Effects of phonon pore scattering and pore randomness on effective conductivity of porous silicon

Jae Dong Chung, Massoud Kaviany*

Department of Mechanical Engineering and Applied Mechanics, The University of Michigan, Ann Arbor, MI 48109-2125, USA

Received 4 January 1999; received in revised form 10 April 1999

Abstract

The observed low effective thermal conductivity of porous silicon makes for its convenient fabrication and integration as a thermal insulation layer in microelectronics. The observed average pore size is controlled by the etching process and ranges between 1 and 100 nm, which on the low end is much less than the bulk phonon mean-free path. This low effective conductivity, i.e., low effective phonon mean-free path, can be explained with the inclusion of the effects of the phonon pore scattering and the pore randomness. The available two-dimensional porous silicon pore-network simulations are used along with the Boltzmann transport equation to determine the effective conductivity. It is shown that the hindering effect of the phonon pore scattering (due to reflection from the solid-pore interface) is significant for small pore size. Also, due to the dendritic structure of the pores, the hindering effect of the pore-network randomness is significant. The predictions are compared with the existing experiments and a good agreement is found.

#1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

Porous silicon layers are formed on the surface of locally or wholly doped silicon wafers. It can also be formed away from the wafer surface by proper, distributed doping. The etching is by anodic dissolution (i.e., electrochemical etching) of the monocrystalline, impurity-doped silicon in aqueous, hydrofluoric acid (HF) solutions. The surface, porous-silicon layer is depicted in Fig. 1. Due to the pore morphology, the effective conductivity is not isotropic and a preferential conduction heat flow occurs along the \( y \)-direction. For the surface, porous layer shown, the porous-layer thickness \( \delta \) may be as small as a few micron and as large as desired (i.e., up to a few hundred micron). Micrograph of a typical porous silicon surface is shown in Fig. 2. Due to its photoluminescence capability and having physical properties greatly different from the bulk material (such as permeability, low effective thermal conductivity, and low density), porous silicon is a promising material in silicon-based integrated circuits and devices. As an example, many thermal sensors operate are based on the measurement of a small amount of thermal energy. This minute amount of energy must cause a relatively large temperature change in the sensing medium. Then this medium must be small and well isolated from its surroundings and a porous-silicon layer can be used. Although complete backside etching is also possible for insulation, it has a mechanical vulnerability disadvantage compared to porous-silicon layer. Other applications, and the current knowledge about porous silicon, are reviewed by Canham [1].

* Corresponding author. Tel.: +1-313-936-0402; fax: +1-313-647-3170.
E-mail address: kaviany@umich.edu (M. Kaviany)

1 Temporary address: Institute of Advanced Machinery and Design, Seoul National University, Seoul, 151-742, South Korea.
The low effective thermal conductivity of porous silicon is expected to be due to the low-dimensionality of the pore walls. This can result in the domination of pore-surface scattering over the intra-solid attenuation. The low conductivity is also due to the tortuous solid conduction path and the presence of a low conductivity gas (i.e., air) in the pores. The random pore structure (i.e., pore morphology), which varies greatly with the wafer-etching conditions, is relatively well documented. Simulations of the pore structure has also been attempted using various models (namely the diffusion-limited and the kinetic-limited models), and good qualitative agreements have been found with the observations. Measurements of the effective thermal conductivity of porous silicon have also been made for several different wafer-etching conditions and also after post-etching oxidation.

The Fourier treatment of the effective conductivity tensor \( \mathbf{K} \), is made by averaging the energy equation over an elementary volume representing the porous medium. When the porous medium has a periodic structure, successful predictions can be made [2]. Recent analytical solutions for periodic structures include that of Bauer [3] for isotropic-periodic, and Lee and Yang [4] for anisotropic-periodic structures. As evident in Fig. 1, porous silicon shows a strong anisotropy and also it is expected that simple, unit-cell based representations will not be realistic and effective.
Kaviany [5] provides a review along with a number of correlations for the effective conductivity and their conditions of applicability. In direct simulation of the effective conductivity tensor rather than imposing a prescribed, simple structure, the pore structure can be scanned from micrographs (two- and three-dimensional structures). This has been done by Nishioka et al. [6] for porous iron (sintered iron particles). The two-dimensional computational grid consists of the same number of pixels as the scanned image. Bakker [7] also discusses the importance of the inclusion of the actual morphology.

Other than the pore morphology, the size effect and the chemical changes (such as oxidization), also influence \( (K) \). The pore-wall length scale of porous silicon can be small compared to the bulk phonon mean-free path \( \lambda_p \). Then, the silicon-pore interface phonon scattering becomes important. This causes a decrease in...
the effective phonon mean-free path \([8]\). This reduction in \((\mathbf{K})\) becomes more pronounced with a decrease in the ratio of the average pore-wall thickness to the bulk mean-free path.

The oxidation of porous silicon (which is rapid due to the availability of a large specific surface area) for the mesoporous silicon is examined by Drost et al. \([9]\), where they show that \((\mathbf{K})\) decreases upon oxidation. This is because, at room temperature, bulk thermal conductivity of pure silicon is about 149 W/m K and that of silicon oxide is about 1.5 W/m K. Benedetto et al. \([10]\) and Gesele et al. \([11]\) have not addressed the oxidation in the discussion of their experimental results.

