Energy and Luminosity Scaling of the Sensitivity to Dimension-Six Operators at High-Energy e^+e^- Colliders

James D. Wells
Leinweber Center for Theoretical Physics
University of Michigan, Ann Arbor, MI 48109

Abstract: There exists in the literature careful simulation studies of the sensitivity to dimension-six operators at high-energy e^+e^- colliders for particular values of the energy and integrated luminosity. It is helpful to know how the sensitivities are altered with changing luminosity and changing energy without a complete re-simulation. This note estimates how the sensitivity changes in two cases: one where the dimension-six operator involves no derivatives and the other with two derivatives. In the case with two derivatives, which is for example applicable to anomalous gauge boson couplings, the sensitivity to the scale increases by the square-root of the increased energy.

Introduction

Our purpose here is to be rather generic, and describe the rough scaling improvements that are to be had on the sensitivity to a dimension-six operator by going to higher center-of-mass energy and/or higher luminosity. Let us suppose that we have a dimension-six operator $\mathcal{O}^{(6)}/\Lambda^2$, with scale suppression Λ, that gives rise to a correction to the cross-section as

$$\sigma = \frac{A}{s}\left(1 + c\frac{s^a v^{2-2a}}{\Lambda^2} + \mathcal{O}(1/\Lambda^4)\right), \tag{1}$$

where $a = 0$ (no derivative) or 1 (two derivatives), $v \simeq 246\,\mathrm{GeV}$ is the Higgs boson vacuum expectation value that we use to normalize the operator when $a = 0$, c is a dimensionless constant made up of parameters in the theory, s is the center-of-mass energy squared of the collision, and A is a universal factor that does not depend on s.

Sensitivity scaling for Λ

Let us suppose that a stage 1 of the collider, where $s = s_1$ and $I = I_1$, with I being the integrated luminosity, one finds after careful analysis a sensitivity to Λ that is Λ_1. At the edge of sensitivity we are justified in ignoring the $1/\Lambda^4$ terms in eq. 1 since they will clearly be subdominant to the already small $1/\Lambda^2$ term.

Let us approximate the sensitivity to be determined by the statistical uncertainty of this total cross-section. This is computed by

$$\frac{1}{\sqrt{N_1}} = \frac{c s_1^a v^{2-2a}}{\Lambda_1^2}. \tag{2}$$

This implies that the number of events needed is

$$N_1 = \frac{\Lambda_1^4}{c^2 s_1^{2a} v^{2(2-2a)}}. \tag{3}$$

We also note that the number of events is equal to the total cross-section multiplied by the integrated luminosity

$$N_1 \simeq \frac{AI_1}{s_1} \tag{4}$$

where we have neglected the small correction term for the cross-section.

At stage 2 of the collider, defined by $s = s_2$ and $I = I_2$, we can estimate the sensitivity to Λ as

$$\frac{1}{\sqrt{N_2}} = \frac{c s_2^a v^{2-2a}}{\Lambda_2^2} \quad \longrightarrow \quad N_2 = \frac{\Lambda_2^4}{c s_2^{2a} v^{2(2-2a)}} \tag{5}$$

However, we also have

$$N_2 = \frac{AI_2}{s_2} = \frac{AI_1}{s_1}\left(\frac{I_2/I_1}{s_2/s_1}\right) = N_1 \frac{I_2/I_1}{s_2/s_1} \tag{6}$$

which implies that

$$\frac{N_2}{N_1} = \frac{I_2/I_1}{s_2/s_1} \tag{7}$$

Now, from Eqs. 3 and 5, we can recast the left-side of eq. 7,

$$\frac{\Lambda_2^4}{\Lambda_1^4}\frac{s_1^{2a}}{s_2^{2a}} = \frac{I_2/I_1}{s_2/s_1} \tag{8}$$

which then allows us to find

$$\Lambda_2 = \Lambda_1 \left[\left(\frac{s_2}{s_1}\right)^{2a-1} \frac{I_2}{I_1}\right]^{1/4}. \tag{9}$$

Summary of Λ sensitivity scaling

In summary, the results are that if there are no derivatives ($a = 0$) in the dimension-six operator then the sensitivity to Λ at collider (s_2, I_2) is

$$\Lambda_2 = \Lambda_1 \left[\frac{s_1 I_2}{s_2 I_1}\right]^{1/4} \quad \text{(no derivatives)}, \tag{10}$$

when the sensitivity at collider (s_1, I_1) is Λ_1. This result makes sense in that if $s_1 = s_2$ the increase in integrated luminosity increases the sensitivity (accesses higher Λ_2). Note, the usual $\sqrt{I}$ dependence on sensitivity is at work here, but it applies to Λ^2, and thus our $I^{1/4}$ scaling on Λ sensitivity is correct. We also see that if the luminosity stays the same the sensitivity is worse for higher energy. This also is a correct result since the total cross-section decreases with higher s and there are less events, and so larger statistical uncertainty, and therefore less sensivity.

