Solution Guide to Homework 12 Physics 451 Fall 2003
Real and Complex Analysis
§17. Evaluating Definite Integrals

§17-1. Show that for $a > 0$ (real) that

$$\int_0^\infty \frac{\cos ax}{(x^2+1)^2} = \frac{\pi}{4}(1+a)e^{-a}.$$

Function is even so change integration to $-\infty$ to $+\infty$ and divide by two. A helpful technique for this is to rewrite $\cos ax$ as $\operatorname{Re} e^{iax}$. Closing the contour in the upper half plane and applying the residue theorem produces answer.

§17-2 (Arfken 7.2.8) Show that for $a > 1$ (real)

$$\int_0^\pi \frac{d\theta}{(a+\cos\theta)^2} = \frac{\pi a}{(a^2-1)^{3/2}}.$$

Rewrite $\cos\theta = (z+z^{-1})/2$ and expand. Contour is a circle of unit radius centered on the origin, and using residue theorem the answer pops out.

§17-3 (Arfken 7.2.20) Show that for $a > 0$ (real)

$$\int_0^\infty \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{4a^3}.$$

Function is even so change integration to $-\infty$ to $+\infty$ and divide by two. Another key to this problem is closing the contour in the *lower half plane*. Applying the residue theorem gives the answer.

§17-4. (Arfken 7.2.21) Show that

$$\int_{-\infty}^\infty \frac{x^2}{1+x^4}dx = \frac{\pi}{\sqrt{2}}.$$

Four roots associated with permutations of

$$z_{\text{roots}} = \frac{1}{\sqrt{2}}(\pm 1 \pm i)$$

Close contour with a big arc at $R \to \infty$ and apply residue theorem with two of the four root singularities enclosed to get the answer.