Here we use existing pore-network simulations for porous silicon and the Boltzmann transport equation for phonons, to allow for the effects of the pore randomness, low dimensionality of the pore wall, and the pore-surface scattering. We also compare the predicted results with those obtained using the bulk conductivity and the Fourier law (i.e., no low dimensionality), and those based on the unit-cell models (i.e., no randomness). Finally, we compare the predictions with the available experimental results.

2. Pore network

The morphology of porous-silicon layers is designated by the average porosity \((\varepsilon)\), the average pore size \((d)\), the pore-size distribution, and the pore geometry. A wide variety of morphologies is achievable depending on the surface crystal plane orientation, impurity type, electrical resistivity of the doped silicon, and the electrochemical parameters such as HF concentration, electrical current density, anodization time, front- and back-side irradiation, and temperature. The pore size can be controlled over three orders of magnitude from nanometers to micrometers and the average porosity of the obtained layer \((\varepsilon)\) can be tuned from 0.10 to 0.90. The porosity decreases substantially upon oxidation.

The pore morphology is characterized by the presence of mesopore and macropore structures. The geometry of the mesopore structures can be divided into columnar (i.e., anisotropic) and equiaxed (i.e., isotropic). For the \(p^+\)-type silicon wafers, \((d)\) is generally small and the structure is equiaxed. For the \(p^+\)-type, \((d)\) is larger and the pores are more columnar. The trend is basically the same for the \(n\)-type silicon \([12,13]\). The \(n\)-type silicon etched with a light illumination is very similar in structure to the \(p\)-type silicon. However, the pore diameters in \(n\)-type silicon (100 nm or more) are considerably larger than the \(p\)-type silicon. An overview of the different morphologies is given in Ref. \([14]\). Fig. 3 presents a summary of the above morphological characteristics of the porous silicon layers with the impurity as a variable.

The mechanisms of formation of porous silicon has been extensively studied, but is not completely clear. The diffusion-limited model, proposed by Smith et al. \([15]\), is one of the accepted models for the pore formation. It explains pore formation from the diffusion of an electroactive species, such as holes and electrons, to or from the silicon interface. The computer simulations of the porous silicon formation using the diffusion-limited model, shows a striking resemblance to the micrographs. Fig. 4 is a typical pore structure obtained from the diffusion-limited simulation. The figure also shows the representative elementary volumes chosen. Each axis gives the grid number in that direction.

A typical pore-size distributions obtained by nitrogen adsorption, for the pore structure in Fig. 2, is shown in Fig. 5(a). The diffusion-limited model does not address the pore size. The dimensionless pore size distribution for the pore network of Fig. 4 is shown in Fig. 5(b). The average pore size for simulation can be estimated using this similarity, although depending on the dopant used and the etching applied, different pore distributions are obtained.

3. Energy equation with Fourier law

On a macroscopic scale, conductive heat transfer can be accurately represented by the Fourier law and energy equation as

\[
\mathbf{q} = -k \nabla T, \quad \nabla \cdot \mathbf{q} = 0.
\]

From the kinetic theory, the thermal conductivity of non-metallic solids can be related to their other physical properties as \([8]\)

\[
k = \rho c_p u_p \hat{\lambda}_p / 3,
\]

where \(\rho c_p\) is the lattice heat capacity, \(u_p\) is the phonon speed, and \(\hat{\lambda}_p\) is the phonon mean-free path. Although the laws of macroscopic heat transport are well known, there exist regimes of size and temperature where such laws are no longer applicable \([16–18]\). Recent experiments show that the phonon scattering at the interface of small, dissimilar materials is considerable enough to reduce the heat transfer rate \([19–21]\).

Here, for prediction of \((\mathbf{K})\), we begin by using the bulk thermal conductivity of silicon along with the Fourier treatment. Then the effect of the pore morphology of porous silicon on the effective thermal conductivity is examined. In the next section, the Boltzmann treatment is applied to include the low-dimensional effect.

Schematic diagrams of two-dimensional unit cell,
and random network models of porous silicon are shown in Fig. 6(a) and (b). The room temperature thermal conductivity of pure silicon ($k_s = 149 \text{ W/m K}$) is used. The doping effect which can be significant at low temperatures [22], is not included. In order to compare the effective thermal conductivity of various porous silicon layers, the average pore size $\langle d \rangle$, the average porosity $\langle e \rangle$, and the pore morphology, are assumed to be independent of depth $d$ [13]. Experiments [11] show that the layer thickness does not influence the effective thermal conductivity $k_{\text{eff}}$.

3.1. Solution method and verification

Fig. 4 depicts a typical, simulated two-dimensional pore structure for porous silicon. The air-filled pores are shown with shade. The pore structure which is available as a digital data, is used for the grid net. The energy equation (1) is discretized using the finite-volume formulation [23] and then solved using the BASIS solver [24].

When the heat flows across the porous silicon layer, the left and the right control surfaces are taken as adiabatic. The lower and the upper surfaces have prescribed temperatures $T_0$ and $T_d$, respectively. The thermal conductivity jump, across the air–silicon interface, is presented by the harmonic mean of $k_l$ (air, $k_l = 0.0267 \text{ W/m K}$) and $k_s$ (silicon, $k_s = 149 \text{ W/m K}$) [23].

The effective thermal conductivity tensor ($K$), and

---

Fig. 3. A classification of the pore morphology, average porosity $\langle e \rangle$, and average pore size $\langle d \rangle$ for an $n$- and $p$-type porous silicon.

