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Neutron-Antineutron Oscillations[1]

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Abstract: The quantum mechanics of neutron-antineutron oscillations is presented, with emphasis on oscillations occurring for freely propagating neutrons with and without an ambient magnetic field. If such oscillations were to be seen it would signal that baryon number is not a conserved quantum number in nature. The basic elements of an experimental program aimed at finding evidence for these oscillations are also presented.

Audience: Supplementary material for upper-level undergraduate or beginning-level graduate quantum mechanics course.

1 Neutron transitioning to its antiparticle

The neutron was discovered by James Chadwick in 1932. Since its discovery we have learned much about it. We know its mass $m_n = 939.6\,\text{MeV}$ [1] (in natural units [2]); we know it has no electric charge; we know that it is a spin-1/2 fermion; we know that its magnetic dipole moment is $\mu_n = -6.02 \times 10^{-14}\,\text{MeV/Tesla}$; and, we know that it decays to a proton, electron and antineutrino with mean lifetime of 880.2 ± 1.0 seconds [1]. We also know that the neutron is not a fundamental particle, being a bound state of two down quarks of charge $-1/3$ and one up quark of charge $+2/3$.

One thing we still do not know is if the neutron can transition to its own antiparticle, $n \to \bar{n}$. This is an active field of physics research with intense theoretical and experimental interest [4], including recent discussions to look for $n{-}\bar{n}$ oscillations at the European Spallation Source (ESS) in Lund, Sweden [5, 6]. Both n and $\bar{n}$ are electrically neutral, and there is no fundamental principle that bars it from making this transition. However, nature could forbid the transition from taking place by charging the neutron under a conserved global quantum number called baryon number B. The proton and neutron are charge $B = +1$ under this new symmetry, while the electron and neutrino have zero baryon charge. Conserving baryon number in a transition still allows neutron decays $n \to pe^-\bar{\nu}$ since the initial state (n) has $B = +1$ and the final state ($pe^-\bar{\nu}$) also has $B = +1$ ($B_p + B_{e^-} + B_{\bar{\nu}} = 1 + 0 + 0 = 1$). However, conservation of baryon number does not allow the $n \to \bar{n}$ transition since the initial state is $B = +1$ and the final state ($\bar{n}$) is baryon number $B = -1$. This is because of the general rule in quantum mechanics that the quantum numbers of an antiparticle ($\bar{n}$) are equal in magnitude but opposite in sign to those of its corresponding particle (n).

Promoting baryon number to a conserved quantum number is a speculation that must be tested. In fact, there are good reasons to believe that baryon number is not rigorously held by nature, but is rather an approximate symmetry, meaning violations of it are small and

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require precision tests to see their effects. One such precision test is the search for protons ($B = +1$) decaying into final states that have no baryon number, such as $\pi^0 e^+ \nu$ ($B = 0$). This is a so-called $\Delta B = 1$ transition. So far there is no experimental evidence for this, although experiments continue their searches. Another precision test, which is the subject of this note, is $n \to \bar{n}$. Since the initial state has $B = +1$ and final state has $B = -1$, neutron-antineutron oscillations like this are called $\Delta B = 2$ transitions. The rates of $\Delta B = 1$ and $\Delta B = 2$ transitions are not necessarily correlated, which means it is important to search for evidences of both independently.

2 Neutron-antineutron oscillations

The transition from $n \to \bar{n}$ is more accurately called oscillations, since once an n turns into an $\bar{n}$ it is able to transition back to n again. The oscillations are governed by solutions to the time-dependent Schrödinger equation subject to an effective Hamiltonion[3] $\mathcal{H}_{\rm eff}$:

$$\mathcal{H}_{\rm eff}|\psi\rangle = i\frac{\partial}{\partial t}|\psi\rangle. \tag{1}$$

If the neutron and antineutron mix then energy eigenstates (or, "mass eigenstates") of $\mathcal{H}_{\rm eff}$ are mixtures of n and $\bar{n}$ which we denote as n_1 and n_2:

$$\begin{pmatrix} |n_1\rangle \\ |n_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |n\rangle \\ |\bar{n}\rangle \end{pmatrix} \tag{2}$$

