Math 224-0 Midterm Exam
Summer 2014
Thursday, July 31, 2014

Instructions

• Read each problem carefully.

• Write legibly and make sure your final answers are clearly indicated.

• Show all of your work on these sheets.

• The exam consists of 5 problems.

• You may not use books, notes or calculators.

• You have 1 hour to complete this exam.

• Good luck!!

You may use the following formulas:

\[
\sum_{i=1}^{n} 1 = n
\]

\[
\sum_{i=1}^{n} i = \frac{n(n + 1)}{2}
\]

\[
\sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}
\]

\[
\sum_{i=1}^{n} i^3 = \left[ \frac{n(n + 1)}{2} \right]^2
\]
1. (16 points) Evaluate the indefinite integral

$$\int \frac{8x + 1}{x^2 + x - 2} \, dx$$
2. (20 points) Evaluate the indefinite integral

\[ \int \sqrt{4 - x^2} \, dx \]

You might find useful the trig identities

\[
\begin{align*}
\sin(2\theta) & = 2 \sin \theta \cos \theta \\
\cos^2 \theta & = \frac{1 + \cos(2\theta)}{2} \\
\sin^2 \theta & = \frac{1 - \cos(2\theta)}{2}
\end{align*}
\]
3. (i) (10 points) Determine whether the improper integral

$$\int_{1}^{\infty} \frac{4}{e^x} \, dx$$

is convergent or divergent.

(ii) (10 points) Determine whether the improper integral

$$\int_{1}^{\infty} \frac{4}{e^x + x} \, dx$$

is convergent or divergent. *Hint:* Use the comparison theorem, and compare it to the integral in part (i).
4. Evaluate the following integrals:
   (i) (8 points)
   \[ \int \frac{x}{\sqrt{x + 10}} \, dx \]

   (ii) (6 points)
   \[ \int \cos(3\theta) \sin(\sin(3\theta)) \, d\theta \]
(iii) (10 points) \[
\int_{1}^{e} x^2 \ln x \, dx
\]
5. (20 points) Evaluate the definite integral

\[ \int_{0}^{3} (x^2 - 1) \, dx \]

using the definition of the integral with equal subdivisions and sample points at right endpoints.