)\,dt \\
&= x + \mu\Delta\left(1 - p(x,\Delta)\right) + \mu\int_0^\Delta t \frac{\partial}{\partial t} p(x,t)\,dt \\
&= x + \mu\Delta - \mu\int_0^\Delta p(x,t)\,dt
\end{aligned}$$

The first equality follows from the linearity of the expectation, the second from the law of total probability, the third from the Strong Markov property of arithmetic Brownian motion, the fourth just rearranges, and the fifth follows from integration by parts. End of proof.

Lemma 2. *If* $\beta = \mu/\rho - K - u_2 \geq 0$*, then* $\dfrac{\partial M(x;u_2)}{\partial x} > 0$ *and* $\dfrac{\partial^2 M(x;u_2)}{\partial x^2} < 0$ *for all* $x > 0$*.*

Proof: (This Lemma is important because it is easy to show that capital issuance can not be optimal unless $\beta \geq 0$.) Let us rewrite the function M starting from (A7) as follows

$$\begin{aligned} M(x;u_2) &= e^{-\rho\Delta} E_x\left[(\beta + X_\Delta)\mathrm{I}_{\{\tau_0 > \Delta\}}\right] \\ &= e^{-\rho\Delta}\left\{\beta\left(1 - p(x,\Delta)\right) + E_x\left[X_\Delta \mathrm{I}_{\{\tau_0 > \Delta\}}\right]\right\} \\ &= e^{-\rho\Delta}\left\{\beta\left(1 - p(x,\Delta)\right) + x + \mu\Delta - \mu\int_0^\Delta p(x,t)\,dt\right\} \end{aligned} \tag{A12}$$

where again $p(x,t) = P\left[\tau_0 \leq t \middle| X_0 = x\right]$ and the third equality utilizes Lemma 1. We know that p satisfies the Kolmogorov backward equation, and that its partial derivatives satisfy

$$\frac{\partial}{\partial x} p(x,t) < 0\,, \quad \frac{\partial^2}{\partial x^2} p(x,t) > 0\,, \quad \frac{\partial}{\partial t} p(x,t) > 0\,,$$

for all $(x,t) \in \mathbf{R}_{++} \times \mathbf{R}_{++}$. We differentiate the final expression in (A12) once and twice with respect to x and obtain

$$\frac{\partial M(x;u_2)}{\partial x} = e^{-\rho\Delta}\left\{-\beta\frac{\partial p(x,\Delta)}{\partial x} + 1 - \mu\int_0^\Delta \frac{\partial p(x,t)}{\partial x}\,dt\right\} > 0$$

$$\frac{\partial^2 M(x;u_2)}{\partial x^2} = e^{-\rho\Delta}\left\{-\beta\frac{\partial^2 p(x,\Delta)}{\partial x^2} - \mu\int_0^\Delta \frac{\partial^2 p(x,t)}{\partial x^2}\,dt\right\} < 0\,,$$

where the inequalities hold for all non-negative β because of the signs of the partials of $p(x,t)$. End of proof.

Lemma 3. *For all* $0 < u_2 \leq u_0$*,* $M(x;u_2) < f_2(x;u_2) = \dfrac{\mu}{\rho} + x - u_2\,.$

Proof. Beginning with (A7), we get

$$\begin{aligned}
M(x,u_2) &= e^{-\rho\Delta}E_x\left[\left(X_\Delta + \frac{\mu}{\rho} - K - u_2\right)\mathrm{I}_{\{\tau_0>\Delta\}}\right] \\
&= e^{-\rho\Delta}\left\{E_x\left[X_\Delta \mathrm{I}_{\{\tau_0>\Delta\}}\right] + E_x\left[\left(\frac{\mu}{\rho} - K - u_2\right)\mathrm{I}_{\{\tau_0>\Delta\}}\right]\right\} \\
&= e^{-\rho\Delta}\left\{x + \mu\Delta - \mu\int_0^\Delta p(x,t)\,dt + \left(\frac{\mu}{\rho} - K - u_2\right)\left(1 - p(x,\Delta)\right)\right\} \\
&\le e^{-\rho\Delta}\left\{\frac{\mu}{\rho} + \mu\Delta + x - u_2 - \mu\int_0^\Delta p(x,t)\,dt - p(x,\Delta)\left(\frac{\mu}{\rho} - u_2\right)\right\} \\
&< e^{-\rho\Delta}\left\{\frac{\mu}{\rho} + \mu\Delta + x - u_2\right\} \\
&< \frac{\mu}{\rho} + x - u_2 = f_2(x,u_2)
\end{aligned}$$

where $p(x,t)$ is as defined in Lemma 1. The third equality utilizes Lemma 1, the first inequality is due to setting K to 0, the second inequality is because $u_2 \le u_0 < \mu/\rho$ (this is due to (A5) and (13iv)), and the third inequality is because

$$e^{-\rho\Delta}\left\{\frac{\mu}{\rho} + \mu\Delta\right\} = e^{-\rho\Delta}\left\{\int_0^\infty e^{-\rho t}\mu dt + \mu\Delta\right\} < \int_0^\infty e^{-\rho t}\mu dt = \frac{\mu}{\rho}.$$ End of proof.