We will come shortly to how the angle θ is determined for this mixing, but let us first describe the full solution of the Schrödinger equation in terms of the energy eigenstates $|n_i\rangle$ (where $i = 1, 2$). First, by definition of $|n_i\rangle$ the time-independent Schrödinger equation acting on the eigenstates $|n_i\rangle$ (where $i = 1, 2$) is

$$\mathcal{H}_{\rm eff}|n_i\rangle = E_i|n_i\rangle. \tag{3}$$

Thus, the general solution to the time-dependent Schrödinger equation is

$$|\psi\rangle(t) = c_1|n_1\rangle e^{-iE_1 t} + c_2|n_2\rangle e^{-iE_2 t}. \tag{4}$$

In order to determine c_1 and c_2 we need a boundary condition on $|\psi\rangle$. Here we consider the neutrons that arise by way of weak-force decays of radioactive nuclei. Thus, they are born as pure neutrons. Therefore, at $t = 0$ we want $|\psi\rangle(0) = |n\rangle$. Since, from eq. 2

$$|n\rangle = \cos\theta|n_1\rangle - \sin\theta|n_2\rangle \tag{5}$$

this sets $c_1 = \cos\theta$ and $c_2 = -\sin\theta$ in Eq. 4. Then, expanding $|n_1\rangle$ and $|n_2\rangle$ in terms of n and $\bar{n}$ one finds

$$|\psi\rangle(t) = \left(\cos^2\theta e^{-iE_1 t} + \sin^2\theta e^{-iE_2 t}\right)|n\rangle + \cos\theta\sin\theta\left(e^{-iE_1 t} - e^{-iE_2 t}\right)|\bar{n}\rangle. \tag{6}$$

[3] We call it "effective Hamiltonian" to signify we are restricting consideration only to terms that act on neutrons and antineutron wave functions.

We compute the probability that $|\psi\rangle(t)$ is measured to be a $\bar{n}$ by the standard probability computation in quantum mechanics,

$$P[\bar{n}(t)] = |\langle\bar{n}|\psi\rangle(t)|^2 = e^{-\Gamma t}\sin^2(2\theta)\sin^2\left(\frac{\Delta E\, t}{2}\right), \quad \text{where,} \tag{7}$$

$$\Gamma = \mathrm{Im}(E_1 + E_2), \quad \text{and} \quad \Delta E = E_1 - E_2. \tag{8}$$

The first term, $e^{-\Gamma t}$, is associated with the lifetime of the neutron. For $t > 1/\Gamma$ the neutron has a high probability of already having decayed, and thus it cannot be an antineutron or neutron leading to $P[n(t)]$ and $P[\bar{n}(t)] \to 0$. The second term, $\sin^2(2\theta)$, is associated with its ability to transition from n to $\bar{n}$. If the theory has perfectly conserved baryon number then there is no mixing between n and $\bar{n}$, and $\theta = 0$ and the transition probability is zero for all time. We will come to a description of this angle θ later. The final term, $\sin^2(\Delta E t/2)$, shows the time dependence of the oscillation.

Let us now discuss in somewhat more detail the origins of the angle θ and the decay width Γ. In the discussion above we introduced the angle θ as the mixing angle of n and $\bar{n}$ that rotates them to energy eigenstates. This comes about due to $|n\rangle$ and $|\bar{n}\rangle$ not being eigenstates of $\mathcal{H}_{\rm eff}$. We can characterize this as

$$\mathcal{H}_{\rm eff}|n\rangle = \left(m_n - i\frac{\Gamma}{2} + \mathcal{E}_n\right)|n\rangle + \delta|\bar{n}\rangle \tag{9}$$

$$\mathcal{H}_{\rm eff}|\bar{n}\rangle = \left(m_n - i\frac{\Gamma}{2} + \mathcal{E}_{\bar{n}}\right)|\bar{n}\rangle + \delta|n\rangle \tag{10}$$

where m_n is mass of the neutron, Γ is the decay width (i.e., neutron lifetime is $\tau_n = 1/\Gamma$), δ is contribution from $\mathcal{H}_{\rm eff}$ that enables $n \leftrightarrow \bar{n}$ transitions, and $\mathcal{E}_n$ and $\mathcal{E}_{\bar{n}}$ are any other additional contributions to the energy of the n and $\bar{n}$ states respectively. If the neutrons were propagating completely freely in space with no other matter around and no magnetic field, etc., $\mathcal{E}_{n,\bar{n}} = 0$. But since that is never the case in experimental configurations, we must keep this term.