In the case of two derivatives in the dimension-six operator the sensitivity to Λ at collider (s_2, I_2) is

$$\Lambda_2 = \Lambda_1 \left[\frac{s_2 I_2}{s_1 I_1}\right]^{1/4} \quad \text{(two derivatives)}, \tag{11}$$

when the sensitivity at collider (s_1, I_1) is Λ_1. In this case, the higher energy helps, even if the total luminosity stays the same. This is because the relative size of the correction is increasing with higher energy. The sensitivity on the scale must necessarily improve. The $I^{1/4}$ scaling with luminosity remains, as expected.

Equivalent sensitivity to κ couplings

Sometimes the sensitivities are quoted for a coupling, which generically I will call κ, related to Λ by $\kappa = 1/\Lambda^2$. Now, when the sensitivity to κ increases with higher luminosity, for example, this is equivalent to saying that we are sensitive to a smaller value of κ. Thus, we find that if we can measure κ down to κ_1 for collider stage 1 with (s_1, I_1) then we can measure κ_2 at collider stage (s_2, I_2) to

$$\kappa_2 = \kappa_1 \sqrt{\frac{s_2 I_1}{s_1 I_2}} \quad \text{(no derivatives)}, \tag{12}$$

$$\kappa_2 = \kappa_1 \sqrt{\frac{s_1 I_1}{s_2 I_2}} \quad \text{(two derivatives)}. \tag{13}$$

The square root derives from the fact that $\Lambda = 1/\sqrt{\kappa}$.

Stage	$\sqrt{s}$ [GeV]	I [ab^{-1}]	Re(λ_L)	Re(λ_R)
1	500	0.5	0.59 (0.59)	3.6 (3.6)
2	800	1	0.24 (0.26)	1.8 (1.6)
3	3000	3	0.036 (0.040)	0.36 (0.25)

Table 1: Sensitivities, in units of 10^{-3}, of the $\lambda_{L,R}$ anomalous triple gauge boson vertices defined in sec. 3.1 of [3]. The values in the parenthesis are derived from the scaling argument of eq. 13, normalized to the 500 GeV stage 1 result.

Example: anomalous triple gauge couplings

The results of eq. 11 and eq. 13 are particularly relevant for anomalous gauge boson couplings analyses. As one can see from eq. (2.1) of the original Hagiwara et al. paper [1], there are two kinds of anomalous couplings that are usually considered: those that parametrize deviations away from the couplings of the Standard Model's dimension-four operators, and those that derive from gauge-invariant dimension-six operators with two derivatives. In the first case, the scaling arguments are not obvious as gauge invariance is disrupted by the altered relations between the renormalizable operators, and each case must be studied to see what the energy dependence of the correction is with respect to the overall cross-section.

In the case of gauge-invariant dimension-six operators, the scaling should follow what we described above for the case of two derivatives. Thus, from eqs. 11 and 13 we see that the higher center of mass energy the greater the sensitivity to the scale of the anomalous coupling operators. It is for this reason that it has been recognized for some time that increased energy of e^+e^- collisions increases the sensitivity to the anomalous couplings [2].

Let us test the scaling argument with simulated results on triple gauge boson vertices from sec. 3.1 of [3]. Table 6.7 on page 168 of [3] shows the sensitivity to the couplings λ_L and λ_R at three different collider stages with $\sqrt{s} = 500$, 800 and 3000 GeV for integrated luminosities of $I = 500\,\mathrm{fb}^{-1}$, $1\,\mathrm{ab}^{-1}$ and $3\,\mathrm{ab}^{-1}$ respectively. Table 1 presented here shows these sensitivities quoted from that document. The sensitivity values in parenthesis are those obtained by apply the scaling results from eq. 13, normalized to the 500 GeV stage 1 values. We see that the scaling argument gives a fairly accurate accounting of the impact of going to higher energy and higher luminosity. The λ's in this table are dimensionless as the dimension-six operator is defined with a $1/M_W^2$ factor, but that does not change the scaling behavior of λ to be the same as the κ scaling in eq. 13.

The comparison of the results of [3] and the $\sqrt{sI}$ scaling argument shows that the scaling factor is approximately correct, and gives a result that matches well, certainly within a factor of two. The utility of this scaling argument is that it helps us estimate what the achievable gains are in the sensitivity to these higher-order dimension-six couplings at various high-energy and high-luminosity options. Furthermore, it enable us to answer a common question when contemplating the machine versus physics requirements: what are the relative trade-offs of higher energy versus higher luminosity. In this particular case, the trade-off can be derived from eqs. 11 and 13, and it suggests that sI is the combined quantity that is most important to maximize.

References

[1] K. Hagiwara, R. D. Peccei, D. Zeppenfeld and K. Hikasa, "Probing the Weak Boson Sector in $e^+e^- \to W^+W^-$," Nucl. Phys. B **282**, 253 (1987).

[2] See, for example, T. Barklow *et al.*, "Anomalous gauge boson couplings," eConf C **960625**, STC127 (1996) [hep-ph/9611454].

[3] E. Accomando *et al.* [CLIC Physics Working Group Collaboration], "Physics at the CLIC multi-TeV linear collider," hep-ph/0412251v1.