Fig. 4. Pore network and the grid system used. Here the average porosities $\langle e \rangle = 0.278$ obtained from diffusion-limited simulations by Smith and Collins [34].
its component across the porous silicon layer \((k)_{yy}\), are determined from

\[
(q) = -(K) \cdot \nabla T, \quad (k)_{yy} = -\frac{(q)_{yy}}{(T_0 - T_\delta)/\delta},
\]

where the \(y\)-direction heat flux \((q)_{yy}\) is found from

\[
(q)_{yy} = \frac{1}{2L} \left( \int_0^L \frac{\partial T}{\partial y} |_{y=0} \, dx + \int_0^L \frac{\partial T}{\partial y} |_{y=d} \, dx \right).
\]

The convergence criterion is that the heat fluxes at \(y = 0\) and \(d\) be different by less than 0.01\%. The lateral effective thermal conductivity \((k)_{xx}\) is found similarly.

Validations were made by comparing the computed \((k)\) for a cubic array of square inclusions with those reported in Ref. [25]. Complete agreement was found.

3.2. Selection of representative elementary volume

The representative elementary volume was selected by progressively increasing the grid net until the computed volume-averaged porosity \((\epsilon)\) and effective conductivity \((k)\) no longer change noticeably. As expected, with the increase in the grid net, the computation time increases substantially. Table 1 and Fig. 7 show the grid size effect on the effective thermal conductivity.
and the average porosity for the asymptotic average porosity \( \langle \varepsilon \rangle = 0.278 \). The corresponding grid nets are shown in Fig. 4, for grid nets A to E. As a compromise, the \( 300 \times 300 \) grid (grid net D) is chosen for this microstructure.

For lower porosities, a larger grid net is needed to reach the asymptotic average porosity and effective conductivity. For example, for \( \langle \varepsilon \rangle = 0.092 \) the computed \( \langle k \rangle_{yy} \) may not reach the asymptotic value even when the largest available grid (i.e., \( 2000 \times 2000 \)) is used. This structure is shown in Fig. 8. Also, note that for the columnar structures, the structural (and consequently the transport) anisotropy is quite substantial.

### Table 1

<table>
<thead>
<tr>
<th>Grid ((M_x \times M_y))</th>
<th>(\langle \varepsilon \rangle^a)</th>
<th>(\langle k \rangle_{yy}) (W/m K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(50 \times 50) (A)</td>
<td>0.262</td>
<td>28.27</td>
</tr>
<tr>
<td>(100 \times 100) (B)</td>
<td>0.277</td>
<td>18.01</td>
</tr>
<tr>
<td>(200 \times 200) (C)</td>
<td>0.279</td>
<td>17.83</td>
</tr>
<tr>
<td>(300 \times 300) (D)</td>
<td>0.277</td>
<td>16.31</td>
</tr>
<tr>
<td>(400 \times 400) (E)</td>
<td>0.277</td>
<td>16.21</td>
</tr>
</tbody>
</table>

\(^a\) The asymptotic average porosity is \( \langle \varepsilon \rangle_{N \to \infty} = 0.278 \).

Fig. 7. The variation of average porosity \( \langle \varepsilon \rangle \) along (a) \( x \)-direction, and (b) \( y \)-direction and their dependence on grid system of Table 1.

Fig. 8. Typical pore morphology \((\langle \varepsilon \rangle = 0.092)\) obtained from diffusion-limited simulation by Smith and Collins [34].

4. Boltzmann transport equation

The Boltzmann transport equation is often used to model transport by particles that follow an arbitrary distribution. In a general form, the equation can be written as [8]

\[
\frac{\partial f}{\partial t} + u_p \cdot \nabla_x f + a_p \cdot \nabla_u f = \left( \frac{\partial f}{\partial t} \right)_s.
\]  

(5)

Using the analogy between photons and phonons as wave packets of energy, the intensity of phonons \( I \) is defined as [26]
\[ I = \frac{1}{4\pi} \sum_{m=1}^{n_{\text{max}}} |\nu_{p,m}|/h_{p,o} \rho_{m}(\omega) \, d\omega \]  

(6)

where \( D \) is the density of the states per unit volume, \( f \) is the phonon distribution function, \( h_{p,o} \) is the Planck's constant, \( |\nu_{p,m}| \) is the magnitude of the phonon group velocity, and \( \omega \) is the phonon frequency. The summation index \( m \) is made over the three phonon polarizations. By multiplying Eq. (5) by \( \nu_{p,m}/h_{p,o} \rho_{m}(\omega) \) and integrating it over all frequencies, the Boltzmann transport equation can be transformed to the equation of phonon radiative transfer [16], i.e.,

\[ \frac{1}{\nu_{p}} \frac{\partial I}{\partial t} + \mu \frac{\partial I}{\partial x} + \zeta \frac{\partial I}{\partial y} = \frac{I h(T(x)) - I}{\nu_{p}(T)}, \]  

(7)

where \( \mu \) and \( \zeta \) are the directional cosines. It is evident that this equation is similar to the photon equation of radiative transfer [27]

\[ \frac{1}{c} \frac{\partial I}{\partial t} + \frac{\partial I}{\partial x} + \zeta \frac{\partial I}{\partial y} = \sigma_{a} I - \sigma_{s} I + \frac{\sigma_{a}}{4\pi} \int_{4\pi} r(s') \Phi(r, s') \, ds', \]  

(8)

where \( c \) is the speed of light, \( \sigma_{a}, \sigma_{s} \), and \( \sigma_{e} \) are absorption, extinction, and scattering coefficients, and \( \Phi \) is the phase function. This similarity gives an easy access to the solution methods which are well developed for the equation of radiative transfer.