The imaginary part $-i\Gamma/2$ of the operator equations above will look mysterious to readers who are not familiar with decaying states in quantum mechanics. A complete justification of that will not be pursued here. We merely note that the final answer for the probability of a neutron state remaining a neutron must incorporate an exponential decay over time according to the well-known poisson-distributed radioactivity law of $e^{-\Gamma t}$, where $1/\Gamma$ is the average lifetime of the neutron (i.e., $1/\Gamma \simeq 880\, s$). As we will see shortly, these imaginary contributions inserted in the equations above provide exactly this factor, which should be viewed here as post facto justification for their inclusion.

The matrix $\langle\mathcal{H}_{\rm eff}\rangle$ in the $\{n, \bar{n}\}$ basis is

$$\langle\mathcal{H}_{\rm eff}\rangle = \begin{pmatrix} m_n - i\frac{\Gamma}{2} + \mathcal{E}_n & \delta \\ \delta & m_n - i\frac{\Gamma}{2} + \mathcal{E}_{\bar{n}} \end{pmatrix}. \tag{11}$$

The eigenvalues are

$$E_{1,2} = m_n - i\frac{\Gamma}{2} + \frac{\mathcal{E}_n + \mathcal{E}_{\bar{n}}}{2} \pm \frac{1}{2}\sqrt{(\mathcal{E}_n - \mathcal{E}_{\bar{n}})^2 + 4\delta^2} \tag{12}$$

For $m_n \gg |\mathcal{E}_n - \mathcal{E}_{\bar{n}}| \gg \delta$, which will be justified later in the nuclear reactor experimental context, one can make the approximations

$$E_1 \simeq m_n + \mathcal{E}_n - i\frac{\Gamma}{2}, \qquad E_2 \simeq m_n + \mathcal{E}_{\bar{n}} - i\frac{\Gamma}{2}, \tag{13}$$

$$\Delta E = E_1 - E_2 = \mathcal{E}_n - \mathcal{E}_{\bar{n}}, \quad \text{and} \quad \sin 2\theta = \frac{2\delta}{\mathcal{E}_n - \mathcal{E}_{\bar{n}}}. \tag{14}$$

Under these assumptions we can now rewrite the transition probability as

$$P[\bar{n}(t)] = e^{-\Gamma t}\left(\frac{2\delta}{\mathcal{E}_n - \mathcal{E}_{\bar{n}}}\right)^2 \sin^2\left(\frac{(\mathcal{E}_n - \mathcal{E}_{\bar{n}})t}{2}\right). \tag{15}$$

As we have emphasized, $\mathcal{E}_{n,\bar{n}}$ are calculable from the experimental environment (see below), leaving δ as the only unknown matrix element parameter. The value of δ can be computed from a more fundamental theory of $\Delta B = 2$ baryon number violation. Such calculations are beyond the scope of this discussion. We only state that its value needs to very small, $\delta < 10^{-29}$ MeV in order not to be in conflict with experiment[4]. How we measure such a small non-zero δ, if it indeed exists, is the subject of the next section.

3 Measuring neutron oscillations at reactors

One method to measure δ, and therefore obtain evidence for neutrons transition to antineutrons, is to produce many neutrons in a nuclear reactor, guide them to a target some distance away where any neutrons that transitioned to antineutrons would annihilate in a spectacular signal announcing their existence[5]. This is what the ILL reactor experiment in Grenoble did [3].

We will write the equations in somewhat general form, but will give numbers applicable to the ILL experiment [3] in order to gain understanding of typical sizes of various important

[4] The value of $\delta < 10^{-29}$ MeV may appear to be the result of very low-energy phenomena, since $\delta \ll m_n$. However, δ more accurately should be thought of as a ratio of the nucleon scale (e.g., $m_n \sim 10^3$ MeV) to a very high suppression scale where baryon number violation is induced (e.g., $\Lambda_B \simeq 10^{10}$ MeV). Raised to appropriate powers one obtains very low values for δ, such as $\delta = m_n^6/\Lambda_B^5 \simeq 10^{-32}$ MeV.