Lemma 4. *If* $\frac{\partial M(x,u_0)}{\partial x}\Big|_{x=0} > \frac{\partial f_1(x,u_0)}{\partial x}\Big|_{x=0}$, *then there exists a solution* (u_1,u_2) *to (A10) satisfying* $0 < u_1 < u_2 < u_0$ *such that* $M(x,u_2) \le f_1(x,u_2)$ *for all* $0 \le x \le u_2$.

Proof. Suppose that the condition $\frac{\partial M(x,u_0)}{\partial x}\Big|_{x=0} > \frac{\partial f_1(x,u_0)}{\partial x}\Big|_{x=0}$ holds.

i) In (A11) we have defined u_0 such that $f_1(0,u_0) = 0$. Therefore $M(0,u_0) = 0 = f_1(0,u_0)$. The condition implies that there is a positive x within $(0,u_0)$ such that $M(x,u_0) > f_1(x,u_0)$. On the other hand, by Lemma 3, $M(u_0,u_0) < f_1(u_0,u_0) = \frac{\mu}{\rho}$. This implies that $M(x,u_0)$ must cross $f_1(x,u_0)$ from above within the interval $0 < x < u_0$.

ii) From (A4) we get that $\frac{\partial f_1(x,u_2)}{\partial u_2} < 0$, so that $M(0,u_2) = 0 < f_1(0,u_2)$ for $0 < u_2 < u_0$. By Lemma 3, we also have $M(u_2,u_2) < f_1(u_2,u_2) = \frac{\mu}{\rho}$ for $0 < u_2 \le u_0$. From (A4) and (A9) we get (given positive Δ)

$$\lim_{u_2 \to 0} M(u_2, u_2) = 0 < \frac{\mu}{\rho} = \lim_{u_2 \to 0} f_1(0, u_2).$$

A necessary condition for the general solution is that $\mu/\rho - K - u_2 \geq 0$. Then by Lemma 2, $M(x,u_2)$ is increasing and concave in x, while $f_1(x,u_2)$ given by (A4) is also increasing in x. Combined with the previous inequality, these imply that one can always find a positive u_2 in the interval $(0,u_0)$ such that $M(x,u_2) < f_1(x,u_2)$ for $0 \leq x \leq u_2$.

iii) Following from i) and ii), by the continuity of $Mf(x,u_2)$ and $f_1(x,u_2)$ with respect to x and u_2, there will exist a u_2 in the interval $(0,u_0)$ such that $M(u_1,u_2) = f_1(u_1,u_2)$ for some $0 < u_1 < u_2$, while $M(x,u_2) \leq f_1(x,u_2)$ for all $0 \leq x \leq u_2$. But at this choice of (u_1,u_2), continuous differentiability of $M(x,u_2)$ and $f_1(x,u_2)$ with respect to x implies that $\frac{\partial Mf(x,u_2)}{\partial x}\Big|_{x=u_1} = \frac{\partial f_1(x,u_2)}{\partial x}\Big|_{x=u_1}$. This is because two continuously differentiable functions which coincide in the interior of their domain, but do not cross, must posses equal derivatives at the point where the functions coincide. End of proof.

Lemma 5. *Assume that a solution to (A10) as described in Lemma 4 exists and that V is defined by (19). Then V is a concave solution to (13) and satisfies Ito's formula.*

Proof: V is concave: By construction $V''(x) = 0$ for $x \geq u_2$. Differentiating f_1 given by (15) three times shows that $\frac{\partial^3 f_1(x;u_2)}{\partial x^3} > 0$ on $x \in (u_1,u_2)$. Therefore f_1 has an increasing second derivative on (u_1,u_2), which combined with the fact that $\frac{\partial^2 f_1(x;u_2)}{\partial x^2}\Big|_{x=u_2} = 0$ implies that f_1 and therefore V is concave on (u_1,u_2). We also know from Lemma 2 that $M(x;u_2)$ is globally concave w.r.t. x, so that V is concave on $(0,u_1)$. Equality of first derivatives of M and f_1 at u_1 then implies that V is globally concave.

V satisfies Ito's formula because each of the component solutions is twice continuously differentiable, and V satisfies the smooth pasting conditions at the barriers u_1 and u_2.

V solves (13):

(13i): $V(0) = 0$ because $M(0;u_2) = 0$.

(13ii): $V(x) = M(x;u_2) = MV(x)$ for $0 \leq x \leq u_1$ by construction. By Lemma 4, $V(x) = f_1(x;u_2) \geq M(x;u_2) = MV(x)$ for $u_1 \leq x \leq u_2$, and by Lemma 3, $V(x) = f_2(x;u_2) > M(x;u_2) = MV(x)$ for $x \geq u_2$.