For steady state, the two equations (7) and (8) are exactly the same if scattering is neglected (i.e., \( \sigma_{s} = 0 \)). This does not mean that Eq. (7) can deal with only non-scattering media. As will be shown later, the main reduction of the heat flow rate is due to the scattering mechanism such as defects, dislocations, boundaries and particle interactions. So the inclusion of scattering effects is important in phonon transport. The various scatterings are modelled using relaxation time \( \tau \), in Eq. (7). In Eq. (7), several bulk material properties are pre- 

scribed, these include the specific heat, the phonon group velocity, and the phonon mean-free-path (or relaxation time).

Once the intensity is found by solving Eq. (7), the radiative heat flux can be determined from

\[ q(r) = \int_{\Phi = \alpha_{e}} \mu I(r, \Phi) \, d\Phi. \]  

(9)

4.1. Solution method and verification

The discrete ordinate method (SN method) is a tool to transform the equation of radiative transfer into a set of simultaneous partial differential equations. This is based on a discrete representation of the directional variation of the radiative intensity \( I \). A solution to the transport problem is found by solving the equation of radiative transfer for a set of discrete directions spanning the entire solid angle. The integrals over the solid angle are approximated by numerical quadrature. This method is widely accepted and a comprehensive discussion is available [28].

For a specific ordinate direction \( m \), defined by \( \Phi_{m} = (\mu_{m}, \zeta_{m}) \), Eq. (7) can be approximated as

\[ \left[ \mu_{m} \frac{\partial}{\partial x} + \zeta_{m} \frac{\partial}{\partial y} + \sigma_{a}(x, y) \right] I_{m}(x, y) = \sigma_{s}(x, y) I_{s}(x, y), \]  

(10)

To solve the discrete ordinate equation, the rectangular enclosure is subdivided into small control volumes in a \( M_{x} \times M_{y} \) net. Within each control volume, the spatially discretized equation for the radiative intensity in the ordinate direction \( \Phi_{m} \) is derived as

\[ \left[ \mu_{m}(A_{x} I_{em} - A_{s} I_{sm}) + \zeta_{m}(B_{x} I_{sm} - B_{s} I_{em}) \right] + \Delta V I_{pm} = \Delta V S_{pm} \]  

(11)

for \( m = 1, \ldots, M \),

where \( e, w, n, s \) are the boundaries in the compass directions and \( p \) is the center point of the control volume. The area and volume elements assume a unit depth in the \( z \) direction.

The number of unknown \( I_{s} \) in Eq. (11) are reduced by using one of several relationships between the control-volume boundary intensities and the center point intensity. The weighted diamond difference scheme is used in this study to relate the intensities in the control volume. The weighted relationship of the cell boundary intensities to the average intensity in the cell is expressed as

\[ I_{pm} = w_{em} I_{em} + (1 - w_{em}) I_{sm} \]

\[ = w_{my} I_{ym} + (1 - w_{ym}) I_{ym}. \]  

(12)

To avoid negative intensities, a scheme suggested by Lathrop [29] is applied in selecting the differencing weights.

If \( I_{em} \) and \( I_{sm} \) are assumed to be known, where the iteration is in the direction with a positive directional cosines and in increasing space dimensions, then Eq. (11) can be reduced to eliminate the intensities \( I_{em} \) and \( I_{sm} \), using Eq. (12). Solving for \( I_{pm} \) yields

\[ I_{pm} = \frac{\mu_{m} A_{x} I_{sm} + \zeta_{m} B_{x} I_{em} + \Delta V S_{pm}}{\mu_{m} A_{x} / w_{em} + \zeta_{m} B_{x} / w_{ym} + \Delta V}, \]  

for \( \mu_{m}, \zeta_{m} > 0 \),

\[ \sum_{m=1}^{M} \frac{\mu_{m}}{w_{em}} A_{x} I_{em} + \sum_{m=1}^{M} \frac{\zeta_{m}}{w_{ym}} B_{x} I_{sm} + \sum_{m=1}^{M} \Delta V S_{pm} = \sum_{m=1}^{M} \frac{\mu_{m}}{w_{em}} A_{x} I_{sm} + \sum_{m=1}^{M} \frac{\zeta_{m}}{w_{ym}} B_{x} I_{em} + \sum_{m=1}^{M} \Delta V S_{pm}. \]  

(13)
where

\[ A = (1 - w_{xm})A_x/w_{xm} + A_u, \]
\[ B = (1 - w_{ym})B_u/w_{ym} + B_x. \quad (14) \]

Special attention is needed for the selection of the ordinates set, \( \mu_m \) and \( \xi_m \). To satisfy the continuity at the interface, the set of ordinates and weights should also satisfy the first moment over a half range, that is,

\[ \int_{\hat{n} \cdot \hat{s} < 0} |\hat{n} \cdot \hat{s}| \, d\Phi = \int_{\hat{n} \cdot \hat{s} > 0} |\hat{n} \cdot \hat{s}| \, d\Phi = \pi = \sum w_{\mu} \hat{n} \cdot \hat{s}. \quad (15) \]

The set of ordinates and weights satisfying Eq. (15) is taken from Fiveland [30].