[5] Another method is to look for transitions of bound-state neutrons in nuclei transitioning to $\bar{n}$, which subsequently annihilates with another neutron in the nucleus. Bounds from this are comparable, and presently even better than the ILL experimental bound [7]. However, it is expected that future experiments involving free neutrons at ESS could do even better [5, 6].

quantities. The key things we need to know to estimate sensitivity to δ are

$$F = \text{Flux of neutrons} \simeq 1.25 \times 10^{11}\,\text{neutrons}/s \tag{16}$$
$$v_{\text{avg}} = \text{average neutron velocity} \simeq 600\,\text{m}/s \tag{17}$$
$$L = \text{distance to annihilation target} \simeq 60\,\text{m} \tag{18}$$
$$B = \text{ambient magnetic field} \simeq 10^{-8}\,\text{T} \tag{19}$$

From the average velocity data, the average time for the neutron to make it to the annihilation target is $t_{\text{avg}} = L/v_{\text{avg}} \simeq 0.1\,s$. This is where the state $|\psi\rangle(t)$ is measured and its wave function collapses to n or $\bar{n}$, at time $= t_{\text{avg}}$ when it interacts with the annihilation target.

We are now also in position to compute $\mathcal{E}_{n,\bar{n}}$ due to the ambient magnetic field. The magnetic moment of the neutron and antineutron is

$$\mu_n = -\mu_{\bar{n}} = -6.02 \times 10^{-14}\,\text{MeV}\,\text{T}^{-1} \tag{20}$$

which gives shifts in the energy for the neutron and antineutron of

$$\mathcal{E}_n = -\mathcal{E}_{\bar{n}} = -\mu_n \cdot B \simeq 6 \times 10^{-22}\,\text{MeV} \tag{21}$$

where the collimated neutrons and antineutrons moments are aligned with the magnetic field. This gives the result that

$$\frac{\mathcal{E}_n - \mathcal{E}_{\bar{n}}}{2} = 6 \times 10^{-22}\,\text{MeV} \left(= \frac{1}{0.66\,s}\right). \tag{22}$$

where the expression in parentheses is the conversion to units of inverse seconds [2]. Since $0.66\,s$ is much larger than $t_{\text{avg}} = 0.1\,s$, we are justified considering the argument of $\sin^2$ function in eq. 15 to be small, and thus can approximate the antineutron probability at the annihilation target to be

$$P[\bar{n}(t_{\text{avg}})] \simeq \delta^2 t_{\text{avg}}^2 = 10^{-18} \left(\frac{10^8\,s}{\tau_{n\bar{n}}}\right)^2 \left(\frac{t_{\text{avg}}}{0.1\,s}\right)^2, \tag{23}$$

where we have made the traditional identification of $\tau_{n\bar{n}} \equiv 1/\delta$. Note, we have also ignored the $e^{-\Gamma t_{\text{avg}}}$ factor in eq. 15 since t_{avg} is much smaller than the neutron lifetime (i.e., $t_{\text{avg}} \ll 1/\Gamma$) which translates to $e^{-\Gamma t_{\text{avg}}} \simeq 1$. If $\tau_{n\bar{n}}$ were about $10^8\,s$, the above equation tells us that we need approximately 10^{18} neutrons produced for one of them to turn into an antineutron when it reaches the annihilation target.

Also, notice that the transition probability dependence on $\mathcal{E}_n - \mathcal{E}_{\bar{n}}$ completely dropped out when expanding eq. 15 to eq. 23. However, this was only because $\mathcal{E}_n - \mathcal{E}_{\bar{n}}$ was very large compared to δ (i.e., $\frac{1}{2}(\mathcal{E}_n - \mathcal{E}_{\bar{n}}) \gg \delta$) and very small compared to the inverse of the time it takes neutrons to reach their annihilation target (i.e., $\frac{1}{2}(\mathcal{E}_n - \mathcal{E}_{\bar{n}}) \ll 1/t_{\text{avg}}$). If either of those two conditions had not held, one would have to retain its non-trivial dependence.