(13iii): For $0 < x < u_1$, we have $V(x) = M(x;u_2)$, and we get from Itô's formula

$$e^{-\rho\tau}M(X_\tau;u_2) = M(x;u_2) + \int_0^\tau (A-\rho)M(X_s;u_2)ds + \int_0^\tau \tfrac{\partial M(X_s;u_2)}{\partial X_s}\sigma dW_s \ ,$$

where τ is a stopping time defined by $\tau = \tau_\varepsilon \wedge \varepsilon$, for some $\varepsilon > 0$ such that $x - \varepsilon > 0$, $x + \varepsilon < u_1$, and $\tau_\varepsilon = \inf\{t \geq 0 : X_t \notin (x-\varepsilon, x+\varepsilon) | X_0 = x\}$. Taking expectations and noting that the last term is a martingale because $M(x;u_2)$ is concave, we obtain

$$E_x\left[e^{-\rho\tau}M(X_\tau;u_2)\right] = M(x;u_2) + E_x\left[\int_0^\tau (A-\rho)M(X_s;u_2)ds\right],$$

where $E_x\left[e^{-\rho\tau}M(X_\tau;u_2)\right]$ is the value of waiting until τ prior to ordering a new capital issue. Because immediate ordering of capital is the optimal action at $x < u_1$, we have

$$M(x;u_2) \geq E_x\left[e^{-\rho\tau}M(X_\tau;u_2)\right].$$

Combining the last two equations and the fact that $V(x) = M(x;u_2)$ for $x < u_1$, we get

$$E_x\left[\int_0^\tau (A-\rho)V(X_s)ds\right] \leq 0 .$$

A limit operation then gives us

$$\lim_{\varepsilon \to 0} \frac{1}{E_x[\tau]} E_x\left[\int_0^\tau (A-\rho)V(X_s)ds\right] = (A-\rho)V(x) \leq 0 ,$$

for $x < u_1$.

For $u_1 < x < u_2$, we have $(A-\rho)\,V(x) = 0$ by construction.

For $x > u_2$, we have $V(x) = \mu/\rho + x - u_2$, $V'(x) = 1$, $V''(x) = 0$, so that

$$(A-\rho)V(x) = \frac{1}{2}\sigma^2 V''(x) + \mu V'(x) - \rho V(x) = \mu - \rho\left(\frac{\mu}{\rho} + x - u_2\right) = u_2 - x < 0$$

for all $x > u_2$.

(13iv): $V'(x) = 1$ for $x \geq u_2$ by construction. That $V'(x) > 1$ for $x < u_2$ follows from the concavity of V (proved above).

(13v): Follows directly from construction.

End of proof.

Proof of Proposition 4. The first-order condition is like in the general case, but with (13i) and (13ii) replaced by (21). (13iii) and (13iv) are solved as in the general case, and smooth pasting at u_2 is enforced as previously. This yields formulas (A4) and (A6). The barrier u_2 is solved by substituting (A4) into the boundary condition (21). Assuming that the maximum in (21) is achieved by the first term, u_2 is determined from the equation

$$g(u_2) \equiv a_1 e^{-d_{1+}u_2} + a_2 e^{-d_{1-}u_2} = \frac{\mu}{\rho} - K - u_2 \,. \tag{A13}$$

$g{:}R_+ \to R$ defined in (A13) has the following properties: i) $g(0) = \mu/\rho$; ii) $g'(u) < 0$; iii) $g(u) \to -\infty$ as $u \to \infty$; iv) $g''(0) = 0$; v) $g'''(u) < 0$. The convergence in iii) is exponential, and g is concave by iv) and v). It follows that when K is positive, we have $g(0) = \dfrac{\mu}{\rho} > \dfrac{\mu}{\rho} - K$, and because the right-hand side of (A13) is linear in u_2, (A13) is solved by a unique $\hat{u} > 0$.

For the maximum in (21) to be achieved by the first term, the solution to (A13) must satisfy

$$g(\hat{u}) = \frac{\mu}{\rho} - K - \hat{u} > 0 \,, \tag{A14}$$

i.e. $g(u_2)$ must be positive at the point of intersection with the function $\mu/\rho - K - u_2$. Setting $g(u) = 0$, we get that $u = u_0$, where u_0 is given by (17). Then a necessary and sufficient condition for (A14) to hold is that $\mu/\rho - K - u_0 > 0$, or equivalently that $K < \mu/\rho - u_0$, which is the condition in the proposition. In this case $\hat{u} < u_0$. End of proof.

Table 1. Bank return parameters

The sample is 61 commercial banks with at least 15 years of annual Compustat data over the period 1983-2002. Each bank's estimate of μ (σ) is the time-series average (standard deviation) of the bank's ROA, as defined in (28). Capital buffers are bank level averages over 1993-2002. Two sided P-values associated with the null hypothesis of zero correlation are in parentheses under the correlation coefficients.