Once all directional intensities for each finite volume have been calculated, the values for the boundary and source terms may be updated, and the procedure is repeated until the convergence criteria are met. Convergence is checked during the iteration process using the cell-average intensities at the current and the previous iterations. It is assumed that the convergence is obtained when the maximum percentage error of the intensities is less than 0.0001%.

In the limit where the domain length scale is large compared to phonon mean-free path, the Fourier law should be recovered. In this optically thick case, the diffusion approximation is valid. Considering the jump boundary condition, Deissler suggested the following solution [31]

\[ \frac{\sigma_{SB}(T^\frac{3}{2} - T_{\text{ref}}^\frac{3}{2})}{\frac{3}{4} \left( \frac{l}{\lambda} \right) + 1} \approx \frac{4\sigma_{SB}T^3\Delta T}{\frac{3}{4} \left( \frac{l}{\lambda} \right) + 1}. \quad (16) \]

Comparing Eq. (16) with Eqs. (1) and (2), the Stefan–Boltzmann constant \( \sigma_{SB} \) for phonon can be obtained as

\[ \sigma_{SB} = \frac{\rho C_p u_p}{16T^2}. \quad (17) \]

As a validation, as shown in Fig. 9, the predicted thermal conductivity of silicon approaches as asymptotic behavior (Fourier behavior) as the domain size increases.

For a further validation, the thermal conductivity of silicon layer is compared with the experiment by Asheghi et al. [32]. Good agreement is found in the experimental error range. Also, the predicted results for the one-dimensional, multi-layer composites are compared with the available results [18]. The boundary and interface between layers are shown in Fig. 10(a). Complete agreement is found, as shown in Fig. 10(b). The room-temperature properties used in the computation of thermal conductivity of GaAs/AlAs and Si/Ge layer composites are given in Table 2. Note that the lattice mismatch between Si/Ge layer composite is much larger than that of GaAs/AlAs, but the agreements are still fair. This agreement gives confidence to the analysis of porous silicon, where phonons are almost totally reflected from silicon–air interface.

### 4.2. Boundary and pore interface conditions

The treatment of boundaries and pore interface in the two-dimensional, unit-cell model (Fig. 6(a)) and the network model (Fig. 6(b)) is similar to that for the one-dimensional, multi-layer composites.

Ziman [8] proposed the following expression for estimating the interface scattering parameter \( p \):

\[ p \approx \exp \left( -16\pi^2 \frac{\delta_i^2}{\lambda^2} \right). \quad (18) \]

where \( \lambda \) is the carrier wavelength and \( \delta_i \) is the mean interface roughness. At room temperature, the dominant phonon wavelength is \( \lambda \sim 10–20 \text{ Å} \). Then, even for a monolayer the roughness is \( \delta_i \sim 3 \text{ Å} \). This gives an interface scattering parameter of nearly zero, and allows for the assumption of a totally diffuse interface.

The interface between layers reflects and transmits

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho C_p \times 10^6 ) (J/m³ K)</th>
<th>( u_p ) (m/s)</th>
<th>( \lambda_p ) (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GaAs</td>
<td>1.71</td>
<td>3700</td>
<td>208</td>
</tr>
<tr>
<td>AlAs</td>
<td>1.58</td>
<td>4430</td>
<td>377</td>
</tr>
<tr>
<td>Si</td>
<td>1.66</td>
<td>6400</td>
<td>409</td>
</tr>
<tr>
<td>Ge</td>
<td>1.67</td>
<td>3900</td>
<td>275</td>
</tr>
</tbody>
</table>
phonons and adds to the resistance to the heat flow. Referring to Fig. 10(a) for a totally diffuse interface, the energy balance at bottom surface gives

\[ I_{p2}^* = \frac{I_{p2}^*}{(\bar{c}_p \rho_p V_p) (\bar{T}_0 - T_L)} + \rho_p \bar{v}_p \frac{\rho_{p,2} C_{p,2} (\bar{T}_0 - T_L)}{4\pi} \]  

Similarly, the energy balance at the internal interface and the top surface, forming one period of the multi-layer composite, gives

\[ I_1^* (\xi = \xi_1) = \frac{R_{d12}}{\pi} \int \Phi' I_1^* (\xi = \xi_1) d\Phi' \]

\[ + \frac{T_{d12}}{\pi} \int \Phi' I_{p2}^* (\xi = \xi_1) d\Phi' \]

where the integration with respect to the solid angle is over the half space and \( R_{dij} \) and \( T_{dij} \) are the reflectivity and transmissivity at an interface for phonons incident from the \( i \) layer towards the \( j \) layer.

The relation between the reflectivity and transmissivity, and the transmissivity at the totally diffuse scattering limit, are obtained based on the diffuse scattering limit model of Swartz and Pohl [33], i.e.,

\[ T_{dij} = R_{dij} = 1 - T_{dij} \]  

\[ T_{dij} = \frac{\rho_{p,i} \mu_{p,i}}{\rho_{p,j} \mu_{p,j}} \]  

4.3. Unit-cell model

The unit cell described in Fig. 6(a) is a starting point for modeling the effective thermal conductivity of a porous medium. When the porous medium has a periodic structure, successful predictions have been made [2–4]. These are based on the Fourier treatment and the effect of the unit cell (or pore) size has not yet been addressed. On the other hand, the existing treatments based on Boltzmann transport equation are for one-dimensional layers, although it gives elegant semi-analytic solution. Considering the practical applications such as the porous silicon, the two-dimensional extension of the Boltzmann approach is needed.

