

Homework Physics 451 Fall 2003  
*Vector Spaces and Tensor Analysis*  
§9. Cartesian Tensors and Pseudotensors

1. (Arfken 2.9.4 & 2.9.13) Show that

$$\epsilon_{ijk}\epsilon_{pqk} = \delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp}$$

and use this identity to prove that

$$(\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D}) = (\vec{A} \cdot \vec{C})(\vec{B} \cdot \vec{D}) - (\vec{A} \cdot \vec{D})(\vec{B} \cdot \vec{C}).$$

2. The moment of inertia of a rigid body has components

$$I_{ij} = \delta_{ij} \int_{\text{body}} r^2 \rho d^3x - \int_{\text{body}} r_i r_j \rho d^3x$$

where  $r_i$  are the components of the position vector  $\vec{r}$ , and  $\rho$  is the density. Show that  $I_{ij}$  is a tensor.

3. Prove the *Cartesian Tensor Erweiterung Theorem*: In Cartesian coordinate systems if  $T_{\dots i}$  is a tensor of rank  $n$ , then  $\partial T_{\dots i} / \partial x_j$  is a tensor of rank  $n + 1$ .