	All banks in the sample (61 banks)			Banks with above average loan loss provisions (29 banks)			Banks with below average loan loss provisions (32 banks)		
Distributions	capital buffer	average ROA (μ)	st.dev. ROA (σ)	capital buffer	average ROA (μ)	st.dev. ROA (σ)	capital buffer	average ROA (μ)	st.dev. ROA (σ)
Minimum	2.14 %	0.31 %	0.22 %	2.91 %	0.32 %	0.41 %	2.14 %	0.31 %	0.22 %
25th percentile	3.69 %	0.73 %	0.48 %	3.64 %	0.63 %	0.65 %	3.85 %	0.85 %	0.43 %
Median	4.70 %	0.91 %	0.60 %	4.09 %	0.81 %	0.78 %	5.31 %	0.97 %	0.49 %
Average	5.01 %	0.92 %	0.76 %	4.61 %	0.81 %	0.95 %	5.37 %	1.02 %	0.58 %
75th percentile	5.95 %	1.11 %	1.05 %	5.27 %	0.97 %	1.19 %	6.36 %	1.18 %	0.60 %
Max	10.43 %	1.88 %	1.89 %	8.11 %	1.35 %	1.89 %	10.43 %	1.88 %	1.33 %
St.dev.	1.67 %	0.32 %	0.40 %	1.42 %	0.29 %	0.42 %	1.82 %	0.31 %	0.27 %
Correlations	capital buffer	average ROA (μ)	st.dev. ROA (σ)	capital buffer	average ROA (μ)	st.dev. ROA (σ)	capital buffer	average ROA (μ)	st.dev. ROA (σ)
capital buffer	1.00	0.15	0.15	1.00	-0.21	0.58	1.00	0.26	0.04
		(0.26)	(0.23)		(0.27)	(0.00)		(0.15)	(0.81)
average ROA (μ)		1.00	-0.37		1.00	-0.33		1.00	-0.17
			(0.003)			(0.08)			(0.34)
st.dev. ROA (σ)			1.00			1.00			1.00

Table 2. Impact of delay and cost on the frequency and size of capital issues

Δ is the delay (in years) between the decision to issue capital and the actual issue. K is the fixed cost of capital issue; u_1 is the optimal capital issue barrier; u_2 is the optimal dividend barrier; $E[s_i]$ is the expected net (of costs) size of a capital issue; all the previous are as a percentage of risk-weighted assets. $K/(K+E[s_i])$ is the expected proportional cost of a capital issue, i.e. total cost divided by gross proceeds; $E(t_i\text{-}t_{i\text{-}1})$ is the expected time (in years) between successive capital issues, conditioned on the assumption that the bank will not be liquidated during the first issue delay. Fixed parameter values: $\mu = 1.0\%$, $\sigma = 1.0\%$, and $\rho = 4.0\%$.

(years)	K	u_1	u_2	$E[s_i]$	$K/(K+E[s_i])$	$E(t_i\text{-}t_{i\text{-}1})$ (years)
0.08	0.10 %	0.9 %	2.4 %	1.5 %	6 %	8
0.08	0.25 %	0.8 %	2.7 %	1.8 %	12 %	18
0.08	0.50 %	0.7 %	2.9 %	2.2 %	19 %	36
0.08	1.00 %	0.6 %	3.1 %	2.5 %	29 %	68
0.5	0.10 %	1.6 %	3.2 %	1.6 %	6 %	11
0.5	0.25 %	1.3 %	3.4 %	2.0 %	11 %	28
0.5	0.50 %	1.1 %	3.5 %	2.4 %	17 %	57
0.5	1.00 %	0.9 %	3.7 %	2.8 %	27 %	122
1	0.10 %	1.7 %	3.5 %	1.8 %	5 %	16
1	0.25 %	1.4 %	3.7 %	2.3 %	10 %	44
1	0.50 %	1.1 %	3.7 %	2.7 %	16 %	103
1	1.00 %	0.6 %	3.8 %	3.2 %	24 %	291

Table 3. Correlations between model capital and explanatory factors

u_1 is the optimal capital issue barrier; u_2 is the optimal dividend barrier; u_0 is the optimal dividend barrier in the model with no capital issuance option. The right-most column applies u_2 (u_0) to large (small) banks. Asset growth, asset size, and loan loss provisions are bank level time averages 1983-2002.

All banks in the sample (61)	actual capital	capital issue barrier (u_1)	dividend barrier (u_2)	dividend barrier (u_0)	difference (actual-u_2)	difference (actual-u_0)	difference (actual-u_2/u_0)
Average	5.01 %	0.80 %	2.40 %	2.72 %	2.61 %	2.29 %	2.37 %
St.dev.	1.67 %	0.78 %	1.63 %	2.09 %	2.20 %	2.52 %	2.43 %
Correlation with							
actual capital	1.00	0.10	0.12	0.12	0.68	0.56	0.56
average ROA (μ)	0.15	-0.47	-0.48	-0.47	0.46	0.49	0.48
volatility ROA (σ)	0.15	0.98	0.99	0.99	-0.62	-0.72	-0.72
asset growth	-0.03	0.07	0.07	0.06	-0.08	-0.07	-0.07
asset size	-0.39	0.11	0.10	0.10	-0.37	-0.34	-0.29
loan loss prov./assets	-0.34	0.49	0.48	0.47	-0.61	-0.62	-0.58