Here we have considered three pore sizes (\( d = 1, 3, 5 \) nm). Given the pore sizes, the unit cell linear dimension \( l \) is determined using the porosity defined as

\[ I_{p2} = I_{p2}^* + \frac{\rho_p \bar{v}_p \bar{c}_p (\bar{T}_0 - T_L)}{4\pi} \]
\[ e = d^2/l^2. \] The effective thermal conductivity is then determined from

\[ (k)_{yy} = \frac{(q)_{yy}}{(T_0 - T_1)/l^2}, \tag{26} \]

where the \( y \)-direction radiation heat flux \((q)_{yy}\) is

\[ (q)_{yy} = \frac{1}{27} \left( \int_0 q_{yy}(x,0) \, dx + \int_0^l q_{yy}(x,l) \, dx \right). \tag{27} \]

The radiation heat flux, \( q_{yy} \), is defined in Eq. (9). The convergence criterion requires that the first and second terms be very close (i.e., much less than 1% for small network grid and as large as a few percent for the largest grid).

### 4.4. Selection of representative elementary volume

For the random pore network model, the representative elementary volume is similar to that of Section 3.2. Table 3 lists the grid systems tested for the pore network with the asymptotic average porosity of \((\varepsilon) = 0.201\). The corresponding selected pore distribution is shown in Fig. 11 for grids A to G. For computational economy, the \( 315 \times 315 \) grid (grid G) is selected for this porosity. Note that every grid point in Fig. 4 corresponds to four subgrids (two in \( x \)-direction and two in \( y \)-direction) in Fig. 11, since additional subgrids are needed to handle the pore interface. Fig. 7(a) and (b) show the porosity convergence for this grid set.

Table 4 shows the effect of the discrete ordinate quadrature sets for a \( 67 \times 67 \) grid, \( d = 1 \text{ nm} \), \((\varepsilon) = 0.201\) on the predicted effective conductivity \((k)_{yy}\)

\[
\begin{array}{l|cc}
\hline
\text{Grid} & \langle k \rangle_{yy} \, (\text{W/m K}) & \langle k \rangle_{yy} \\
\hline
67 \times 67 & 8.8659 & 21.4229 \\
87 \times 87 & 6.0112 & 15.1683 \\
131 \times 131 & 5.5386 & 13.9619 \\
159 \times 159 & 6.3194 & 15.6150 \\
203 \times 203 & 4.8340 & 11.9502 \\
251 \times 251 & 4.2461 & 10.3032 \\
315 \times 315 & 3.3201 & 8.3454 \\
\hline
\end{array}
\]

To show the effect of pore-network randomness, we use the Fourier treatment of Section 3 and to show the effect of pore size, we use the Boltzmann treatment of Section 4. Geselle et al. [11] suggest a simple relation for the combined effects of the morphology and the size. They suggest that Eq. (2) be modified as

\[ \langle k \rangle = \frac{1}{3} (1 - (\varepsilon))^3 \rho c_v u_p l, \tag{28} \]

where they suggest that the pore-network randomness effect be modeled with \((1 - (\varepsilon))^3\) and the pore-size effect with \(l\). However, this does not correctly predict the experiments for low porosity and does not satisfy the \((\varepsilon) \to 0\) asymptote, unless \(l\) is continuously changed with \(l \to l_p\) for \((\varepsilon) \to 0\).

To explain the effect of pore-network randomness and the pore size on \((k)\), four different computations are made, namely, (1) Fourier treatment plus unit cell, (2) Fourier treatment plus network, (3) Boltzmann

<table>
<thead>
<tr>
<th>( (k)_{yy} ) (W/m K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_4 )</td>
</tr>
<tr>
<td>( S_6 )</td>
</tr>
<tr>
<td>( S_8 )</td>
</tr>
<tr>
<td>( S_{10} )</td>
</tr>
</tbody>
</table>
5.1. Effect of pore-network randomness

To demonstrate the effect of pore-network randomness, \( k_{yy} \) is determined based on the Fourier treatment using the simulated pore-network model. The pore structure and the distribution of the normalized temperature \( (T - T_0)/(T_{\delta} - T_0) \), for grid D in Fig. 4, are shown together in Fig. 12. The air-filled pores are shown with shade. Fig. 12 shows how the high conductivity solid fingers the heat into the layer, while the low conductivity air causes a large temperature drop. The isothermal lines follow the high-conductivity solid-phase distribution. Also note that the random pore structure requires a large representative elementary volume for achieving an asymptotic (grid-size independent) behavior.

For a variety of applications, high porosity (e.g., \( \psi > 0.4 \)) porous silicon is more desirable and most experiments have been conducted for this range. However, the simulation of Smith and Collins [34] are for relatively low porosities (i.e., \( \psi < 0.28 \)). For comparison with the experiments, the low-porosity simulations can be slightly extrapolated. Two additional pore networks with slightly larger porosities (\( \psi = 0.322 \) and \( 0.380 \)) are formed using the available network for the largest porosity (\( \psi = 0.278 \)). Different enlargement factors are applied to the solid phase and the pore volume. Then the enlarged domain size is scaled to its initial value. For validation of this process, a pore network with \( \psi = 0.252 \), is produced using the available pore network with \( \psi = 0.208 \). It is found that the predicted effective thermal conductivities are in good agreement.

The uncertainty associated with extensive extrapolations is partly due to the highly irregular pore struc-
ture and partly due to lack of availability of a large set of simulated networks.

