Homework Physics 451 Fall 2003
Vector Spaces and Tensor Analysis
§7. Geometrical Vector Integral Theorems

1. (Arfken 1.11.6) The electric displacement vector $\vec{D}$ satisfies the Maxwell equation $\vec{\nabla} \cdot \vec{D} = \rho$ where ρ is the charge density (per unit volume). At the boundary between two media there is a surface charge density σ (per unit area). Show that a boundary condition for $\vec{D}$ is

$$(\vec{D}_2 - \vec{D}_1) \cdot \hat{n} = \sigma,$$

where $\hat{n}$ is a unit vector normal to the surface and out of medium 1. (See the hint and figure Arfken provides for this problem.)

2. (Arfken 1.13.9) The magnetic induction $\vec{B}$ is related to the magnetic vector potential $\vec{A}$ by $\vec{B} = \vec{\nabla} \times \vec{A}$. By Stokes's theorem

$$\int \vec{B} \cdot d\vec{\sigma} = \oint \vec{A} \cdot d\vec{r}.$$

Show that each side of this equation is invariant under the gauge transformation $\vec{A} \rightarrow \vec{A} + \vec{\nabla}\phi$. Give an example of a fairly complicated function that could be added to A for this gauge transformation.