Banks with above average loan loss provisions (29 banks)	actual capital	capital issue barrier (u_1)	dividend barrier (u_2)	dividend barrier (u_0)	difference (actual-u_2)	difference (actual-u_0)	difference (actual-u_2/u_0)
Average	4.61 %	1.20 %	3.24 %	3.78 %	1.37 %	0.84 %	1.00 %
St.dev.	1.42 %	0.81 %	1.69 %	2.24 %	1.46 %	1.83 %	1.72 %
Correlation with							
actual capital	1.00	0.54	0.57	0.58	0.32	0.07	0.02
average ROA (μ)	-0.21	-0.38	-0.44	-0.42	0.30	0.35	0.33
volatility ROA (σ)	0.58	0.98	0.99	0.99	-0.58	-0.75	-0.79
asset growth	0.19	-0.02	-0.01	-0.01	0.19	0.16	0.16
asset size	-0.43	-0.11	-0.12	-0.11	-0.28	-0.19	-0.10
loan loss prov./assets	-0.16	0.20	0.20	0.20	-0.38	-0.36	-0.27

Figure 1. Illustration of model structure

x is the current level of the capital buffer; u_1 denotes the capital issue barrier; u_2 denotes the dividend barrier; Δ is the delay (in years) between the decision to issue capital and the actual issue; all these are as a percentage of risk-weighted assets.

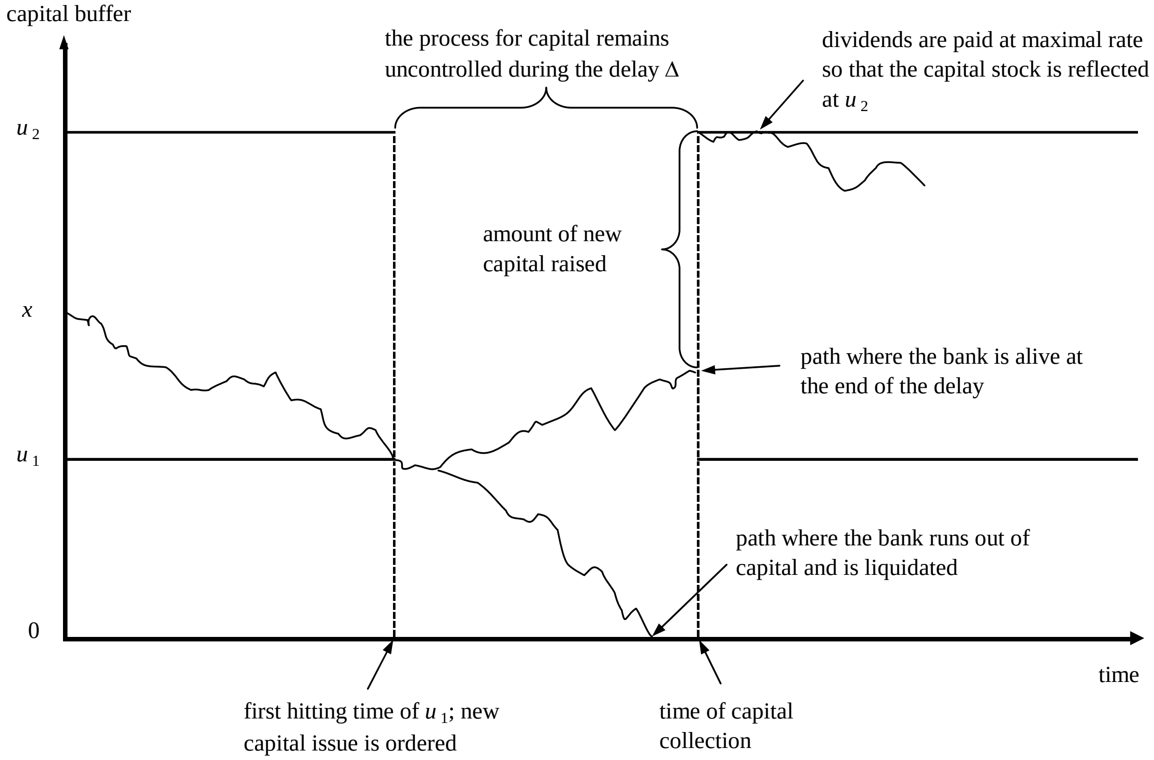


Figure 2. Construction of the value function

The picture shows the components of the value function (19). Parameter values: $\mu = 1.0\%$, $\sigma = 1.0\%$, $\rho = 4.0\%$, $\Delta = 0.5$ years, and $K = 1.0\%$. The optimal capital issue barrier u_1 is 0.90% and the optimal dividend barrier u_2 is 3.66%. These are marked with vertical dotted lines. The optimal dividend barrier in the absence of the capital issue option, u_0, is 3.80%.