Fig. 13 shows the predicted variation of the effective thermal conductivity \( k_{\text{eff}} \) with respect to porosity. By noting the difference with the results of the unit-cell model, we note that the heat transfer is significantly hindered by the pore-network randomness.

5.2. Effect of phonon pore scattering

The mean-free path can be determined using Eq. (2). At room temperature bulk thermal conductivity of silicon is \( k_s = 149 \, \text{W/m K} \), and its volumetric heat capacity is \( \rho c_v = 1.659 \times 10^5 \, \text{J/m}^3 \, \text{K} \) [35]. A special average of the transverse and the longitudinal phonon velocities, \( u_{\text{ph}} = \frac{1}{2}(u_{\text{ph}}^T + u_{\text{ph}}^L) = 6.53 \times 10^3 \, \text{m/s} \), is used [36]. This gives the bulk phonon mean-free path \( \lambda_{\text{p},0} \approx 40 \, \text{nm} \). According to Flick et al. [37], the size effects on heat conduction becomes important when \( l < 7\lambda_{\text{p},0} \approx 280 \, \text{nm} \) and this is larger than the average pore size \( \langle d \rangle \) for porous silicon considered here. The small pore wall thickness introduces an additional resistance to heat flow.

To apply the Boltzmann transport equation, the wall-pore dimension should be larger than the phonon wavelength. At room temperature, the phonon wavelength for silicon is approximately \( \lambda_{\text{ph}}/T \approx 1.1 \, \text{nm} \) [17]. Also, the quantum size effect is negligible in this large length scale. We also note that as the linear dimension of the representative elementary volume increases, the optical thickness increases. Also, when the transmission through the network is very small, numerical difficulties are encountered [38]. The average pore size for porous silicon can be controlled over three orders of magnitude from order of 1–100 nm.

Based on the experimental results and to demonstrate the size effect, three pore sizes \( \langle d \rangle = 1, 3 \) and \( 5 \, \text{nm} \) are used.

Fig. 14 shows the predicted effective thermal conductivity, defined in Eq. (26), for the unit-cell model using the Fourier and Boltzmann treatments. Since the effective phonon mean-free path \( \lambda_{\text{p}} \) is limited by the pore boundary scattering, the reduction of effective thermal conductivity is apparent as the pore size \( \langle d \rangle \) is decreased. Abrupt changes near \( \varepsilon = 0.2 \) is found. This occurs as the ratio of pore size to the unit cell size reaches a threshold value where a significant amount of the phonon energy is reflected at the pore boundary.

Note that the pore boundary scattering is dominant here, because of the long bulk mean-free path \( \lambda_{\text{p},0} \) compared to the unit cell size \( \langle l \rangle \).

Fig. 15 shows the temperature distributions obtained from Boltzmann treatment (Fig. 15(a)–(e)) with respect to pore size \( d \). The pore size varies between \( d = 1–500 \, \text{nm} \). The asymptotic temperature distribution \( \langle d \rightarrow \infty \rangle \) obtained from the Fourier treatment is also shown (Fig. 15(f)). The results are for \( \varepsilon = 0.16 \). The agreement between the two treatments, for large \( d \), is clear in Fig. 15. From the Boltzmann treatment with a small pore size, a temperature jump is found at the interface. This is similar to that for the one-dimensional, multilayer composite. Due to the total reflection at the pore boundary, some localized hot and cold regions are found below and above the pore.

To account for the size effect, Majumdar [16] suggests a size-affected, mean-free path, given by
Fig. 15. The temperature distributions obtained from the Boltzmann treatment for (a) $d = 1$, (b) 10, (c) 25, (d) 50, and (e) 500 nm. The asymptotic temperature distribution ($d \to \infty$) obtained from the Fourier treatment is also shown (f).
\[ \lambda_p = \frac{\lambda_{p,0}}{1 + \frac{4\lambda_{p,0}}{3} l'} \]  

which may be used to determine \( k_s \) in Eq. (2). For a one-dimensional layer, the characteristic length \( l' \) is the layer thickness. But, for a two-dimensional unit cell, we use an equivalent length defined as \( l'^2 = l^2 - d^2 \). This is an equivalent length after the pore is removed from the unit cell. Using this modified \( k_s \) in the Fourier treatment of unit-cell model, the predicted effective conductivities are shown in Fig. 14. The good agreement with the results of the Boltzmann treatment at the higher porosity is noted, although the asymptotic behavior at low porosity is not in agreement. Also shown in Fig. 13 is the result using the Eq. (29) with \( l = 10 \text{ nm} \). This gives \( \lambda_p = 0.158\lambda_{p,0} \), which gives a solid conductivity \( k_s \approx 23.5 \text{ W/m K} \). This characteristic length represents experimental samples A, B and a in Fig. 17.

5.3. Lateral effective conductivity

Due to the anisotropic pore morphology, the effective conductivity is not isotropic and a preferential conduction occurs along the \( y \) direction, shown in Fig. 1. The predicted variation of the lateral effective conductivity \( (k)_{xx} \) with respect to porosity is shown in Fig. 16. Here the ratio \( (k)_{xx}/(k)_{yy} \) is also used to emphasize the anisotropy. There are no available experimental data for comparison. The overall trend in the porosity dependency is similar to that of \( (k)_{yy} \). The pore branches completely break the solid-phase connectivity and the resulting serial arrangement of phases leads to a very low thermal conductivity. This is not expected in three-dimensional structures, where the solid-phase continuity results in higher effective conductivity.

