Homework 1 Physics 452 Winter 2004
Due at beginning of class Wed Jan 14, 2004

1.1 (Arfken 15.8.4) Using partial fraction expansions, show that

$$\begin{aligned} \text{(a)}\quad \mathcal{L}^{-1}\left[\frac{1}{(s+a)(s+b)}\right] &= \frac{e^{-at}-e^{-bt}}{b-a}, \quad a \neq b. \\ \text{(b)}\quad \mathcal{L}^{-1}\left[\frac{s}{(s+a)(s+b)}\right] &= \frac{ae^{-at}-be^{-bt}}{a-b}, \quad a \neq b. \end{aligned}$$

1.2 Find the Laplace Transforms of $\cos^2 at$ and $\sin^2 at$.

1.3 Find the Laplace Transform of $t^{-1/2}$. *Hint:* It is useful to make the change of variable substition $u = \sqrt{t}$ in the Laplace Transform Integral and then utilize the identity

$$\int_0^\infty e^{-x^2} dx = \sqrt{\pi}/2.$$

1.4 Given $f(t)$ of exponential order, show that $F(s) = \mathcal{L}[f(t)] \to 0$ as $s \to \infty$.

1.5 Given $F(s) = \mathcal{L}[f(t)]$ show by induction that

$$\mathcal{L}\left[\frac{d^n f}{dt^n}\right] = s^n F(s) - s^{n-1} f(0) - s^{n-2} \left.\frac{df}{dt}\right|_{t=0} - \ldots - \left.\frac{df^{n-1}}{dt}\right|_{t=0}$$