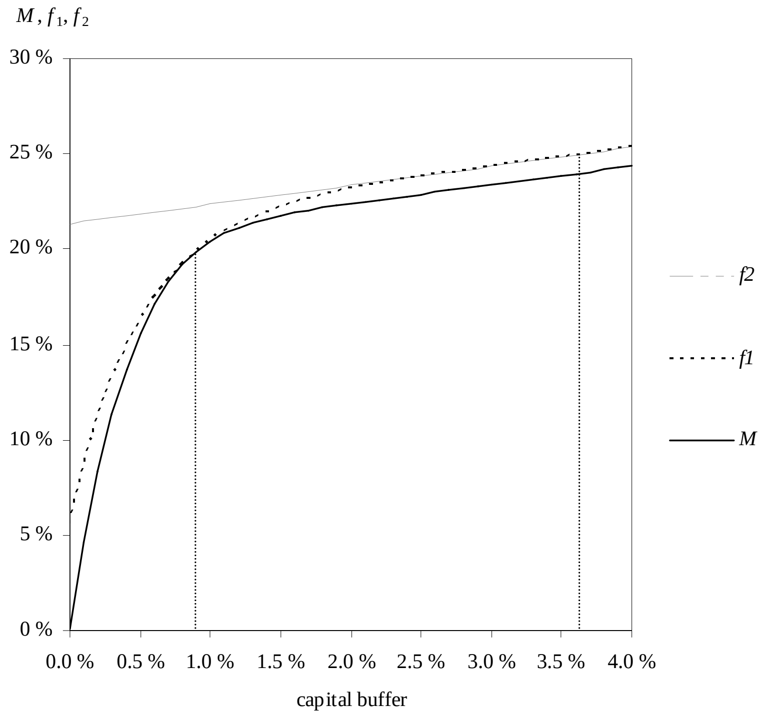
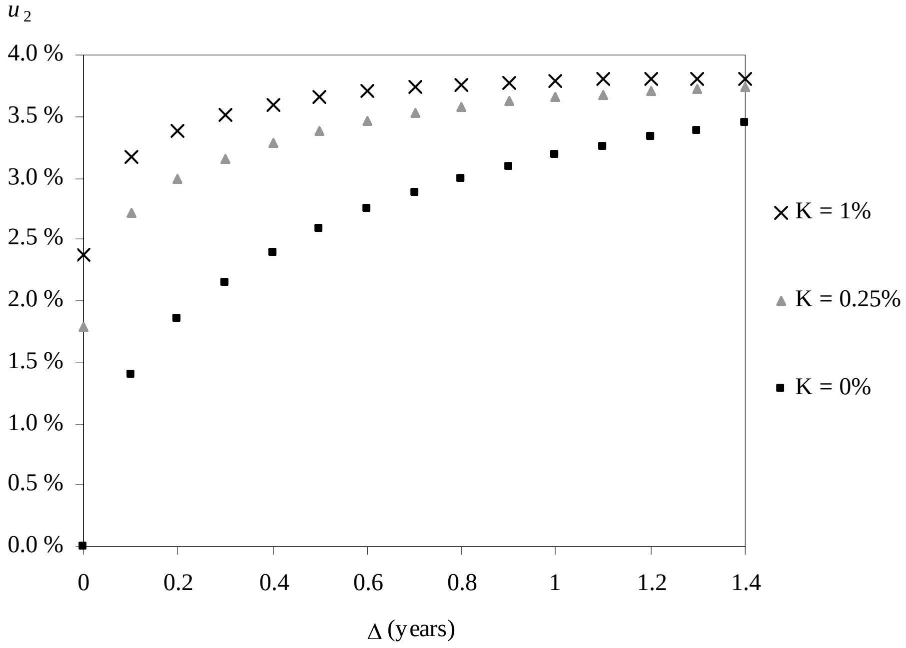


Figure 3. Optimal capital issue and dividend barriers

The upper picture shows the optimal dividend barrier u_2 as a function of the capital market imperfections Δ and K. The lower picture shows the optimal capital issue barrier u_1. Fixed parameter values: $\mu = 1.0\%$, $\sigma = 1.0\%$, and $\rho = 4.0\%$.