6. Comparison with experiments

The available experimental results are too few and not consistent. They also lack sufficient geometrical information for a direct comparison with the predictions. Drost et al. [9] used a dynamic conductivity measurement technique based on the thermal waves traveling in the specimen. However, as mentioned by Benedetto et al. [10], the measurement technique by Drost et al. is rather complicated and cannot be used for different sample types. Also, the undesirable effect of SiC layers, which are needed to cover the porous silicon, in the dynamic conductivity measurement, should be removed. Benedetto et al. [10] measured \( (k)_{yy} \) for porous-silicon layers using a photoacoustic technique and Gesele et al. [11] used the 3Ω method. The measurements are summarized in Table 5. Among the parameters, the porosity, the morphology, and the pore size are used in the prediction. The pore sizes reported by Drost et al., and Gesele et al., are well documented. However, the pore sizes in the experiment of Benedetto et al., are not given and are assumed to be similar to those in experiments of Beale et al. [13].

The experimental results for \( (k)_{yy} \) are shown along with the various predictions in Fig. 17. Samples A, B, II, III, and ‘a’ have similar pore structure as shown in Fig. 4, which is a highly-branched, columnar structure. Sample III shows a large difference between the oxidized and the fresh samples and the high values are

![Graph showing predicted variation of lateral effective thermal conductivity with porosity.](image)
Table 5

The doping and etching parameters, and experimental results for various porous silicon samples

<table>
<thead>
<tr>
<th>Sample</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistivity, $\rho$ ($\Omega \text{ cm}$)</td>
<td>0.01</td>
<td>0.01</td>
<td>10</td>
<td>10</td>
<td>38.0–52.0</td>
<td>1–2</td>
<td>0.010–0.018</td>
<td>0.01</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HF concentration (%)</td>
<td>20</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>–</td>
<td>–</td>
<td>48</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current density (mA/cm²)</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>5</td>
<td>8</td>
<td>6</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Etching time (s)</td>
<td>1800</td>
<td>1800</td>
<td>720</td>
<td>3000</td>
<td>420</td>
<td>900</td>
<td>240</td>
<td>–</td>
<td>–</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Illumination applied</td>
<td>–</td>
<td>–</td>
<td>None</td>
<td>None</td>
<td>None</td>
<td>Halogen</td>
<td>None</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Porosity, $\varepsilon$</td>
<td>0.60</td>
<td>0.50</td>
<td>0.40</td>
<td>0.40</td>
<td>0.45</td>
<td>0.64</td>
<td>0.64</td>
<td>0.71</td>
<td>0.79</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer thickness, $d$ (nm)</td>
<td>23</td>
<td>10</td>
<td>75</td>
<td>175</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>21</td>
<td>31</td>
<td>31</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>Silicon wafer type</td>
<td>p</td>
<td>p</td>
<td>n</td>
<td>n</td>
<td>p</td>
<td>p</td>
<td>p</td>
<td>p</td>
<td>p</td>
<td>p</td>
<td>p</td>
<td></td>
</tr>
<tr>
<td>Morphology</td>
<td>Columnar</td>
<td>Equiaxed</td>
<td>Columnar</td>
<td>Equiaxed</td>
<td>Columnar</td>
<td>Equiaxed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pore size, $d$ (nm)</td>
<td>&gt; 12</td>
<td>~ 3</td>
<td>~ 500/200</td>
<td>~ 3</td>
<td>&gt; 100</td>
<td>9.0 ± 3.0</td>
<td>1.7 ± 0.5</td>
<td>2.0 ± 0.3</td>
<td>2.7 ± 0.3</td>
<td>4.5 ± 0.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\langle k \rangle_{yy}$ (W/m K)</td>
<td>2.5</td>
<td>3.9</td>
<td>31.2</td>
<td>10.4/31.2</td>
<td>1.2/1.3</td>
<td>1.75/1.85</td>
<td>80/2.7</td>
<td>0.79</td>
<td>0.196</td>
<td>0.135</td>
<td>0.058</td>
<td>0.035</td>
</tr>
</tbody>
</table>

a A–D: Benedetto et al. [10]; I–III: Drost et al. [9]; a–e: Gessele et al. [11].

The upper parts are for specimen as prepared and the lower parts are for oxidized specimen.
(ε = 0.177, 0.201 and 0.271) and two different pore sizes ((d) = 1 and 3 nm) are simulated, the significant reduction in the heat flow rate by pore randomness and pore scattering, is apparent.

7. Conclusions

Direct simulation of conduction through anisotropic porous silicon layers is made, using two-dimensional matrix structures along with the Boltzmann transport equation. The observed low conductivity, i.e., lower effective phonon mean-free path, can be explained with the inclusion of the effects of the phonon pore scattering and the pore randomness. The hindering effect of the phonon pore scattering (due to the reflection from the solid-pore interface) is significant. Also, due to the dendritic structure of the pores, the hindering effect of the pore-network randomness is also significant. The predictions are compared with the existing experiments and considering the lack of structural data in the experiments, a good agreement is found.

Acknowledgements

The authors are thankful to Professor Collins for the digital data from his simulations. We would also like to thank Dr. Amir A.M. Oliveira for his assistance with the pore-size distribution for porous silicon. The financial support of the National Science Foundation through grant CST-97-14157, is greatly appreciated. The first author (J.D. Chung) would like to express his appreciation to Korea Science and Engineering Foundation for partial financial support.

References