Let us now do an approximate calculation for the required value of $\tau_{n\bar{n}}$ to obtain one $\bar{n}$ on target for arbitrary flux F and running time $T_{\rm run}$. This requires solving for $\tau_{n\bar{n}}$ in the equation $P[\bar{n}(t_{\rm avg})]FT_{\rm run} \simeq 1$. The result is

$$\tau_{n\bar{n}} \simeq (2\times 10^8\, s)\left(\frac{F}{1.25\times 10^{11}\,\text{neutrons/s}}\right)^{1/2}\left(\frac{T_{\rm run}}{1\,\text{yr}}\right)^{1/2}. \qquad (24)$$

Thus, for some flux F and run-time $T_{\rm run}$ the sensitivity to $\tau_{n\bar{n}}$ is approximately given by the above equation. Keep in mind that the ILL values for $t_{\rm avg}$ and magnetic field were used to obtain the coefficient $2\times 10^8\,\text{s}$, which approximately the sensitivity that ILL obtained: $\tau_{n\bar{n}} > 0.86\times 10^8\,\text{s}$ at 90% C.L. [3].

4 Oscillations of freely propagating neutrons

In our derivation above of the sensitivity to neutron-antineutron oscillations, we introduced the "oscillation time" $\tau_{n\bar{n}}$, which was defined to be the inverse of the matrix element $\tau_{n\bar{n}} \equiv 1/\delta$, where $\langle n|\mathcal{H}_{\rm eff}|\bar{n}\rangle = \delta$. A confusion might be that upon inspecting eq. 15 one notes that δ plays no role in the oscillation but rather only in the amplitude of the probability. The oscillation is completely controlled by $\mathcal{E}_n - \mathcal{E}_{\bar{n}}$ which is set by the magnetic field of the experimental environment. So why does one call $\tau_{n\bar{n}}$ the "oscillation time" for neutron-antineutron oscillations?

The answer lies in the analysis of propagating *free* neutrons. In that case there are no environmental contributions to the energy and thus $\mathcal{E}_n = \mathcal{E}_{\bar{n}} = 0$. This requires a new computation of the eigenvalues and eigenvectors of the Hamiltonian, which is now

$$\langle \mathcal{H}_{\rm eff}^{\rm free}\rangle = \begin{pmatrix} m_n - i\frac{\Gamma}{2} & \delta \\ \delta & m_n - i\frac{\Gamma}{2}\end{pmatrix} \qquad (25)$$

The solution to this is maximal mixing, and yields

$$\begin{pmatrix} |n_1\rangle \\ |n_2\rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} |n\rangle \\ |\bar{n}\rangle \end{pmatrix}, \quad \text{with eigenvalues} \quad E_{1,2} = m_n - i\frac{\Gamma}{2} \pm \delta \qquad (26)$$

Carrying out the steps as we did before, one finds that the quantum state $|\psi\rangle(t)$ that starts out as a neutron at $t = 0$ is

$$|\psi\rangle(t) = \left(\frac{e^{-iE_1 t} + e^{-iE_2 t}}{2}\right)|n\rangle + \left(\frac{e^{-iE_1 t} - e^{-iE_2 t}}{2}\right)|\bar{n}\rangle \qquad (27)$$

Computing the probability of this state being $\bar{n}$ at time t yields

$$P[\bar{n}(t)] = |\langle\bar{n}|\psi\rangle(t)|^2 = e^{-\Gamma t}\sin^2\left(\frac{(E_1 - E_2)}{2}t\right) = e^{-\Gamma t}\sin^2(\delta t). \qquad (28)$$

It is here that we see that $\tau_{n\bar{n}} \equiv 1/\delta$ is controlling the oscillation of neutron to antineutron, and why it gets its name "oscillation lifetime." In the case of neutrons in a relatively strong magnet field, the oscillation time was overwhelmed by the magnetic field contributions, and the role of δ in the time-varying oscillations was lost. In free space propagation, on the other hand, δ is dominant in determining the energy difference in eigenstates and therefore dictates the oscillation frequency. Therefore, $\tau_{n\bar{n}} \equiv 1/\delta$ is rightly designated the oscillation lifetime.

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