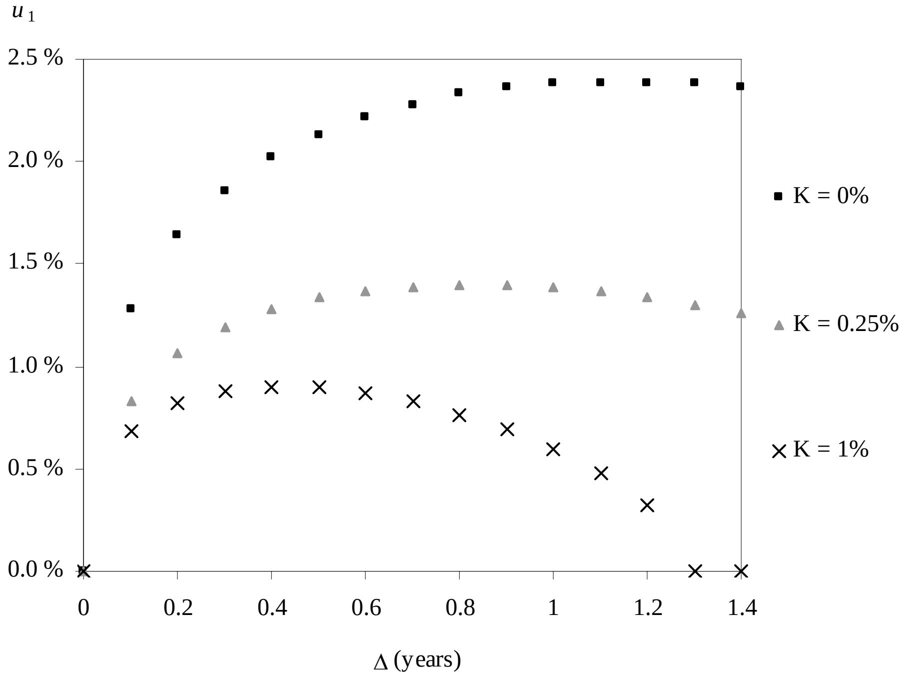


Figure 4. Value of the capital issue option

Value of the capital issue option as a percentage of risk-weighted assets, as a function of the capital buffer. Fixed parameter values: $\mu = 1.0\%$, $\sigma = 1.0\%$, $\rho = 4.0\%$, and $\Delta = 0.5$ years. The rectangle along each line indicates the capital issue barrier u_1, and the cross indicates the dividend barrier u_2.

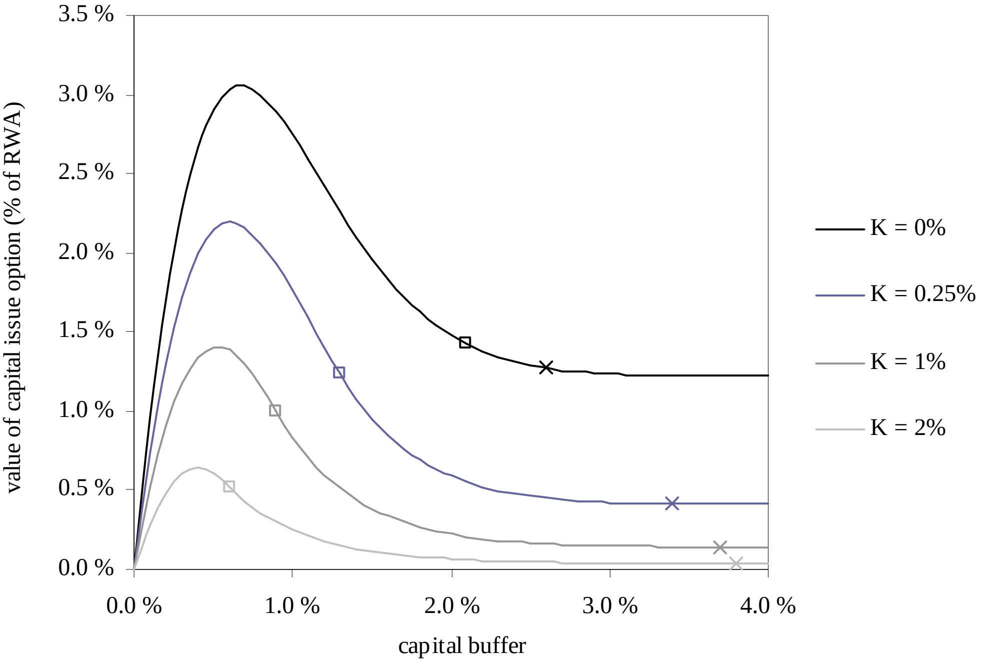


Figure 5. Actual vs. model capital ratios for the sample banks

The left picture shows average actual capital buffer (over 1993-2002) plotted against model dividend barrier (u_2). The right picture shows maximum actual capital buffer (over 1993-2002) plotted against model dividend barrier (u_2). The filled (empty) squares are banks with above (below) average loan loss provisions. A linear least squares fit is drawn into the pictures for both groups of banks.

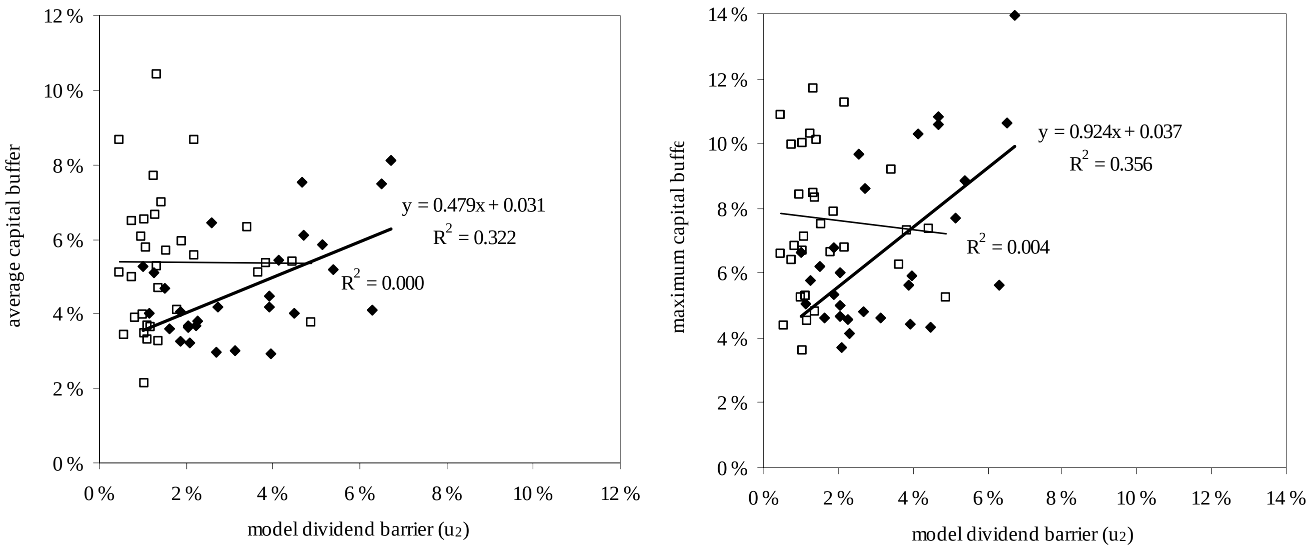


Figure 6. Drivers of the model capital buffer

The left picture shows model dividend barrier (u_2) plotted against σ-estimates for the sample of 61 banks. The banks that have higher (lower) than average loan loss provisions have been marked with squares (crosses). The right picture shows u_2 plotted against the corresponding μ-estimates. The observations have been marked depending on the level of the σ-estimate: 0-25 percentile (cross); 26-50 percentile (triangle); 51-75 percentile (circle); 76-100 percentile (square). The correlation in the left (right) picture is 99% (-48%). The correlations among the subsamples in the right picture are -28% (cross), -79% (triangle), -41% (circle), and -6% (square).

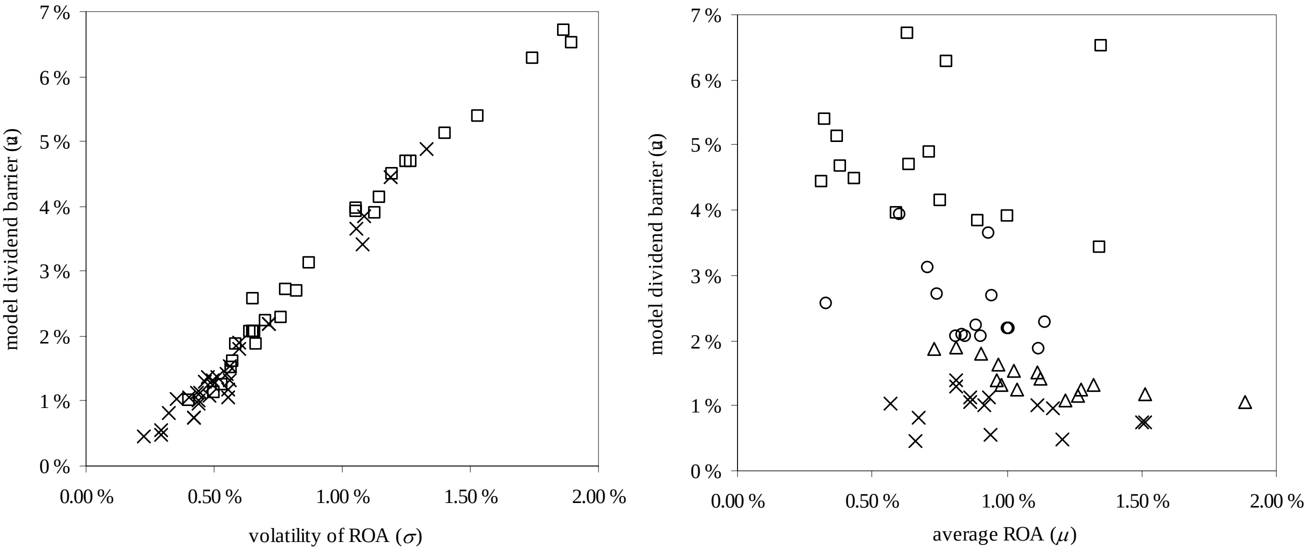


Figure 7. Implied volatilities for large and small banks

Dividend barrier (u_2) is plotted against volatility (σ) for large banks ($\mu = 0.86\%$, $\rho = 4.0\%$, $\Delta = 0.5$, $K = 0.25\%$) and small banks ($\mu = 0.77\%$, $\rho = 4.0\%$, capital market imperfections prohibitively high). The dotted lines indicate the implied volatilities, 1.06% and 1.19%, corresponding to the actual capital ratios of large and small banks, 3.81% and 5.26%, respectively. The average σ-estimate for large (small) banks is 0.82% (1.06%). All estimates are based on the subsample of banks with higher than average loan loss provisions (29 banks in total).